SENSITIVITY OF OFDM/CDMA TO CARRIER PHASE JITTER

Heidi Steendam, Marc Moeneclaey

Communications Engineering Lab.
University of Ghent
B-9000 GENT, BELGIUM

ABSTRACT

In this paper, we investigate the influence of carrier phase jitter on the performance of an OFDM/CDMA system. This carrier phase jitter reduces the performance of the system, defined as the signal-to-noise ratio at the input of the decision device. When all users have the same power level and phase jitter spectrum, it is shown that for the highest load, the degradation only depends on the jitter variance but not on the specific shape of the jitter spectrum.

INTRODUCTION

Orthogonal frequency-division multiplexing code-division multiple-access (OFDM/CDMA) has recently received a considerable attention owing to its possibility to achieve a high capacity per unit bandwidth, which provides us an excellent candidate for high data rate applications [1]-[6].

OFDM/CDMA consists of a combination of two well known and studied modulation techniques. In CDMA, the different users are modulated by codes. All users can transmit at the same time and, as each user makes use of the entire frequency band, a high capacity of the channel is reached. A drawback of CDMA is that the performance rapidly decreases when the number of users increases.

Orthogonal frequency-division multiplexing (OFDM) multiplexes the incoming streams on orthogonal carriers. These carriers are spectrally overlapping, in contrast to FDMA which uses non-overlapping channels. Moreover, OFDM is less complex to realize, as it can be implemented using a fast Fourier transform algorithm (FFT).

It is known that multi-carrier systems are more sensitive to carrier phase jitter than single carrier systems [7]. This carrier phase jitter is generated by the phase locked loop (PLL) which converts the received IF signal to the base-band signal. In the OFDM/CDMA system, the carrier phase jitter occurring in the PLL systems gives rise to the multi-user interference (MUI).

In this paper, we derived the degradation of the OFDM/CDMA system caused by carrier phase jitter. An expression for the degradation is presented in terms of the phase jitter spectra and the power levels.

SYSTEM DESCRIPTION

The conceptual block diagram of the OFDM/CDMA transceiver is shown in Fig. 1. A data symbol $a_{n,m}$ with unit energy and at symbol rate 1/T, transmitted by the n^{th} user during the m^{th} symbol interval, is multiplied with a CDMA chip sequence $\{c_{n,\ell}\}$ ℓ=0,…,N-1, N denoting the number of chips. Sequences belonging to different users are assumed to be orthogonal. The resulting samples, at a rate N/T, are modulated on N equidistant orthogonal carriers using an inverse discrete Fourier transform. The resulting time domain samples are fed to p(t), a unit energy square root Nyquist filter with respect to the interval T/N. The complex envelope s(t) of the transmitted signal is disturbed by additive noise and carrier phase jitter. This carrier phase jitter is the phase error between the carrier used for up-converting the baseband OFDM/CDMA signal at the transmitter and the phase-locked carrier used for down-converting at the receiver. All OFDM carriers exhibit an identical carrier phase jitter as they are up-converted by the same oscillator. The phase error $\phi_n(t)$ of user n is modeled as a stationary zero mean process having a bandwidth much smaller than N/T. The additive noise n(t) has a power spectrum $S_n(f)$. The complex envelope of the received signal r(t) is given by :

$$r(t)=\sum_{m}\sum_{n=0}^{N-1}\frac{\sqrt{E_{s,n}}}{N}a_{n,m}\sum_{k,\ell=0}^{N-1}c_{n,\ell}e^{j2\pi\frac{k\ell}{N}}p\left(t-(k+mN)\frac{T}{N}\right)e^{j\phi_n(t)}+n(t) \quad (1)$$

The receiver consists of a filter $p^*(-t)$ matched to the transmit pulse whose output is sampled at the chip rate at the instants {kT/N}. In order to detect the symbol $a_{n,m}$, the matched filter output samples are fed to the discrete Fourier transform and the resulting frequency domain samples are correlated with the chip sequence corresponding to the n^{th} user.

SENSITIVITY TO CARRIER PHASE JITTER

In this section, we compute the degradation (in dB) of the signal-to-noise ratio (SNR) at the input of the decision device when carrier phase jitter is present. We consider the detection of the symbol $a_{n,0}$.

As the spectrum of the phase jitter is much smaller than N/T, the variation of the phase jitter over the impulse response duration of the matched filter, which is in the order of T/N, can be neglected. The input of the decision device is fed with the sample $\hat{a}_{n,0}$:

$$\begin{aligned}\hat{a}_{n,0} &= \sqrt{E_{s,n}}\, a_{n,0}\, \underline{c}_n^T E[B_n]\underline{c}_n^* + \sqrt{E_{s,n}}\, a_{n,0}\, \underline{c}_n^T \left(B_n - E[B_n]\right)\underline{c}_n^* \\ &\quad + \sum_{\substack{\ell=0\\ \ell\neq n}}^{N-1} \sqrt{E_{s,\ell}}\, a_{\ell,0}\, \underline{c}_\ell^T E[B_n]\underline{c}_n^* + W_{n,0}\end{aligned} \tag{2}$$

The vector $\underline{c}_n$ contains the chip sequence of user n, where $|c_{n,i}|=1$, i=0,…,N-1. The elements of the matrix B_n are given by :

$$\left(B_n\right)_{k,\ell} = \frac{1}{N^2}\sum_{i=0}^{N-1} e^{j2\pi\frac{i(k-\ell)}{N}}\, e^{j\phi_n\left(\frac{iT}{N}\right)} \tag{3}$$

and the additive noise samples $W_{n,0}$ are given by :

$$W_{n,0} = \frac{1}{N}\sum_{k,\ell=0}^{N-1} c_{n,\ell}^*\, e^{-j2\pi\frac{k\ell}{N}} \int_{-\infty}^{+\infty} n(t)\, p^*\left(t-\frac{kT}{N}\right)dt \tag{4}$$

The sample at the input of the decision device can be decomposed into four uncorrelated contributions. The first term consists of the mean value, with respect to the carrier phase jitter, of the useful component. The second contribution contains a zero-mean disturbance caused by the fluctuation of the useful component. The third contribution is zero-mean multi-user interference caused by the phase jitter that provokes a loss of orthogonality between the different users. The fourth term is caused by the additive noise.

For small phase jitter $\phi_n(t)$, we can use the approximation $\exp(j\phi_n(t)) \cong 1+j\phi_n(t)$, which reduces the matrix elements $(B_n)_{k,\ell}$ for $k\neq\ell$ to :

$$\left(B_n\right)_{k,\ell} \cong \frac{1}{N^2}\sum_{i=0}^{N-1} j\phi_n\left(\frac{iT}{N}\right) e^{j2\pi\frac{i(k-\ell)}{N}} \tag{5}$$

Similarly, for $k=\ell$

$$\left(B_n\right)_{k,k} \cong \frac{1}{N} + \frac{1}{N^2}\sum_{i=0}^{N-1} j\phi_n\left(\frac{iT}{N}\right) \tag{6}$$

from which we can derive

$$E\left[\left(B_n\right)_{k,k}\right] \cong \frac{1}{N} \tag{7}$$

We define the signal-to-noise ratio (SNR) at the input of the decision device as the ratio of the power of the average useful component to the power of the remaining contributions. In the presence of phase jitter, the SNR is reduced as compared to the case of no jitter $E_{s,n}/E[|W_{n,0}|^2]$. This degradation, expressed in dB, is given by :

$$D_n = 10\log\left(1 + \frac{E_{s,n}}{E\left[\left|W_{n,0}\right|^2\right]} E\left[\left|\underline{c}_n^T\left(B_n - E\left[B_n\right]\right)\underline{c}_n^*\right|^2\right] + \sum_{\substack{\ell=0 \\ \ell\neq n}}^{N-1} \frac{E_{s,\ell}}{E\left[\left|W_{n,0}\right|^2\right]} E\left[\left|\underline{c}_\ell^T\left(B_n - E\left[B_n\right]\right)\underline{c}_n^*\right|^2\right]\right) \tag{8}$$

where the power of the fluctuation of the useful component yields :

$$E_{s,n} E\left[\left|\underline{c}_n^T\left(B_n - E\left[B_n\right]\right)\underline{c}_n^*\right|^2\right] = E_{s,n} \int_{-\infty}^{+\infty} S_{\phi,n}(f)\left|H_0(f)\right|^2 df \tag{9}$$

and the power of the multi-user interference :

$$\sum_{\substack{\ell=0 \\ \ell\neq n}}^{N-1} E_{s,\ell} E\left[\left|\underline{c}_\ell^T\left(B_n - E\left[B_n\right]\right)\underline{c}_n^*\right|^2\right] = \frac{1}{N-1} \sum_{\substack{\ell=0 \\ \ell\neq n}}^{N-1} E_{s,\ell} \int_{-\infty}^{+\infty} S_{\phi,\ell}(f)\left(1 - \left|H_0(f)\right|^2\right) df \tag{10}$$

$S_{\phi,\ell}(f)$ denoting the phase jitter spectrum of user ℓ. The filter $|H_0(f)|^2$ is a low-pass filter :

$$\left|H_0(f)\right|^2 = \left|\frac{1}{N} \frac{\sin(\pi f T)}{\sin(\pi f T/N)}\right|^2 \tag{11}$$

This implies that the fluctuation of the useful component and the multi-user interference mainly contain the low frequency components (<1/T) and the high frequency components (>1/T) of ϕ, respectively. The additive noise yields the power $E[|W_{n,0}|^2]$:

$$E\left[\left|W_{n,0}\right|^2\right] = \int_{-\infty}^{+\infty} S_n(f)\left|P(f)\right|^2 df \tag{12}$$

where P(f) is the Fourier transform of p(t).

In the following we consider the degradation of the SNR under the assumption that all N users have the same jitter spectrum $S_\phi(f)$ and the same energy per symbol E_s. Under these assumptions, the power of the fluctuation of the useful component (9) yields :

$$E\left[\left|\underline{c}_n^T\left(B_n - E\left[B_n\right]\right)\underline{c}_n^*\right|^2\right] = E_s \int_{-\infty}^{+\infty} S_\phi(f)\left|H_0(f)\right|^2 df \tag{13}$$

The power of the multi-user interference (10) is given by :

$$\sum_{\substack{\ell=0 \\ \ell\neq n}}^{N-1} E_{s,\ell} E\left[\left|\underline{c}_\ell^T\left(B_n - E\left[B_n\right]\right)\underline{c}_n^*\right|^2\right] = \frac{M}{N-1} E_s \int_{-\infty}^{+\infty} S_\phi(f)\left(1 - \left|H_0(f)\right|^2\right) df \tag{14}$$

where M denotes the number of disturbing users.

Note that for the number of disturbing users equal to N-1, the sum of the powers of the fluctuation of the useful component and multi-user interference only depends on the jitter variance but not on the specific shape of the jitter spectrum : for N-1 disturbing users, the degradation is given by :

$$D_n = 10\log\left(1+\frac{E_s}{E\left[\left|W_{n,0}\right|^2\right]}\sigma_\phi^2\right) \tag{15}$$

where the jitter variance σ_ϕ^2 is defined by :

$$\sigma_\phi^2 = \int_{-\infty}^{+\infty} S_\phi(f)df \tag{16}$$

COMPUTATIONAL RESULTS

In the simulations, we made use of the orthogonal Walsh-Hadamard chip sequences. We subjected the system to phase jitter $\phi(t)$, having a power spectral density $S_\phi(f)$ shown in Fig. 2. The psd is dimensioned in such a way that the jitter variance equals σ_ϕ^2. Fig. 3 shows the degradation as function of the normalized number of disturbing users M/(N-1) for $f_L=0$, $f_H=B$, N=32, $\sigma_\phi^2=10^{-4}$ and the $SNR_{\phi=0}$, which corresponds to $E_s/E[|W_{n,0}|^2]$, equals 25dB. Enlarging the jitter bandwidth B reduces the power of the fluctuation of the useful component and increases at the same time the power of the multi-user interference, which engenders a stronger dependency on the number of disturbing users. For N-1 disturbing users, the degradation becomes only dependent on the jitter variance and not on the shape of the jitter spectrum.

The scatter diagrams for OFDM/CDMA-QPSK, shown in Figs. 4 and 5, are made in absence of additive noise, for $f_L=0$, $f_H=2.56/T$ and $f_L=2.56/T$, $f_H=5.12/T$ respectively. In both cases, the jitter variance was taken σ_ϕ^2=-21dB and the number of carriers N=128. Figs. 4 and 5 demonstrate that the spectral distribution of the jitter determines the outlook of the scatter diagram. This can be explained as follows : the fluctuation of the useful component, which is mainly determined by the low frequency components (<1/T) of the phase jitter, gives rise to an essentially angular displacement of the sample at the input device, while the radial displacement is negligible. As easily can be proved, the term of the multi-user interference has uncorrelated real and imaginary part, which implies that this contribution shows a circular distribution. In Fig. 4, the jitter spectrum mainly contains low-frequency components, which signifies that the phase jitter essentially provokes a fluctuation of the useful component, whereas in Fig. 5, no low-frequency components are present, so that the multi-user interference becomes the major contribution. This considerations explain easily the nature of the scatter diagrams.

CONCLUSIONS

We have investigated the influence of carrier phase jitter on the performance of an OFDM/CDMA system. The results can be summarized as follows :

- The degradation, caused by the phase jitter, depends on the number of disturbing users. For a given jitter variance, the degradation is for phase jitter with low spectral contents less dependent of the number of disturbing users than for phase jitter without low frequency components.
- For N-1 disturbing users, the degradation only depends on the jitter variance, not of the spectral contents of the jitter.
- The jitter components at low frequencies (<1/T) cause essentially an angular displacement of the sample at the input of the decision device, while the jitter components at higher frequencies give rise to an angular as well as a radial displacement.

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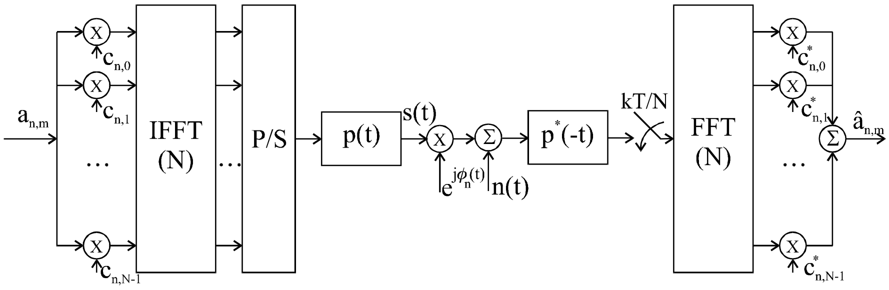

Fig. 1 : Conceptual block diagram of an OFDM/CDMA transceiver

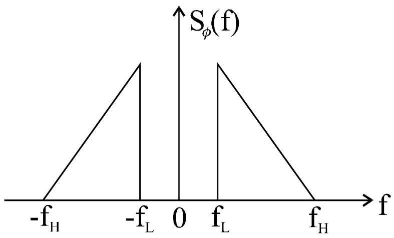

Fig. 2 : Phase jitter spectrum

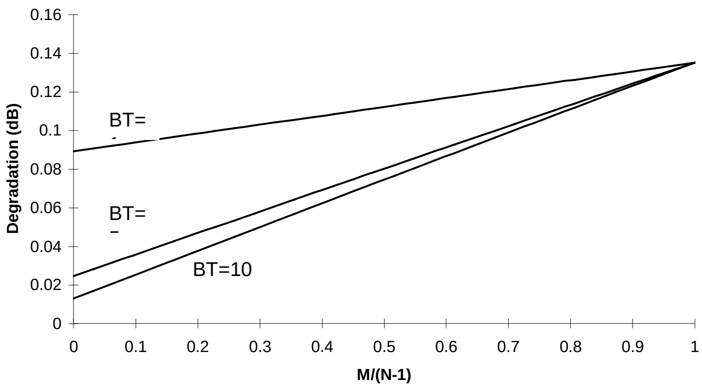

Fig. 3 : Degradation as function of M : σ_ϕ^2=10-4, $SNR_{\phi=0}$=25dB

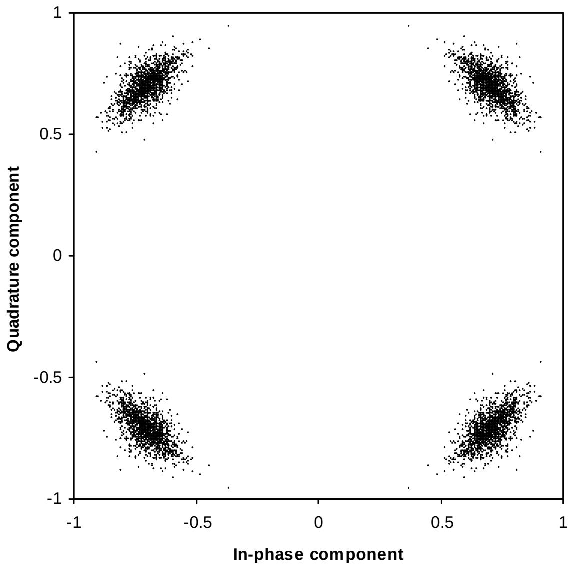

Fig. 4 : Scatter diagram for f_L=0, f_H=2.56/T, N-1 disturbing users

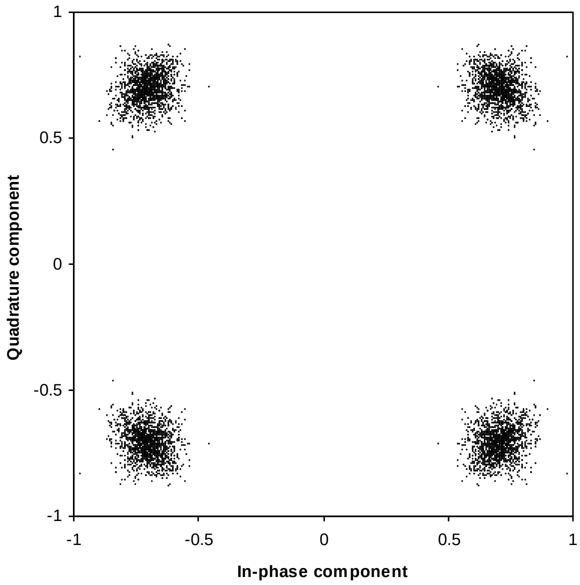

Fig. 5 : Scatter diagram for f_L=2.56/T, f_H=5.12/T, N-1 disturbing users