MC-CDMA Performance in the Presence of Timing Errors

H. Steendam and M. Moeneclaey

Department TELIN, University of Gent, St. Pietersnieuwstraat 41, 9000 Gent, Belgium

Abstract

In this contribution, we investigate the influence of timing errors on the performance of a multicarrier code-division multiple-access (MC-CDMA) system. We show that a constant timing offset introduces no degradation of the signal-to-noise ratio (SNR) at the input of the decision device as it can be compensated without loss of performance. On the other hand, in the presence of time-varying timing errors performance degrades. We show that the MC-CDMA system is very sensitive to a clock frequency offset: the system performance rapidly degrades for an increasing number of carriers. It is also shown that the timing jitter gives rise to a performance degradation that is essentially independent of the number of carriers. Moreover, this degradation caused by the timing jitter does not depend on the spectral contents of the jitter.

I. INTRODUCTION

During the last decade, we witnessed a widespread deployment of telecommunication services. However, due to the limited spectral resources of the existing channels, the need of bandwidth efficient access techniques arises. Particularly multicarrier (MC) systems received considerable attention in the context of high data rate communications as they combine a high spectral efficiency with an immunity to channel dispersion [1-2]. Furthermore, recent developments in DSP techniques have made MC systems suitable for implementation. Recently, different combinations of orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA) have been proposed [3-5]. Among these combinations we find multicarrier CDMA (MC-CDMA), which has been investigated in the context of high data rate communication over dispersive channels and has been proposed for downlink communication in mobile radio. In the MC-CDMA system, the data symbols are first multiplied by a higher rate chip sequence and then modulated on the orthogonal carriers.

The use of a large number of carriers makes the MC system highly sensitive to some types of timing errors [6-9]. In the case of the OFDM system, it was shown in [6-7] that for a large number of carriers the system severely suffers from a clock frequency offset between a free-running local oscillator for timing recovery and the transmitter sampling clock. In order to avoid this strong dependency, it was proposed in [7-9] to use a phase-locked local oscillator for the timing recovery, to eliminate the clock frequency offset. The timing jitter resulting from the PLL gives rise to a performance degradation that is independent of the jitter spectrum and the number of carriers.

In this contribution we investigate the sensitivity of MC-CDMA to timing errors. In section II, the MC-CDMA system is described in the presence of timing errors, by including the timing error in an equivalent time-varying impulse response. We consider the case of downlink communication, which implies that the signals transmitted to the various users are synchronised at the basestation. The sensitivity of the MC-CDMA system to a constant timing offset, a clock frequency offset and timing jitter is presented in section III. Conclusions are drawn in section IV.

II. SYSTEM DESCRIPTION

The conceptual block diagram of the MC-CDMA system is shown in figure 1. The data symbol $a_{i,m}$ to be transmitted to user m during the ith symbol interval is multiplied with a higher rate chip sequence $\{c_{n,m}|n=0,...,N-1\}$, where $c_{n,m}$ is the nth chip of the sequence of length N for spreading the data $a_{i,m}$. Sequences corresponding to different users are assumed to be orthogonal (e.g. Walsh Hadamard codes). The resulting samples are modulated on the N orthogonal carriers, yielding the time-domain samples

$$s_{i,k} = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{m}\sqrt{E_{sm}}\,a_{i,m}c_{n,m}e^{j2\pi\frac{kn}{N}} \qquad (1)$$

where E_{sm} is the energy per symbol when transmitting to user m.

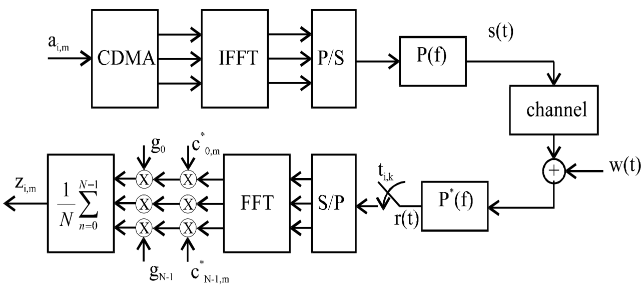

Figure 1 : MC-CDMA transceiver

Intersymbol interference, caused by the presence of a dispersive channel, can be avoided by cyclically extending the transmitted signal with a guard interval νT : the transmitted samples during the ith symbol interval are $\{s_{i,k}|k=-\nu,...,N-1\}$, where the first ν samples are a duplication of the last ν samples, $\{s_{i,k}|k=-\nu,...,-1\}= \{s_{i,k}|k=N-\nu,...,N-1\}$. The resulting time-domain samples are fed to a unit-energy square-root Nyquist filter with Fourier transform $P(f)$ and then applied to a dispersive channel with transfer function $H_{ch}(f)$. The transmitted signal is disturbed by additive white Gaussian noise $w(t)$, with uncorrelated real and imaginary parts, each having a power spectral density of $N_0/2$.

At the receiver, the signal is applied to a filter matched to the transmit filter and sampled at the instants $t_{i,k}=kT+iNT+\varepsilon_{i,k}T$, where $\varepsilon_{i,k}$ is the normalised timing error of the kth sample in the ith symbol interval ($|\varepsilon_{i,k}|<1/2$). The resulting samples can be written as

$$r_{i,k} = \sum_{m} h_{eq}\left(kT - mT; t_{i,k}\right) s_{i,m} + w_{i,k} \tag{2}$$

where $w_{i,k}$ is the value of the matched filter output noise and $h_{eq}(t;t_{i,k})$ is the equivalent *time-varying* impulse response and its Fourier transform with respect to the variable t, i.e. $H_{eq}(f;t_{i,k})$, is given by

$$H_{eq}\left(f;t_{i,k}\right) = H\left(f\right) e^{j2\pi f \varepsilon_{i,k} T} \tag{3}$$

where $H(f)$ consists of the cascade of the transmit filter, the channel transfer function and the receiver filter, i.e. $H(f)=|P(f)|^2 H_{ch}(f)$. Assuming the duration of $h_{eq}(t;t_{i,k})$ does not exceed the duration of the guard interval, the signal is only disturbed by the signal transients during the guard interval, such that the adjacent symbols will have no influence on the considered symbol interval outside the guard interval. The receiver selects the N samples outside the guard interval for further processing and disregards the other ν samples. The samples $r_{i,k}$ *outside* the guard interval are given by

$$r_{i,k} = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m} \sqrt{E_{sm}}\, a_{i,m} c_{n,m} H_n\left(t_{i,k}\right) e^{j2\pi \frac{nk}{N}} + w_{i,k} \qquad k = 0,...,N-1 \tag{4}$$

where

$$H_n\left(t_{i,k}\right) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} H_{eq}\left(\frac{n}{NT} + \frac{m}{T}; t_{i,k}\right) \tag{5}$$

The selected samples are demodulated with a fast Fourier transform (FFT). The nth output of the FFT is multiplied with the chip $c^*_{n,m}$ of the considered user m and applied to a one-tap zero-forcing equaliser with coefficient $g_{n,i}$, the filter coefficient belonging to the nth FFT output during the ith symbol interval. The resulting samples are summed to obtain the samples at the input of the decision device :

$$\begin{aligned} z_{i,m} &= \frac{1}{N} \sum_{n=0}^{N-1} g_{n,i} c^*_{n,m} \sum_{k=0}^{N-1} r_{i,k} e^{-j2\pi \frac{kn}{N}} \\ &= \sqrt{E_{sm}}\, a_{i,m} I_{i,m,m} + \sum_{\ell \neq m} \sqrt{E_{s\ell}}\, a_{i,\ell} I_{i,\ell,m} + W_{i,m} \end{aligned} \tag{6}$$

where $W_{i,m}$ is a zero-mean complex-valued Gaussian noise term and

$$I_{i,\ell,m} = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{n,n'=0}^{N-1} g_{n',i} c_{n,\ell} c^*_{n',m} e^{j2\pi \frac{k(n-n')}{N}} H_n\left(t_{i,k}\right) \tag{7}$$

For $\ell \neq m$, the quantity $I_{i,\ell,m}$ denotes the multi-user interference (MUI) at the input of the decision device of the mth user, sent to the ℓth user during the ith symbol interval. When the timing error can be modelled as random processes, the quantities $I_{i,\ell,m}$ are random variables and can be decomposed as

$$I_{i,\ell,m} = E\left[I_{i,\ell,m}\right] + \left(I_{i,\ell,m} - E\left[I_{i,\ell,m}\right]\right) \tag{8}$$

where the first contribution is the average of $I_{i,\ell,m}$ with respect to the timing error and the second term the zero-mean fluctuation of the useful component of the quantity $I_{i,\ell,m}$ about its average. Defining the signal-to-noise ratio (SNR) at the input of the decision device as the ratio of the power of the average useful component to the sum of the powers of the other contributions, the SNR of the mth user is given by

$$\begin{aligned} SNR_m = E_{sm}\left|E\left[I_{i,m,m}\right]\right|^2 \Bigg(& E\left[\left|W_{i,m}\right|^2\right] + \sum_{\ell \neq m} E_{s\ell}\left|E\left[I_{i,\ell,m}\right]\right|^2 \\ & + E_{sm} E\left[\left|I_{i,m,m} - E\left[I_{i,m,m}\right]\right|^2\right] + \sum_{\ell \neq m} E_{s\ell} E\left[\left|I_{i,\ell,m} - E\left[I_{i,\ell,m}\right]\right|^2\right] \Bigg)^{-1} \end{aligned} \tag{9}$$

where the first noise contribution in the denominator of (9) is additive white Gaussian noise, the second term is the power of the average multi-user interference, the third contribution is the power of the fluctuation of the useful component caused by the random character of the timing error and the last term is the power of the fluctuation of the multi-user interference.

In the case of an ideal channel and in the absence of timing errors, the quantities $H_m(t_{i,k})$ yield $H_m(t_{i,k})=1$. It can be verified that no scaling or rotation of the FFT outputs is necessary ($g_{n,i}=1$), as the channel introduces no attenuation of the useful component nor any multi-user interference, reducing the quantities (7) to $I_{i,\ell,m}=\delta_{\ell,m}$. The SNR (9) yields $SNR_m=E_{sm}/N_0$, hence the degradation caused by a non-ideal channel and/or the presence of timing errors yields $D_m=10\log((E_{sm}/N_0)/SNR_m)$ (dB).

III. Timing Errors

In this section, we investigate the sensitivity of MC-CDMA to timing errors. As we consider the case of downstream communication, the signals transmitted to the different users are synchronised at the base station. In the following, we separately consider the cases of a constant timing offset, a clock frequency offset and timing jitter in the case of an ideal channel ($H_{ch}(f)=1$), so we can concentrate on the influence of the timing errors only. The transmit and receiver filters are square-root raised-cosine filters with rolloff α.

A. Constant Timing Offset

For a constant timing offset $\varepsilon_{i,k}=\varepsilon$, the quantities (5) are independent of the sampling instants $(k+i(N+\nu))T$, reducing the quantities (7) to

$$I_{i,\ell,m} = \frac{1}{N}\sum_{n=0}^{N-1} c_{n,\ell} H_n(t_{i,k}) g_{n,i} c^*_{n,m} \qquad (10)$$

Due to the bandwidth limited character of the transmit and receiver filters, the equivalent filter (3) is also bandwidth limited: $H_{eq}(f;t_{i,k})=0,\ |f|>(1+\alpha)/2T$. Considering the frequencies n/T *outside* the rolloff area, the sum (5) reduces to one contribution:

$$H_n(t_{i,k}) = H\left(\frac{\mathrm{mod}(n;N)}{T}\right) e^{j2\pi\varepsilon\frac{\mathrm{mod}(n;N)}{N}} \qquad \frac{n}{T} \notin \left(\frac{1-\alpha}{2T}, \frac{1+\alpha}{2T}\right) \qquad (11)$$

where mod $mod(x;N)$ is the modulo of x with respect to N, yielding a result in the interval $[-N/2,N/2]$. For frequencies n/T *inside* the rolloff area, the sum (5) reduces to two contributions for which it can be verified that

$$\left|H_n(t_{i,k})\right| < \left|H\left(\frac{\mathrm{mod}(n;N)}{T}\right)\right| \qquad \frac{n}{T} \in \left(\frac{1-\alpha}{2T}, \frac{1+\alpha}{2T}\right) \qquad (12)$$

From (11), (12) and figure 2 it follows that for frequencies outside the rolloff area, the nth FFT output exhibits a constant amplitude $|H(mod(n;N)/T)|=1$ and is rotated over an angle $2\pi\varepsilon mod(n;N)/N$, and for frequencies inside the rolloff area, the FFT outputs are rotated over some angle and attenuated as compared to the FFT outputs outside the rolloff area.

The zero-forcing equaliser multiplies the FFT outputs with $g_{n,i}=1/H_n(t_{i,k})$ reducing the quantities (10) to $I_{i,\ell,m}=\delta_{\ell,m}$. Outside the rolloff area, the equaliser rotates the FFT outputs over an (estimate of the) angle $-2\pi\varepsilon mod(n;N)/N$. Inside the rolloff area, the equaliser scales and rotates the FFT outputs. In this way, the equaliser compensates for the rotation and attenuation of the FFT outputs and avoids MUI. However, the scaling of the FFT outputs inside the rolloff area affects the noise power:

$$E\left[\left|W_{i,m}\right|^2\right] = N_0 \frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{\left|H_n(t_{i,k})\right|^2} \geq N_0 \qquad (13)$$

The noise power level is increased as compared to a zero timing offset, resulting in a performance degradation. This degradation is merely caused by the carriers inside the rolloff area. For $\alpha=0$, a constant timing offset introduces no degradation. We can avoid this sensitivity to a constant timing offset by not using the carriers inside the rolloff area.

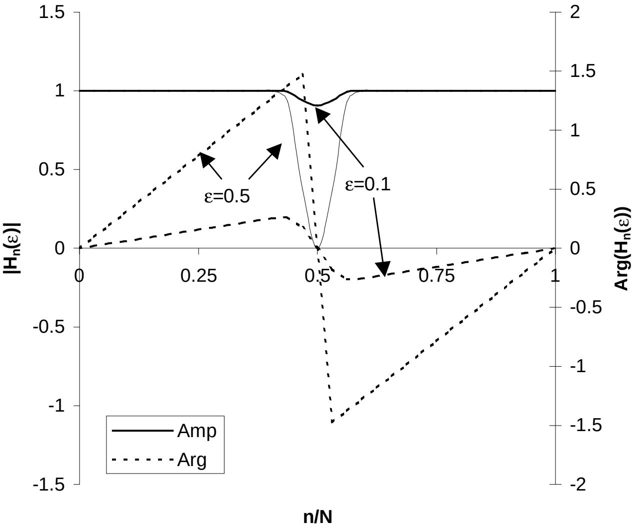

Figure 2 : Dependency of $|H_n(\varepsilon)|$ and $Arg(H_n(\varepsilon))$ on the carrier index n, $\alpha=0.15$

B. Clock Frequency Offset

When the sampling is performed by means of a free-running clock with a relative clock frequency offset of $\Delta T/T$, the normalised timing error linearly increases in time and is given by $\varepsilon_{i,k}=(k+i(N+\nu))\Delta T/T$.

In the case of $\alpha=0$ and for a slowly varying timing error, i.e. $\Delta T/T<<1$, the quantities (10) yield

$$I_{i,\ell,m} = \frac{1}{N}\sum_{n,n'=0}^{N-1} g_{n',i} c_{n,\ell} c^*_{n',m} G\left(\frac{n-n'}{N} + \frac{\mathrm{mod}(n;N)}{N}\frac{\Delta T}{T}\right) e^{j2\pi i\frac{N+\nu}{N}\mathrm{mod}(n;N)\frac{\Delta T}{T}} \qquad (14)$$

where

$$G(x) = \frac{1}{N}\sum_{k=0}^{N-1} e^{j2\pi kx} \qquad (15)$$

The equaliser multiplies the FFT outputs with

$$g_{n,i} = \frac{e^{-j2\pi i\frac{N+\nu}{N}\mathrm{mod}(n;N)\frac{\Delta T}{T}}}{G\left(\frac{\mathrm{mod}(n;N)}{N}\frac{\Delta T}{T}\right)} \qquad (16)$$

With these equaliser coefficients, we observe in (14) that the equaliser compensates for the attenuation and rotation of the useful component, however the equaliser is not able to eliminate the MUI ($I_{i,\ell,m}\neq 0$ for $\ell\neq m$). It can also be verified that the scaling of the FFT outputs results in an increase of the noise power. The degradation of the SNR at the input of the decision device of user m, caused by a clock frequency offset is given by

$$D_m = 10\log\left(\frac{E\left[\left|W_{i,m}\right|^2\right]}{N_0} + \frac{E_s}{N_0}\sum_{\ell \neq m}\left|I_{i,\ell,m}\right|^2\right) \qquad (17)$$

Assuming the energy per symbol transmitted to each user equals E_s, the degradation at the input of the decision device is user-independent and is shown in figure 3 for a maximal load, i.e. the number of active users $M=N$. From figure 3, it follows that the MC-CDMA system is very sensitive to a clock frequency offset. In order to obtain a small degradation, the clock frequency offset must be limited to small offsets, i.e. $\Delta T/T<<1/N$. However, it is possible to avoid this strong degradation by using a PLL-like timing correction mechanism in front of the FFT.

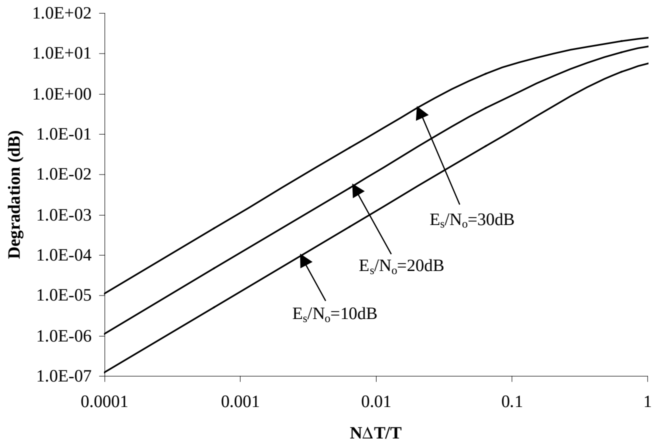

Figure 3 : Clock frequency offset

C. Timing Jitter

In order to get rid of a constant timing offset and a clock frequency offset, we can perform synchronised sampling, e.g. by means of a PLL. The normalised timing error resulting from the PLL can be modelled as a zero-mean stationary Gaussian process with jitter spectrum $S_\varepsilon(f)$ and jitter variance σ_ε^2. For slowly varying timing errors, the bandwidth f_B of the jitter spectrum is limited, i.e. $f_B T<<1$. The quantities (7) are random processes and for small jitter variances $\sigma_\varepsilon^2<<1$, they can be approximated by

$$I_{i,\ell,m} = \frac{1}{N}\sum_{n,n'=0}^{N-1} g_{n',i}\, c_{n,\ell}\, c_{n,m}^{*}\left(\delta_{n,n'} + j2\pi\tilde{H}_n \frac{1}{N}\sum_{k=0}^{N-1}\varepsilon_{i,k}\, e^{j2\pi\frac{k(n-n')}{N}}\right) \qquad (18)$$

where

$$\tilde{H}_n = \frac{1}{T}\sum_{m=-\infty}^{+\infty}\left(\frac{n}{N}+m\right)H\left(\frac{n}{NT}+\frac{m}{T}\right) \qquad (19)$$

For small jitter variances $\sigma_\varepsilon^2<<1$, the equaliser is essentially the same as for zero timing jitter, i.e. $g_{n,i}=1$. From (18) it follows that for $\ell=m$ the useful component $I_{i,m,m}$ consists of an average component and a random fluctuation about its average and for $\ell\neq m$ (18) indicates that the timing jitter introduces MUI. Assuming all users exhibit the same jitter spectrum $S_\varepsilon(f)$ and the same energy per symbol E_s, the powers of the average useful component, the fluctuation of the useful component, the multi-user interference and the noise are given by

$$\begin{aligned}
&E_{sm}\left|E\left[I_{i,m,m}\right]\right|^2 = E_s \\
&E_{sm}E\left[\left|I_{i,m,m}-E\left[I_{i,m,m}\right]\right|^2\right] = (2\pi)^2 E_s\left|\frac{1}{N}\sum_{n=0}^{N-1}\tilde{H}_n\right|^2 \int_{-\infty}^{+\infty} S_\varepsilon(f)\left|G(fT)\right|^2 df \\
&\sum_{\ell\neq m} E_{s\ell}E\left[\left|I_{i,\ell,m}\right|^2\right] = \frac{M-1}{N-1}(2\pi)^2 E_s \left\{\frac{1}{N}\sum_{n=0}^{N-1}\left|\tilde{H}_n\right|^2\int_{-\infty}^{+\infty}S_\varepsilon(f)df - \left|\frac{1}{N}\sum_{n=0}^{N-1}\tilde{H}_n\right|^2\int_{-\infty}^{+\infty}S_\varepsilon(f)\left|G(fT)\right|^2 df\right\} \\
&E\left[\left|W_{i,k}\right|^2\right] = N_0
\end{aligned} \qquad (20)$$

From (20) it follows that the sum of the powers of the fluctuation of the useful component and the multi-user interference linearly increases with the number of active users M. For the maximum load $M=N$, this sum is independent of the spectral contents of the jitter and only depends on the jitter variance

$$\sigma_\varepsilon^2 = \int_{-\infty}^{+\infty} S_\varepsilon(f)df \qquad (21)$$

For $\alpha=0$, the maximal load ($M=N$) and large N ($N\rightarrow\infty$), the degradation of the SNR in the presence of timing jitter is essentially independent of the number of carriers and is given by

$$D_m = 10\log\left(1+\frac{\pi^2}{3}\frac{E_s}{N_0}\sigma_\varepsilon^2\right) \qquad (22)$$

The degradation (22) is shown in figure 4. The degradation of figure 4 yields an upper bound for the degradation in the case of $M<N$ active users.

IV. CONCLUSIONS

In this contribution, we have investigated the sensitivity of MC-CDMA to timing errors in downlink communication. We have shown that a constant timing offset can be compensated without loss of performance, when not using the carriers inside the rolloff area, while MC-CDMA performance degrades for time-varying timing errors. In the presence of a clock frequency offset, MC-CDMA performance rapidly degrades for an increasing number of carriers. To avoid this strong sensitivity, the use of a phase-locked local oscillator is suggested for timing recovery. For the maximum load, the timing jitter resulting from the PLL gives rise to a degradation that is independent of the spectral contents of the jitter. Moreover, this degradation caused by the timing jitter is essentially independent of the number of carriers. An overview of the effect of the various timing errors is shown in table 1.

constant timing offset	no degradation
clock frequency offset	highly sensitive, degradation increases with N
timing jitter	degradation essentially independent of N

Table 1 : Sensitivity of MC-CDMA to timing errors

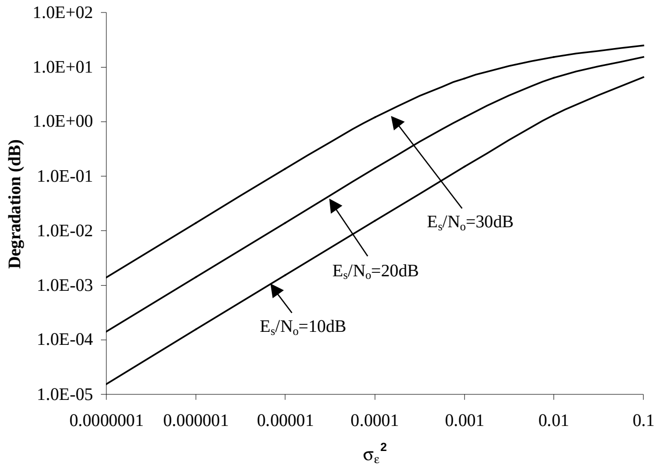

Figure 4 : Timing jitter

V. REFERENCES

[1] J.A.C. Bingham, “Multicarrier Modulation for Data Transmission, An Idea Whose Time Has Come”, IEEE Comm. Mag., Vol. 31, no. 5, May 90, pp. 5-14

[2] I. Kalet , “The Multitone Channel”, IEEE Trans. on Comm., Vol. 37, No. 2, Feb 89, pp. 119-124

[3] Multi-Carrier Spread-Spectrum, Eds. K. Fazel and G.P. Fettweis, Kluwer 1997

[4] S. Hara, R. Prasad, “Overview of Multicarrier CDMA”, IEEE Comm. Mag., Vol. 35, no. 12, Dec 97, pp. 126-133

[5] N. Morinaga, M. Nakagawa , R. Kohno, “New Concepts and Technologies for Achieving Highly Reliable and High-Capacity Multimedia Wireless Communications Systems”, IEEE Comm. Mag., Vol. 35, no. 1, Jan 97, pp. 34-40

[6] T. Pollet, P. Spruyt, M. Moeneclaey, “The BER Performance of OFDM Systems Using Non-Synchronised Sampling”, Proc. Globecom 1994, San Fransisco CA, Nov 94, Paper S7.6 , pp. 253-257

[7] M. Moeneclaey, “The effect of synchronization errors on the performance of orthogonal frequency-division multiplexed (OFDM) systems”, Proc. COST 254 "Emerging techniques for Communication Terminals", Toulouse, France, Jul 97, pp. 41-45

[8] L. Tomba and W.A. Krzymien, “A Model for the Analysis of Timing Jitter in OFDM Systems”, Proc. ICC 1998, Atlanta GA, Jun 98, Paper S35.1, pp. 1227-1231

[9] T.N. Zogakis, J.M. Cioffi, “The Effect of Timing Jitter on the Performance of a Discrete Multitone System”, IEEE Trans. on Comm., Vol. 44, No. 7, Jul 96, pp. 799-808