Optimization of OFDM on Frequency-Selective Time-Selective Fading Channels

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ABSTRACT

In mobile radio communication, the fading channels generally exhibit both time-selectivity and frequencyselectivity. Orthogonal frequency division multiplexing has been proposed to combat the frequency-selectivity, but its performance is also affected by the timeselectivity. In this paper, we investigate how various parameters, such as the number of carriers, the guard time length and the sampling offset between receiver and transmitter, affect the system performance. Further, we determine the optimum values of the above parameters, which minimize the degradation of the signal-tonoise ratio at the input of the decision device.

1. INTRODUCTION

Due to the enormous growth of wireless services (cellular telephones, wireless LAN's,...) during the last decade, the need of a modulation technique that can transmit reliably high data rates at a high bandwidth efficiency arises [1]-[3]. In a mobile radio channel, the signal is disturbed by multipath fading which generally exhibits both time-selectivity and frequency-selectivity. The signal power is carried by a large number of paths with different strengths and delays. For GSM, typical multipath intensity profiles are defined for rural areas (RA), urban areas (TU) and hilly terrain areas (HT) [4]-[5].

The influence of the intersymbol interference caused by the frequency-selectivity can be reduced by increasing the duration of a transmitted symbol. This can be accomplished by using orthogonal frequency division multiplexing (OFDM) : the symbol sequence to be transmitted is split into a large number of lower speed symbol streams, which each modulate a different carrier; the carrier spacing is selected such that modulated carriers are orthogonal over a symbol interval. In addition, a guard interval (cyclic prefix) is inserted in order to combat the frequency-selectivity of the channel [6]-[7]. The transmitter and receiver for OFDM can be implemented efficiently by using Fast Fourier Transform (FFT) techniques. OFDM has been proposed and/or accepted for various applications, such as broadcasting of digital audio (DAB) and digital television (DTTB) [8], mobile radio [1]-[2],[9], and transmission over twisted pair cables (ADSL) [10]-[11].

Increasing the duration of a transmitted symbol however, makes the system more sensitive to the timeselectivity of the channel. As the time-selectivity affects the orthogonality of the carriers, a larger symbol duration gives rise to intercarrier interference (ICI). The lengthening of the symbol duration, introduced to combat the frequency-selectivity, therefore is limited by the time-selectivity. In this contribution we analyse the effect of the number of carriers, the guard time duration and the sampling offset between receiver and transmitter, on the performance of the OFDM system. The OFDM system is described in section II. In section III we consider the degradation of the signal-to-noise ratio at the input of the decision device as performance measure of the OFDM system. Section IV focuses on two limiting cases, i.e. the time-flat channel (for a small number of carriers) and the frequency-flat channel (for a large number of carriers), and analyses the resulting interference powers. The problem of frame synchronisation is considered in section V. Numerical results, including performance optimizations, are presented in section VI. Finally, conclusions are drawn in section VII.

2. SYSTEM DESCRIPTION

In OFDM, the available bandwidth is partitioned into N subchannels that are made orthogonal by using carriers with a spacing equal to the subchannel symbol rate. A binary message is coded and mapped to a sequence of complex data symbols, which are split into frames of N symbols: a_{i,n} denotes the n-th symbol of the i-th frame $(0 \le n \le N-1, -\infty < i < +\infty)$. The n-th carrier is modulated by the symbols $\{a_{i,n}| \rightarrow i < +\infty\}$, and the modulated carriers are summed before transmission. In a practical implementation, the N samples of the transmitted signal corresponding to the i-th frame are generated by feeding $\{a_{in}|n=0,...,N-1\}$ to an inverse discrete Fourier transform (IDFT) (see Figure 1). The loss of orthogonality between the carriers, caused by the dispersive channel, introduces intercarrier and intersymbol interference (ISI and ICI) at the receiver. To combat this interference, each frame is preceded by a guard interval of v samples containing a cyclic extension of the transmitted time domain samples (cyclic prefix). The i-th transmitted frame (including the prefix) contains N+v time domain samples, of which the m-th sample is given by :

$$g_i(m) = \sqrt{\frac{E_s}{N+\nu}} \sum_{n=0}^{N-1} a_{i,n} e^{j2\pi \frac{nm}{N}} m = -\nu, \dots, N-1$$
(1)



Figure 1 : An OFDM transceiver

Assuming the data symbols are statistically independent and have a unit average energy, i.e. $E[a_{i,n}a^*_{j,k}]=\delta_{i,j}\delta_{n,k}$, the transmitted average energy per symbol equals E_s .

Many wireless communication channels can be modeled as multipath Rayleigh fading channels, having an impulse response $h(k;\ell)$, represented by a tapped delay line where the k-th coefficient is a Gaussian random process with time variable ℓ . Bello [4] introduced the wide-sense stationary uncorrelated scattering (WSSUS) model to easily describe fading channels. This model, which is valid for most radio channels, assumes that the signal variations arriving at different delays are uncorrelated and that the correlation properties of the channel are stationary. The autocorrelation function, considering these assumptions, yields :

$$E[h(k_1, \ell_1)h^*(k_2, \ell_2)] = \delta(k_1 - k_2)R(k_1; \ell_1 - \ell_2)$$
(2)

The channel, having an autocorrelation function $R(k;\ell)$, can be characterized by a *multipath intensity profile* R(k;0) and a *Doppler spectrum* $S_D(e^{j2\pi fT})$, with

$$S_D\left(e^{j2\pi jT}\right) = \sum_{\ell=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} R(k;\ell) e^{-j2\pi j\ell T}$$
(3)

Without loss of generality, we assume that the largest value of the multipath intensity profile occurs at k=0, i.e. $R(0;0)\ge R(k;0)$. In addition, the received signal is corrupted by complex-valued additive white Gaussian noise (AWGN) with a power spectral density N₀.

For each transmitted frame of N+v samples, the receiver selects N consecutive samples to be processed further and drops the other v samples (guard time removal). The indices of the N remaining samples corresponding to the j-th frame are $\{k-k_0+j(N+v)|k=0,...,$ N-1}. The sampling offset k_0 is assumed to be provided by a frame synchronization algorithm, which selects k_0 such that the signal-to-noise ratio at the input of the decision device is maximum. The remaining samples r(k) of the j-th frame, given by

$$r(k) = \sum_{i=-\infty}^{+\infty} \sum_{m=-\nu}^{N-1} g_i(m) h(k-m-i(N+\nu);k) + n(k)$$

$$k = -k_0 + j(N+\nu), \dots, -k_0 + j(N+\nu) + N - 1$$
(4)

are demodulated using a discrete Fourier transform DFT. Each of the N outputs of the DFT is scaled and rotated (single-tap equalization per DFT output) and applied to the decision device.

3. SYSTEM PERFORMANCE

In the following, we concentrate on the detection of the data symbols during the frame j=0. Due to the loss of orthogonality caused by the fading channel, the outputs of the discrete Fourier transform are disturbed by interference, which adds to the channel noise. The power P(n) at the n-th output of the DFT can be decomposed as

$$P(n) = E_{s} \frac{N}{N+\nu} (P_{U} + P_{ICI} + P_{ISI}) + N_{0}$$
(5)

The useful power P_U denotes the contribution from the symbol $a_{0,n}$. The intercarrier interference (ICI) power

 P_{ICI} contains the contributions from the other symbols transmitted in the considered frame (i=0), whereas the intersymbol interference (ISI) power P_{ISI} contains the contributions from all symbols transmitted in other frames (i \neq 0). Finally, N₀ denotes the contribution from the additive noise. Assuming all N carriers are modulated, one obtains

$$P_{U} = E\left[\left|\gamma_{n,n,0}(k_{0})\right|^{2}\right]$$

$$P_{ICI} = \sum_{\substack{\ell=0\\\ell\neq n}}^{N-1} E\left[\left|\gamma_{\ell,n,0}(k_{0})\right|^{2}\right]$$

$$P_{ISI} = \sum_{\substack{i=-\infty\\i\neq 0}}^{+\infty} \sum_{\ell=0}^{N-1} E\left[\left|\gamma_{\ell,n,i}(k_{0})\right|^{2}\right]$$
(6)

where $\gamma_{\ell,n,i}(k_0)$, given by

$$\gamma_{\ell,n,i}(k_0) = \frac{I}{N} \sum_{m=-\nu}^{N-1} \sum_{k=0}^{N-1} e^{-j2\pi \frac{k\ell-nm}{N}} h(k-k_0-m-i(N+\nu);k-k_0)$$
(7)

denotes the signal component at the n-th DFT output during the frame j=0, caused by the symbol $a_{\ell,i}$ which is transmitted on the ℓ -th carrier during the i-th frame.

The signal-to-noise ratio (SNR) at the output of the DFT is defined as the ratio of the power of the useful component to the power of the remaining contributions :

$$SNR = \frac{E_s \frac{N}{N+\nu} P_U}{E_s \frac{N}{N+\nu} (P_{ICI} + P_{ISI}) + N_0}$$
(8)

In the presence of the fading channel, the SNR is reduced as compared to the case of an AWGN channel. The AWGN channel yields $P_U=1$, $P_{ICI}=P_{ISI}=0$, so that for v=0 the SNR equals E_s/N_0 . It can be verified that the sum $P_U+P_{ICI}+P_{ISI}$ of the useful power and the interference powers is independent of the considered carrier; assuming the impulse response $h(k;\ell)$ has a unit average

energy (i.e.
$$\sum_{k=-\infty}^{+\infty} R(k;0) = 1$$
), we obtain $P_U + P_{ICI} + P_{ISI} = 1$.

Under these considerations, the degradation of the SNR expressed in dB is given by :



Figure 2 : The weight function w(k;0)

For large E_s/N_0 , the SNR (8) is limited by $P_U/(1-P_U)$ which indicates that the performance is limited by the interference. Hence, increasing E_s/N_0 far beyond (($(1-P_U)N/(N+v)$))⁻¹ yields only a marginal performance improvement.

In a further analysis of the powers in (6), taking into account the above-mentioned considerations, it can be verified that:

$$P_U = \frac{1}{N} \sum_{k=-\infty}^{+\infty} \sum_{\ell=-\infty}^{+\infty} w(k;\ell) R(k-k_0;\ell)$$
(10a)

$$P_{ICI} = \sum_{k=-\infty}^{+\infty} w(k;0) R(k-k_0;0) - P_U$$
(10b)

$$P_{ISI} = \sum_{k=-\infty}^{+\infty} (1 - w(k;0)) R(k - k_0;0)$$
(10c)

where $w(k; \ell)$ is a two-dimensional weight function

$$w(q;r) = \frac{1}{N} \begin{cases} N - |r| & 0 \le q \le v \\ 0 \le |r| \le N \\ N - q + v - |r| & v \le q \le N + v \\ N - q + v - |r| & 0 \le |r| \le N - q + v \\ N + q - |r| & 0 \le |r| \le N + q \\ 0 & elsewhere \end{cases}$$
(11)

and w(k;0) is shown in Figure 2. According to (10), the intersymbol interference power P_{ISI} is independent of the time correlation properties of the channel; taking into account the weight function w(k;0) from Figure 2, it follows that P_{ISI} is determined only by the tails of the multipath intensity profile of the fading channel. The intercarrier interference P_{ICI} consists of two contributions: the first contribution is independent of the time correlation properties of the fading channel and is mainly determined by the central part (the 'body') of the multi-path intensity profile; the second contribution (-P_U) depends on both the dispersive and time-correlation characteristics of the fading channel.



Figure 3 : The OFDM signal

Let us consider quantitatively the effect of the system parameters (i.e. the guard time length ν and the number of carriers N) on the performance degradation (9)

• For given N, increasing v reduces the amount of channel distortion on the N samples that are kept by the receiver for further processing (see Figure 3). Hence, increasing v reduces P_{ICI} and P_{ISI} , so that P_U moves closer to 1. On the other hand, increasing v reduces the power efficiency through the factor N/(N+ ν), because the receiver keeps only N of the N+ ν received samples. Note that for an AWGN channel with ν >0 the degradation (9) becomes $-10\log(N/(N+\nu))$, which reflects the power efficiency loss caused by the guard interval.

• For given v, the dispersive channel introduces a given amount of linear distortion, which is basically confined to a few samples at the edges of the block of N samples that are processed by the receiver. Increasing N reduces the relative importance of these distorted samples and in addition, increases the power efficiency. On the other hand, increasing N makes the system more sensitive to the time-selectivity of the channel, because the transmitted frames get longer. The time-selectivity affects the orthogonality of the transmitted frames, and therefore introduces ICI at the DFT output.

From the above considerations, it follows that an optimum set (v,N) exists, which minimizes the degradation (9)

4. LIMITING CASES : TIME-FLAT CHANNEL AND FREQUENCY-FLAT CHANNEL

When the duration of a transmitted frame is small as compared to the coherence time of the fading channel, the variation in time of the channel during a frame can be neglected : the channel can be approximated by a time-flat channel with $R(k;\ell)=R(k;0)$. For a time-flat channel, the general expression (10a) of the power P_U simplifies to :

$$P_{U} = \sum_{k=-\infty}^{+\infty} w^{2}(k;0) R(k-k_{0};0)$$
(12)

where we have taken into account that $\frac{1}{N} \sum_{\ell=-\infty}^{+\infty} w(k;\ell) = w^2(k;0)$. Taking into account that $P_{ISI}+P_{ICI}=1-P_U$, it follows from (11) that

$$P_{ISI} + P_{ICI} = \sum_{k=-\infty}^{+\infty} (1 - w^2(k;0)) R(k - k_0;0)$$
(13)

Considering the nature of w(k;0), (12) and (13) indicate that the useful power is mainly determined by the 'body' of the autocorrelation function, while the total interference power only involves the head and tail of R(k;0) (see Figure 2).

Assuming the tails of the multipath intensity profile are much shorter that N samples, (13) is well approximated by

$$P_{ISI} + P_{ICI} \cong \frac{2}{N} \sum_{k=0}^{+\infty} k \left(R \left(-k_0 - k; 0 \right) + R \left(\nu - k_0 + k; 0 \right) \right) (14)$$

which indicates that the total interference is proportional to 1/N.

When N increases for a given v and a given channel autocorrelation function, the number of samples affected by the channel dispersion is small as compared to the total number (N) of samples that are processed per frame : the effect of the channel dispersion becomes negligible. When the frame duration becomes comparable to the coherence time of the channel, the effect of the time variations of the channel becomes noticeable: the total interference (ISI+ICI) increases with N. The ISI power (10c) is independent of the coherence time of the channel and is proportional to 1/N when N is much larger that the duration of the tails of the multipath intensity profile (see (14)). Hence the total interference is mainly ICI when the time variations of the channel become dominant.

When the frame duration is large as compared to the coherence time and the delay spread, it is shown in Appendix A that the fading channel can be approximated by a frequency-flat channel with $R(k;\ell) = \tilde{R}(\ell)\delta(k)$, where

$$\widetilde{R}(\ell) = \sum_{k=-\infty}^{+\infty} R(k;\ell)$$
(15)

Hence, the frequency-flat model corresponds to a single coefficient $\tilde{h}(0; \ell)$ with the same Doppler spectrum as the actual channel. The resulting useful power P_U is given by

$$P_{U} = \frac{1}{N} \sum_{\ell=-N}^{+N} \left(1 - \frac{|\ell|}{N} \right) \widetilde{R}(\ell)$$
(16)

5. FRAME SYNCHRONIZATION

The receiver makes a detection of the symbols $\{a_{0,n}|n=0,...,N-1\}$ by processing the samples $\{r(m-k_0)|m=0,...,N-1\}$. The sampling offset k_0 determines how much these samples are affected by the channel dispersion, and should be selected such that the degradation (9) is minimal. As $P_U+P_{ISI}+P_{ICI}=1$, minimizing the degradation (9) is equivalent to minimizing the interference $P_{ISI}+P_{ICI}$ or maximizing the useful power P_U .

As the impact of the channel dispersion increases when the frame gets shorter, we will determine the optimum sampling offset k_0 under the assumption that the frame duration is much less that the coherence time of the channel; hence the time-flat channel model applies. The resulting k_0 might be no longer optimum when the frame length is in the order of the coherence time; however in this case the value of k_0 is less critical, because the effect of channel dispersion is less important than the effect of the time variation of the channel.

When the time-flat model applies, the optimum value k_0 minimizes the interference power given by (13). In most cases of practical interest, N is much larger than the tails of the multipath intensity profile, so that minimizing (13) is essentially equivalent to minimizing (14). Minimization of (14) yields an optimum value of k_0 that does not depend on N. Noting that the maximum of the intensity profile R(k;0) occurs at k=0, the optimum value of k_0 is easily determined in the following cases :

• For causal channels (i.e. R(k;0)=0 for k<0), the optimum value of k_0 is $k_0=0$

• For anti-causal channels (i.e. R(k;0)=0 for k>0), the optimum value of k_0 is $k_0=v$

• For symmetric channels (i.e. R(k;0)=R(-k;0)), the optimum value of k_0 is $k_0=v/2$

In other cases, the optimum value of k_0 has to be obtained by numerically minimizing (13) or (14).

6. NUMERICAL RESULTS

In the computations, a 5MHz channel bandwidth and a 1GHz carrier frequency have been assumed. The Doppler spreading for a typical outdoor radio channel can be calculated straightforwardly from the expression $f_D = (v/c)f_C$, v representing the velocity of the mobile (135 km/hr), c the velocity of light and f_C the center frequency of the mobile radio channel (1 GHz). The resulting coherence time T₀, according to the rule of thumb $T_0=0.5/f_D$ [12]-[13], equals 4 ms. In the literature [4]-[5], typical channel impulse responses are defined. For a typical urban (TU) area, a delay spread of 5 µs is taken. Considering the 5MHz channel bandwidth, it follows that the duration of a sample is 0.2 µs. The proposed autocorrelation function exhibits an exponentially decaying multipath intensity profile and a Gaussian time correlation profile:

$$R(k; \ell) = C \exp(-\frac{k}{y_0}) \exp(-\frac{\ell^2}{2\sigma_0^2}) \quad k \ge 0, -\infty < \ell < +\infty (17)$$

where C is a constant of normalization. Defining the delay spread as the time at which the multipath intensity profile does not falls 20 dB below the level of the strongest component, the parameter y_0 is found to be about 5 samples. The coherence time T_0 is fixed to the duration of twice the spreading of the Gaussian time correlation profile, yielding σ_0 10000 samples.

In Figures 4 and 5 we compare the total interference power $P_{ISI}+P_{ICI}$ for the following cases :

(a) the frequency-selective time-selective channel with autocorrelation function $R(k;\ell)$ from (17)

(b) the limiting case of the time-flat channel with autocorrelation function R(k;0)

(c) the limiting case of the frequency-flat channel with autocorrelation function $\tilde{R}(\ell)\delta(k)$, with $\tilde{R}(\ell)$ given by (15)

(d) the sum of the total interference powers resulting from (b) and (c)

Figure 4 shows the total interference power for v=40 as function of N, whereas Figure 5 shows the total interference power for N=256 as function of v.

We observe from Figure 4 that the total interference power for the limiting cases of the time-flat channel (b) and the frequency-flat channel (c) converge to the total interference power of the frequency-selective timeselective channel (a), for small N and large N, respectively. The total interference power for the time-flat channel is proportional to 1/N; this agrees with the result (14). Note that the total interference power for the frequency-flat channel already approaches the total interference power for the frequency-selective timeselective channel, for values of the frame duration that are considerably less that the coherence time of the channel; the resulting total interference power is proportional to N². Finally, we have added the total interference power resulting from the time-flat channel (b) and the frequency-flat channel (c). The resulting sum (d), which can be computed much more efficiently than the exact result (a), turns out to be an accurate approximation of the total interference power for the actual frequency-selective time-selective channel (a).

Figure 5 shows that, for the time-flat channel, the total interference power decreases with increasing v; this is because the effect of channel dispersion is reduced by increasing the guard interval. This decrease with v is exponential, because of the exponentially decaying multipath intensity profile. For the frequency-flat channel, the total interference power does not depend on v : the interference is caused solely by the time-variations of the channel, which cannot be counter-acted by a guard interval. Again, the sum of the total interference powers resulting from the time-flat and frequency-flat limits of the channel is a very good approximation of the total interference power for the frequency-selective time-selective channel.

Figure 6 shows the effect of the sampling offset k_0 on the degradation of the SNR, for $E_s/N_0=20$ dB, N=256 and various values of v. In Figure 6, we observe that the minimum degradation occurs at $k_0=0$. Note that the optimum values of k_0 , found in Figure 6, agree with the optimum values for a time-flat channel, determined in section V.



Figure 4 : Interference power as function of N (v=40)



Figure 5 : Interference power as function of v (N=256)

Figure 7a displays the optimum values N_{opt} and v_{opt} as a function of E_s/N_0 , for $y_0=5$ and $\sigma_0=10000$. We observe that for increasing E_s/N_0 , N_{opt} and v_{opt} are decreasing and increasing, respectively. This behavior can be explained as follows. For very large E_s/N_0 , the degradation (9) converges to $-10\log(P_U/(1-P_U))+$ $10\log(E_s/N_0)$, in which case the minimization of the degradation is equivalent to the maximization of P_U ; let us denote by $(N_{opt}(\infty), v_{opt}(\infty))$ the corresponding optimum system parameters. For very small E_s/N_0 , the degradation (9) converges to $-10\log(P_UN/(N+v))$, in which case the minimization of the degradation is equivalent to the maximization of $P_UN/(N+v)$; let us denote by $(N_{opt}(0), v_{opt}(0))$ the corresponding system parameters. As N/(N+v) is decreasing with N and decreasing with v, it follows that $N_{opt}(\infty) < N_{opt}(0)$ and $v_{opt}(\infty) > v_{opt}(0)$. The range of E_s/N_0 displayed in Figure 7a is an "intermediate" range, in which (N_{opt}, v_{opt}) has reached neither its limit $(N_{opt}(0), v_{opt}(0))$ for low E_s/N_0 nor its limit $(N_{opt}(\infty), v_{opt}(\infty))$ for high E_s/N_0 .

Figure 7b shows the optimum system parameters (N_{opt}, v_{opt}) as function of y_0 (which is proportional to the delay spread), for $E_s/N_0=30dB$ and $\sigma_0=10000$. When y_0 increases, v_{opt} varies in proportion to y_0 ; N_{opt} also increases with y_0 , in order to compensate for the power efficiency reduction caused by the increase of v_{opt} . However, as increasing N_{opt} enhances the interference caused by the time variations of the channel, the increase of N_{opt} is not linear with y_0 .

Figure 7c shows the optimum system parameters (N_{opt}, v_{opt}) as a function of σ_0 (which is proportional to the coherence time), for $E_s/N_0=30dB$ and $y_0=5$. The optimum guard time duration v_{opt} does not depend on the coherence time, because the guard interval has no impact on the interference caused by the time-selectivity. The optimum value N_{opt} is a compromise between the following phenomena:

(a) Increasing N reduces both the power efficiency loss and the interference caused by the frequency-selectivity(b) Decreasing N reduces the interference caused by the time-selectivity

Hence, N_{opt} increases with σ_0 .



Figure 6 : Degradation as function of k_0 : asymmetric profile

7. CONCLUSIONS

In this paper, we have first investigated the effect of the number of carriers N and the guard time duration v on the performance of an OFDM system operating on a frequency-selective time-selective fading channel. Our main conclusions are the following.

• For short frames, the time-selectivity of the channel can be ignored. The frequency-selectivity of the channel yields equal portions of ISI and ICI. The total interference power decreases with ν , and is proportional to 1/N. • For long frames, the frequency-selectivity of the channel can be ignored. The time-selectivity of the channel yields ICI but no ISI. The ICI power does not depend on ν , and increases with N.

• The total interference power for a channel with both frequency-selectivity and time-selectivity is well ap-

proximated by adding the total interference powers that result from the time-flat limit and the frequency-flat limit of the considered channel. The computation of this approximated total interference power is much faster that the computation of the correct total interference power.

Further, we have determined the optimum values of the timing offset (k_0) , the number of carriers (N) and the guard time duration (v), that minimize the degradation of the SNR, caused by ISI and ICI.

• The optimum timing offset is determined mainly by the guard time duration v and the multipath intensity profile R(k;0). Assuming R(0;0) \geq R(k;0), the optimum timing offsets are k₀=0 for a causal profile and k₀=v/2 for a symmetric profile.

 \bullet The optimum number of carriers increases with the delay spread and the coherence time, but decreases with $E_{\rm s}/N_0.$

• The optimum guard time duration increases with the delay spread and with E_s/N_0 , but is independent of the coherence time.

APPENDIX A

When the frame length is large as compared to the coherence time and the delay spread, the weight function $w(k;\ell)$ (11) can be approximated by :

$$w(k;\ell) = w(k;0) - \frac{|\ell|}{N} \qquad \forall k,\ell \qquad (A1)$$

which reduces the useful power $P_U(9)$ to :

$$P_{U} = \frac{1}{N} \sum_{\ell=-\infty}^{+\infty} \left(1 - \frac{\ell}{N} \right) \widetilde{R}(\ell) - \frac{1}{N} \sum_{\ell=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} (1 - w(k;0)) R(k - k_{0}; \ell)$$
(A2)

defining $\widetilde{R}(\ell)$ as :

$$\widetilde{R}(\ell) = \sum_{m=-\infty}^{+\infty} R(m;\ell)$$
(A3)

For large N, the first contribution of (A2) behaves inversely proportional to N, while the second contribution behaves inversely proportional to N². In addition, if the guard interval is of the order of the delay spread, the second contribution is negligible as compared to the first contribution. The useful power therefore can be obtained using expression (9) where the autocorrelation function is substituted by (A3), which corresponds to the autocorrelation function of a frequency-flat fading channel. In a similar way, it can be found that the intercarrier interference power and the intersymbol interference power for large N converge to values that correspond to a channel autocorrelation function $\tilde{R}(\ell)\delta(k)$.

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Figure 7 : Optimal system parameters