Guard time optimization for OFDM transmission over fading channels

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Abstract

In this paper, we consider the transmission of an orthogonal frequency division multiplexed (OFDM) signal over a channel with time-varying frequency-selective fading and additive white Gaussian noise (AWGN). Due to channel dispersion, intersymbol and intercarrier interference will occur at the receiver. To reduce this interference, a guard time is added as a cyclic prefix. The system performance is defined as the signal-to-noise ratio (SNR) at the receiver, after demodulation. We investigate the system performance for various delay spreads and coherence times, and optimize this performance as function of the guard time.

1. Introduction

Wireless communications is beginning to exercise a great influence on our daily lives. The enormous increase of wireless and portable telephones is leading to a number of initiatives for new systems and services. In the future, we will be witness of a widespread deployment of a diversity of wireless services : wireless LAN's and wireless PBX (private branch exchange) systems for indoor areas, offices and commercial buildings, PCS (personal communication services) for in and around buildings in residential and office areas, cellular telephone services in all outdoor areas, ... [1]

Because of the increasing demand of wireless communications, there is need of a modulation technique that can transmit reliably high data rates at a high bandwidth efficiency. An excellent candidate is orthogonal frequency division multiplexing (OFDM) [2],[3]. This technique has been developed in the 60's but only recently the implementation became technical feasible and cost competitive. Under the name discrete multitone (DMT), it is used as basis for a worldwide ADSL standard [4]-[7]. OFDM is an effective method to combat interference in signaling over multipath radio channels [8].

OFDM partitions a given bandwidth in a set of orthogonal subchannels. These subchannels can be modulated and demodulated using discrete Fourier transforms (DFT). In practice, the DFT's are implemented by efficient fast Fourier transform (FFT) techniques. Interference, caused by the time-dispersive channel is reduced by adding a guard time that consists of a cyclic prefix. Inserting this guard time reduces interference but at the same time, it decreases transmission efficiency.

In this paper, the influence of the guard time on the system performance is investigated. We derive an optimal guard time that minimizes the performance degradation.

2. An OFDM transceiver

Orthogonal frequency division multiplexing (OFDM) is a modulation technique that results in a high bandwidth efficiency and a relatively low complexity. OFDM partitions the given bandwidth in equidistant orthogonal subchannels, which can be modulated and demodulated easily using discrete Fourier transforms (DFT).

In Fig. 1, an OFDM transceiver is depicted. A set of N subsymbols $a_{i,n}$, where the index n $\in$ [0,N-1] refers to the n^{th} subchannel and the index i to the i^{th} transmitted symbol, is modulated using an inverse DFT. To reduce the interference, caused by the time-dispersive channel, a guard time interval ν is added as a cyclic prefix. The transmitted samples $g_i(m)$, corresponding to the i^{th} symbol interval, are given by :

$$g_i(m) = \sqrt{\frac{E_s}{N+\nu}} \sum_{n=0}^{N-1} a_{i,n} e^{j2\pi \frac{nm}{N}} \quad m = -\nu, \ldots, N-1 \qquad \textbf{(1)}$$

The subsymbols $a_{i,n}$ are statistically independent and have a unit average energy :

$$E\left[a_{i,n} a^{*}_{j,m}\right] = \delta_{i,j}\delta_{n,m} \qquad \textbf{(2)}$$

At the receiver, the cyclic prefix is removed before demodulation by a DFT. The DFT is followed by a frequency domain equalizer, not shown in Fig. 1. In practice, the DFT's are implemented by fast Fourier transforms (FFT).

3. Channel description

In wireless communications, signal propagation is blocked by objects and the signal power is carried by a large amount of paths with different strengths and delays. Describing this propagation in a deterministic way is too complex. Therefore, we use a statistical analysis. The resulting received signal, due to the summation of all reflected signals, can be approximated using the central limit theorem by a complex Gaussian random variable. The amplitude of this complex Gaussian variable has a Ricean distribution or, if the mean value is zero (no direct path), a Rayleigh distribution. The characteristics of the transmission media are constantly changing and involve randomly time-varying impulse responses $h(k;\ell)$:

$$x_{out}(k) = \sum_{\ell=-\infty}^{+\infty} x_{in}(k-\ell)h(k;\ell) \qquad \textbf{(3)}$$

where $x_{in}(k)$ are the input samples and $x_{out}(k)$ are the output samples. The impulse response is described by its autocorrelation function :

$$R_{hh}(k_1,k_2;\ell_1,\ell_2) = E\left[h(k_1;\ell_1)h^{*}(k_2;\ell_2)\right] \qquad \textbf{(4)}$$

The wide-sense stationary uncorrelated scattering (WSSUS) model [1] makes physical assumptions that are valid for most radio transmission channels :

- the signal variations on paths arriving at different delays are uncorrelated
- the correlation properties of the channel are stationary

With this WSSUS model, the autocorrelation function becomes :

$$R_{hh}(k_1,k_2;\ell_1,\ell_2) = R(k_1;\ell_1-\ell_2)\delta(k_1-k_2) \quad \textbf{(5)}$$

In what follows, we assume that the transmitted samples are only disturbed by WSSUS multipath time-varying frequency-selective Rayleigh fading and additive white Gaussian noise (AWGN).

4. Performance

At the input of the receiver, we obtain the samples z(ℓ), given by :

$$z(\ell) = \sqrt{\frac{E_s}{N+\nu}} \sum_{i=-\infty}^{+\infty} \sum_{n=0}^{N-1} \sum_{m=-\nu}^{N-1} \left(a_{i,n} e^{j2\pi \frac{nm}{N}} h(\ell - m - i(N+\nu);\ell) \right) + w(\ell) \quad \textbf{(6)}$$

where w(ℓ) is white Gaussian noise with independent real and imaginary parts, each having a variance of $N_0/2$. Assuming that the impulse response h(k;ℓ) has an average energy equal to 1, we get :

$$\sum_{k=-\infty}^{+\infty} R(k;0) = 1 \quad \textbf{(7)}$$

At the output of the FFT we find :

$$W_k^{(j)} = \sqrt{\frac{E_s}{N(N+\nu)}} \sum_{i=-\infty}^{+\infty} \sum_{n=0}^{N-1} a_{i,n}\gamma_{i,k,n} + w_k \quad \textbf{(8)}$$

where

$$\gamma_{i,k,n} = \sum_{m=-\nu}^{N-1} \sum_{\ell=0}^{N-1} \left(e^{-j2\pi \frac{k\ell - nm}{N}} h((\ell - m) - i(N+\nu);\ell) \right)$$

The average total power P(k) at the output k of the FFT is given by :

$$\begin{aligned} P(k) &= E\left[\left|W_k^{(j)}\right|^2\right] \\ &= \frac{E_s}{N(N+\nu)} E\left[\left|\gamma_{0,k,k}\right|^2\right] && \text{(a)} \\ &+ \frac{E_s}{N(N+\nu)} \sum_{\substack{n=0 \\ n\neq k}}^{N-1} E\left[\left|\gamma_{0,k,n}\right|^2\right] && \text{(b)} \\ &+ \frac{E_s}{N(N+\nu)} \sum_{\substack{i=-\infty \\ i\neq 0}}^{+\infty} \sum_{n=0}^{N-1} E\left[\left|\gamma_{i,k,n}\right|^2\right] && \text{(c)} \\ &+ N_0 && \text{(d)} \end{aligned} \quad \textbf{(10)}$$

The average total power consists of 4 contributions. The first component (a) is the useful power. Besides the useful power, we find intercarrier interference (ICI) (b) and intersymbol interference (ISI) (c). The last contribution (d) is white Gaussian noise.

Assuming that the impulse response h(k;ℓ) has a limited duration :

$$h(k;\ell) = 0 \qquad k < 0 \;\text{ and }\; k \geq N+\nu \quad \textbf{(11)}$$

the summation over i in the ISI contribution (c) reduces to i=-1, which means that only the previous symbol interferes during the present symbol interval.

We define $P_U(k)$ and $P_I(k)$ as the normalized useful power and the total interference power, respectively :

$$P_U(k) = \frac{1}{N^2} E\left[\left|\gamma_{0,k,k}\right|^2\right]$$

$$P_I(k) = \frac{1}{N^2} \sum_{\substack{n=0 \\ n\neq k}}^{N-1} E\left[\left|\gamma_{0,k,n}\right|^2\right] + \frac{1}{N^2} \sum_{n=0}^{N-1} E\left[\left|\gamma_{-1,k,n}\right|^2\right] \quad \textbf{(12)}$$

The total power P(k) becomes thus :

$$P(k) = E_s \frac{N}{N+\nu} P_U(k) + E_s \frac{N}{N+\nu} P_I(k) + N_0 \quad \textbf{(13)}$$

When all carriers are modulated and using assumption (7), we can find that $P_U(k)$ and $P_I(k)$ are independent of the index k, and that $P_U(k)+P_I(k)=1$. In the following, we drop the index k. We obtain the signal-to-noise ratio (SNR) at the output of the FFT :

$$SNR = \frac{E_s}{N_0} \frac{N}{N+\nu} \frac{P_U}{1+\frac{E_s}{N_0}\frac{N}{N+\nu}P_I} \quad \textbf{(14)}$$

We define the degradation of the SNR as

$$Deg = \left(\frac{N}{N+\nu} \frac{P_U}{1+\frac{E_s}{N_0}\frac{N}{N+\nu}P_I} \right)^{-1} \quad \textbf{(15)}$$

Note that for $E_s/N_0 \to \infty$, the SNR is limited by P_U/P_I.

5. Computation results

For calculations, we used an exponential multipath intensity profile and a Gaussian time-correlation profile. This yields the following autocorrelation function :

$$R(k;\Delta\ell) = \begin{cases} Ce^{-\frac{k}{y_0}} e^{-\frac{1}{2}\frac{\Delta\ell^2}{\sigma^2}} & 0 \le k < N+\nu \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(16)}$$

where C is a constant of normalization (see (7)). The exponential multipath profile is extinct after about $6y_0$, which defines the delay spread. The variance σ is proportional to the coherence time.

In Fig. 2, the interference power P_I is shown for several values of y_0 and σ as a function of the guard time ν. Note that for increasing guard time, P_I decreases and reaches an asymptotic value for large ν, which depends only on σ. The interference power P_I depends only on y_0 at small guard times. As the sum of P_U and P_I is constant, similar conclusions can be drawn for the useful power.

The degradation (15) is plotted for various E_s/N_0 in Fig. 3. For small E_s/N_0, the degradation behaves like $(P_U N/(N+\nu))^{-1}$. The useful power P_U decreases for decreasing ν at small guard times, so degradation increases. For large guard times, P_U reaches its asymptotic value, so that the degradation increases like $(N/(N+\nu))^{-1}$ for growing ν. A minimum degradation is achieved for a guard time approximately equal to the delay spread.

For large E_s/N_0, the degradation approximates $(E_s/N_0)(P_I/P_U)$. At small guard times, P_I/P_U increases for decreasing ν, so the degradation increases. For large guard times, P_I/P_U achieves its asymptotic value so the degradation also reaches a lower limit. This lower limit is obtained from a guard time approximately equal to the delay spread. As the interference power P_I (and the useful power P_U) have a similar behavior for constant y_0 or constant σ, it is obvious that for large E_s/N_0 values, the degradation will show that behavior as well (Fig. 4) : for large guard times, the asymptotic value is determined by σ, and for small guard times, the degradation for $\nu \rightarrow 0$ is determined by y_0. For large E_s/N_0, i.e. $E_s/N_0 >> (P_I N/(N+\nu))^{-1}$, the SNR (14) approaches P_U/P_I, which indicates that the performance is limited by the interference. Hence, increasing E_s/N_0 far beyond $(P_I N/(N+\nu))^{-1}$ yields only a marginal performance improvement.

6. Conclusions

We have investigated the performance of the transmission of an OFDM signal over a channel with WSSUS time-varying frequency-selective fading and additive white Gaussian noise. We have determined the guard time that minimizes the SNR degradation. This optimal guard time is calculated for several delay spreads and coherence times and approximates the delay spread.

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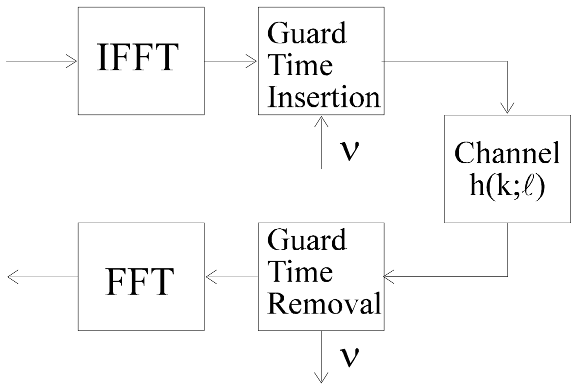

Fig. 1 : An OFDM transceiver

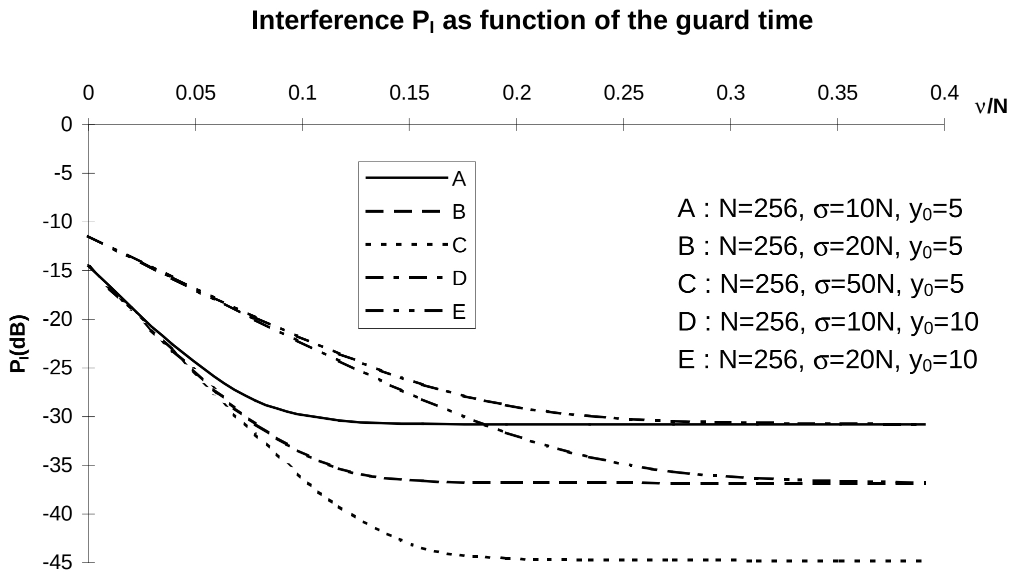

Fig. 2 : Interference power P_I

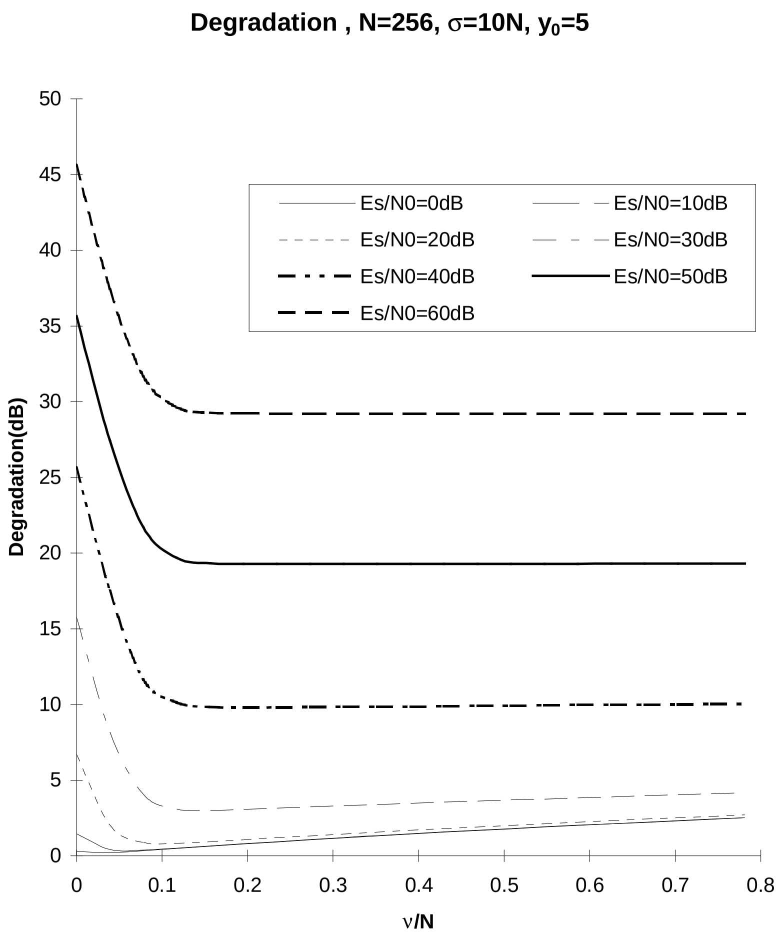

Fig. 3 : Degradation

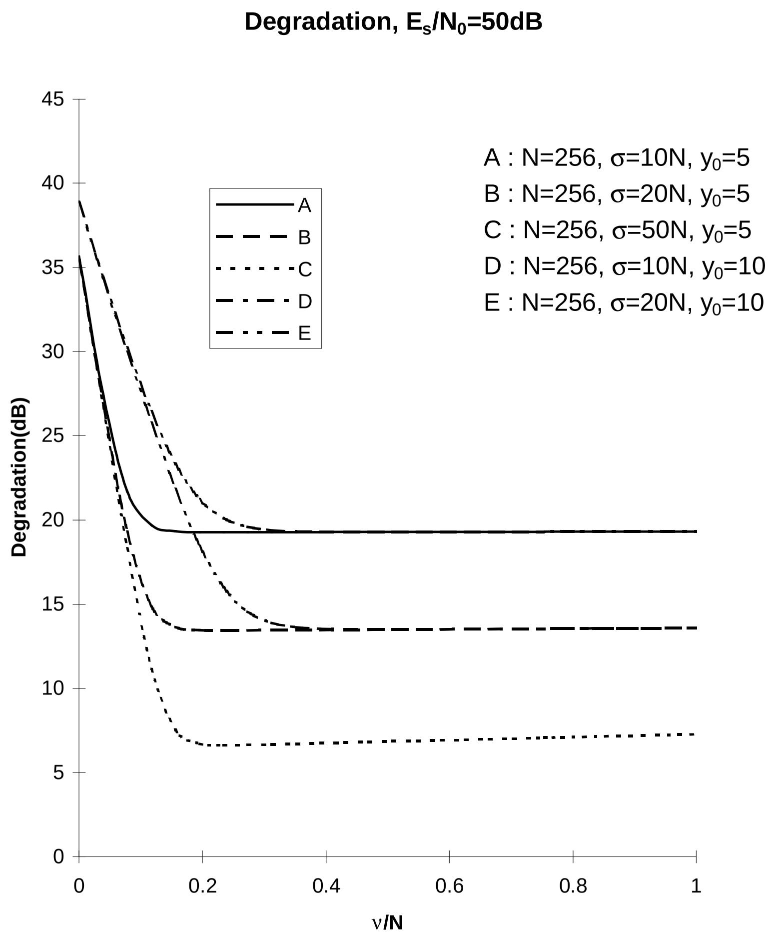

Fig. 4 : Degradation