A New Embedded Lossless/Quasi-Lossless Image Coder based on the Hadamard Transform (Revised, November 4, 1997)

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This paper is a revised version of the paper with the same title in the proceedings of ICIP97 (which contains a serious error in fig. 1 and in table 1).

Abstract

This paper presents a lossless block-transform coder based on the Hadamard transform that achieves a compression ratio as good as that of the current lossless JPEG-standard and about 10 to 20% lower than the currently best state-of-the-art technique Calic. The main advantage of the proposed technique is that it produces an embedded data stream which may be truncated to achieve lossy compression. Preliminary results obtained on the "lena" image show that the proposed scheme produces an image quality that is comparable to that of lossy JPEG for compression ratios up to 4, and only slightly worse for compression ratios in the range 4 to 10. This means that the proposed coder performs well over a wide range of compression ratios and is therefore a useful alternative for the lossy and lossless JPEG-standards in nearly-lossless coding or when lossless and lossy decoding must be supported simultaneously. Finally, the transform part of the proposed scheme requires fewer computations than lossy JPEG (and only slightly more than lossless JPEG).

1 Introduction

Most lossless compression schemes are based on prediction techniques, or simple variants thereof. Many lossless schemes share the same principle: they compute a linear estimate to predict (final or intermediate) pixel values and round the estimate to the nearest integer to achieve losslessness. Simple examples of such quasi-linear "predictive" techniques include the S+P transform [1], of which many variants have been published under different names, e.g., the "lossless integer wavelet transform" [2] and the modified Haar transform [3]. Interpolation based wavelet techniques [4, 5] are less trivial examples of such quasi-linear predictive techniques. Finally, the current state-of-the-art lossless compression techniques such as Calic [6], Loco-I [7] and Btpc [**?**] are based on more complicated (non-linear) predictive techniques.

In contrast, most lossy image coding schemes (e.g., JPEG [8], wavelet coders, ...) are not based on prediction but rather on general transforms such as the DCT, the wavelet transform, ...; they apply the linear transform to the input data and then quantize the resulting coefficients. In theory, these techniques can be made lossless by sufficiently finely quantizing the coefficients, or by losslessly coding the error image, but unfortunately both of these strategies result in a negligibly small compression ratio (of the order of 1, or even less). Therefore it is not surprising that lossless coding schemes are hardly ever based on non-trivial linear transforms, but rather on predictive techniques or simple variants thereof.

One noteworthy exception (i.e., an example of a lossless coder based on a non-trivial transform) is the recently introduced rounding transform [9], a technique which is able to losslessly recover the input data from the quantized transform coefficients for transforms that satisfy certain criteria. In theory, the class of "rounding" transforms is fairly general, but the results published so far are restricted to some very simple transforms, which are similar in nature to the predictive schemes discussed above.

In section 2 of this paper, we present a block-transform coder based on the Hadamard transform and a sophisticated coefficient quantization/dequantization scheme. The scheme cannot be interpreted in terms of simple prediction and rounding operators. The quantized coefficient data is organized into an embedded data stream which may be decoded completely to achieve lossless compression but which may also be truncated to achieve a much higher compression ratio, at the expense of a lossy reconstruction. In the latter case, the coefficients reconstructed from the truncated data stream approximately equal quantized version of the coefficients of the traditional lossy Hadamard transform [10]. Therefore, in lossy mode the coding properties of the proposed scheme are similar to those of the lossy Hadamard transform.

The experimental results in section 3 of this paper show that in lossless mode the proposed scheme achieves a compression ratio which is as good or even better (depending on the mode of operation) than the current lossless JPEG-standard. The achieved compression ratio is still 10 to 20% lower than the cur-

rently best state-of-the-art technique Calic [6], but the proposed technique has not been optimized to the same degree as Calic and has the advantage of producing an embedded data stream (see above). Predictive techniques such as Calic, Loco-I and Btpc can also operate in lossy mode, but the proposed scheme is based on transform coding and it is known that transform coders produce much better images than lossy predictive techniques at higher compression ratios [10]. In fact, on images of size 512×512, the compressed image quality produced by the proposed scheme is indistinguishable of that of lossy JPEG for compression ratios up to 4, and is only slightly worse for compression ratios in the range 4 to 20. (see sect. 3).

In conclusion, the proposed coder performs well over a wide range of compression ratios and is therefore a useful alternative for the lossy and lossless JPEG-standards when lossless and lossy decoding must be supported simultaneously. Furthermore, the computational complexity of the transform part of the proposed scheme (less than 10 additions per pixel and some bit shift operations) is quite low.

2 Lossless Hadamard coding

2.1 The principle of lossless coding

Let H be the Hadamard matrix [11, p. 209] of order $n = 2^k$. In the following, $\mathbf{x}$ denotes a column vector of dimension n and x_j its j-th component; Z_n is the space of all $\mathbf{x}$ with integer components; finally, A is an $n \times n$ matrix and $A_{i,j}$ one of its components. In this paper we only consider the case $n = 8$, which is most appropriate for image coding; however, all of the derivations in this section can be repeated for any other value of n, with similar results.

In transform coding it is most useful to order the base vectors, i.e., the rows of H in order of increasing sequency:

$$\mathrm{H} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \quad (1)$$

Typical highly correlated input (column) vectors $\mathbf{x}$ are then transformed into coefficient vectors $\mathbf{a} = H\mathbf{x}$ which tend to be arranged in order of decreasing magnitude.

We assume that each of the coefficients a_j is quantized by rounding it to an integer multiple $k_j\Delta_j$ of a quantization step Δ_j. Specifically, we assume that k_j is the largest integer for which $k_j\Delta_j \leq a_j$ (similar results hold for other rounding schemes, but lead to more complicated notations).

It can be shown that the unquantized coefficient vector $\mathbf{a}$ and hence the input vector $\mathbf{x}$ can be unambiguously recovered from the quantized coefficient vector $\mathbf{k}$ if $\Delta_1 = 8$, $\Delta_2 = \Delta_3 = \Delta_4 = 4$, $\Delta_5 = \Delta_6 = \Delta_7 = 2$ and $\Delta_8 = 1$. The following equations may be used for this purpose:

$$\begin{aligned} a_8 &= k_8 \\ a_7 &= 2k_7 + a_8 \pmod 2 \\ a_6 &= 2k_6 + a_7 \pmod 2 \\ a_5 &= 2k_5 + (a_8 + a_7 - a_6) \pmod 2 \\ a_4 &= 4k_4 + (a_8 + a_7 - a_6) \pmod 4 \\ a_3 &= 4k_3 + (a_6 + a_5 - a_4) \pmod 4 \\ a_2 &= 4k_2 + (a_8 + a_5 - a_4) \pmod 4 \\ a_1 &= 8k_1 + (a_8 + a_7 - a_6 + a_5 \\ &\quad -a_4 + a_3 - a_2) \pmod 8 \end{aligned} \quad (2)$$

From an implementational point of view, it is worth noting that the multiplications may be implemented as bit shifts and that the remaining computations involve simple operations on the least significant bits of the coefficients a_l, which could even be implemented using lookup tables, if desired.

2.2 Theoretical performance

A useful performance measure of different quantization schemes on Hadamard transformed data is $\mathcal{P} = \log_2\left(\prod_{j=1}^{n} \Delta_j\right)$. Indeed $\mathcal{P}$ is a (very) rough estimate[1] of the number of bits that are saved by quantizing the coefficients $\mathbf{a}$. For the scheme proposed in this paper $\mathcal{P} = n\log_2(n)/2$, i.e., $\mathcal{P} = 12$ for $n = 8$. On the other hand, with classical quantization/dequantization schemes, the Hadamard transform can be lossless only when $\Delta_i = 1$, $i = 1, \ldots, 8$. Indeed, such classical schemes reconstruct $\mathbf{x}$ by simply inversely transforming scaled versions $k_j\Delta_j$ of the quantized coefficients and rounding the result. In order for such schemes to be lossless, Δ_j must not exceed 1, hence $\mathcal{P} = 0$. Therefore, for $n = 8$ the scheme in this paper theoretically requires about $12/8 = 1.5$ fewer bits per coefficient than classical schemes. However, the actual improvement is usually larger (see sect. 3 for an example).

The quantization/dequantization scheme proposed in this paper has another interesting property: it strips off more of the least significant bits from low order coefficients than it does from high order coefficients. As low order unquantized coefficients tend to have a larger dynamic range than high order coefficients, the scheme tends to equalize the dynamic range of the quantized coefficients of different orders. Typically, this results in quantized coefficients which can all be represented using 8 bits when the original data has a resolution of 8 bits.[2] This is an important advantage because the quantized coefficients are entropy coded in practice and because the complexity of entropy coders tends to grow with the dynamic range of the coefficient data.

[1] The estimate would be exact if the least significant bits of the coefficients a_j were completely random.

[2] In fact, it is easily verified that even in the worst case, k_1 requires no more than 8 bits, while k_2 and k_3 require no more than 9 bits.

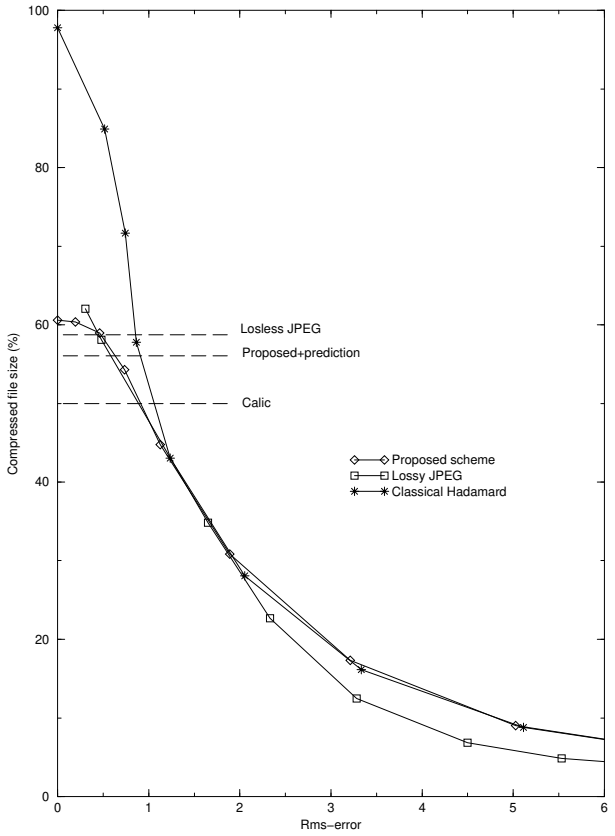

Figure 1: Rate-distortion curves for three compression methods obtained on the "lena" image. The dashed lines indicate the compressed file size for three lossless compression methods.

2.3 The image codec

The currently implemented image coder operates on blocks of 8×8 pixels. The columns of each block are transformed and rounded as explained in section 2.1. The resulting quantized coefficient vectors **k** are grouped into an intermediate coefficient block and the rows of this block are then processed in a similar fashion. This results in a final coefficient block for each input block.

The final coefficient blocks are then encoded using an adaptive arithmetic coder in several passes; in each of these passes one bit of the magnitude (absolute value) of each coefficient is encoded, starting with the most significant bits. The sign bit of a coefficient is encoded along with the first non-zero magnitude bit of this coefficient. When encoding a bit of a coefficient, all of the previously encoded bits of that coefficients are used as a context. The arithmetic coder is inspired by the ideas in [12].

The data stream produced by the encoder is clearly embedded: if it is truncated only information about some least significant bits of the coefficients is lost.

3 Experimental results

The proposed lossless Hadamard transform was applied to three gray scale images (8 bit per pixel), which were subsequently decoded to verify its losslessness. Table 1 shows that the compression ratios for the proposed scheme are very close to those of lossless JPEG (Ljpg) and about 10 to 20% lower than those of Calic, which is the best lossless technique we know of. The table also shows that the compression ratio of the proposed scheme exceeds that of Ljpg, when the Hadamard transform is preceded by an additional prediction step. However, the extra prediction step probably limits the usefulness of the proposed scheme for progressive decoding (as it does in e.g., Calic and Ljpg); therefore, we omit the prediction step in the remainder of this paper.

Fig. 1 displays the rate-distortion curves for the "lena" image[3] of lossy JPEG, the classical lossy Hadamard scheme, and the proposed scheme (note: the same arithmetic coder was used in both Hadamard schemes). For compression ratios in the range 2 to 4, the Hadamard schemes perform as well as lossy JPEG from an root mean squared error (Rmse) point of view. For higher compression ratios, lossless JPEG produces a lower Rmse. However, for compression ratios up to 10, the difference in *subjective* image quality between all of these techniques remains small: at an Rmse of 5, we found that JPEG produced slightly more detailed images than the proposed scheme, but that block distortion was more disturbing in the background regions of the JPEG image. The images are not included as the differences are invisible on paper print outs.

Note that JPEG cannot achieve lossless coding (the RMS-error is non-zero, even when the quality factor is set to its highest possible value). The classical Hadamard scheme can achieve lossless operation, but only at the expense of no compression (compression factor 1.02). The proposed scheme achieves lossless operation using about 3 bits fewer per pixel and also performs better in lossy mode. The savings of 3 bits is almost double the theoretically expected 1.5 bits. This can probably be explained by the fact that the coded coefficients are obtained after *two* instead of one transformations.

In principle, techniques such as Calic, Loco-I and Btpc can also be operated in lossy (or rather near-lossless) mode. Unfortunately, the implementation made available by the authors of Calic, has no such possibility so we were not able to test this mode of operation. However, it is to be expected that the image quality of Calic (being a predictive technique) will drop below that of the proposed scheme at high compression ratios. This is because it is generally agreed that transform coders outperform predictive schemes at high compression ratios [10]. On the other hand, we have found that the Btpc technique clearly outperforms both lossy JPEG and our scheme for compression ratios up to 20 on the "lena" image. This is probably due to its multi-resolution properties which enable it to utilize long-range correlations in images.

[3] There exist different versions of this image; we used the one from ipl.rpi.edu, which is most difficult to code by any method.

Table 1: Lossless Compression ratios for different images

Image	Proposed	Proposed+Pred.	Ljpg	Calic
lena (512x512)	1.66	1.78	1.7	2.0
mr-chest (256x256)	1.52	1.66	1.56	1.80
angio-chest (512x512)	2.28	2.48	2.29	2.60

However, it should be noted that our scheme can also be turned into a multiresolution scheme.

4 Conclusion

The compression scheme presented in this paper clearly demonstrates the feasibility of lossless coding schemes based on non-trivial orthogonal transforms. The proposed Hadamard coder achieves efficient coding in both lossy and lossless operation and may be useful in progressive transmission.

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