Influence of Transmitter and Receiver Orientation on the Channel Gain for RSS Ranging-Based VLP

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***Abstract*—In this work, a set of general expressions taking into account the impact of transmitter and receiver tilt in visible light positioning on the channel gain is elaborated. A rigorous approach involving Euler rotations results in a compact expression for the modified channel gain that can be interpreted graphically. The relative modification of this gain is numerically evaluated for a number of representative configurations. A first order approximation in case of small tilt angles leads to a number of interesting conclusions that can be utilized directly when applying received signal strength visible light positioning.**

I. Introduction

The topic of accurate indoor positioning has been investigated for several decades [1]. Multiple signals of different nature are hereby analysed. A majority of the research efforts can be found in the usage of ubiquitous radio frequency (RF) signals emitted by WiFi, Bluetooth or other RF communication technologies [2]. More recently though, a technique based on the modulation of artificial visible light has emerged, exploiting the omnipresence of indoor illumination infrastructures [3], [4]. Several papers on the potential accuracies [5]–[7] and multiple access techniques [8] of this novel technique, called Visible Light Positioning (VLP), have been published. In all these publications, the signal transmitting light emitting diode (LED) is pointing perfectly downwards while the receiving photo diode (PD) is oriented upwards. In reality though, one can imagine that this situation is more an exception than a rule. Regarding the LED, attachment of this device at the ceiling or on a dedicated rail can easily introduce a small tilt due to the mechanical fixation. At the receiver side, the time varying presence of small tilts is obvious, even if the PD is attached on the roof of e.g., a forklift truck. When the vehicle is charged with a large weight, one can imagine that the roof becomes tilted towards the increased weight lifted by the arms of the truck.

In this work, we quantify the tilting impact on the channel gain in received signal strength (RSS) ranging-based VLP. In such an approach, we first determine the distance between the PD and each LED, and the position is computed using multilateration. In section II, a theoretical description of the tilt-modified channel gain is elaborated. In section III, multiple representative configurations are studied, including tilt of the receiver and transmitter, and a first order approximation of the normalized channel gain in case of small tilts is described.

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II. System Description

To determine the distance between an LED and the PD, we evaluate the RSS. The RSS signal at the PD output is given as [6]

$$r(t) = R_p h_c s(t) + n(t), \tag{1}$$

where R_p is the responsivity of the PD, h_c the channel gain, $s(t)$ the transmitted optical power signal and $n(t)$ the additive noise. This noise is typically modelled as zero-mean Gaussian noise with power spectral density σ^2. From the RSS signal, we extract an RSS value, by correlating $r(t)$ with $s(t)$ in the interval $[0, T]$. Assuming perfect synchronization, the RSS value yields

$$r = R_p h_c P_s + n, \tag{2}$$

where $P_s = \int_0^T s^2(t)\mathrm{d}t$ and the noise term $n = \int_0^T s(t)n(t)\mathrm{d}t$ is zero-mean Gaussian noise with variance $\sigma^2 P_s$. Further, in (2), the channel gain can be written as

$$h_c = \frac{(m+1)A}{2\pi d^2} \cos^m \phi_S \cos\phi_R T(\phi_R) g(\phi_R), \tag{3}$$

where A is the surface of the PD, m is the Lambertian order of the LED, d is the distance between the LED and the PD, $T(\phi_R)$ is the signal transmission gain, and $g(\phi_R)$ the concentrator gain. Further, the angles ϕ_S and ϕ_R correspond to the angle of radiation and the angle of incidence, respectively and are defined as in Fig. 1; $\cos\phi_S$ and $\cos\phi_R$ are found by (4) and (5), where $\mathbf{n}_S$ and $\mathbf{n}_R$ are the unit normal vectors of the LED and photo diode. In Fig. 1, the global coordinate system (GCS) that is linked to the room has been introduced. The vectors $\mathbf{r}_S$ and $\mathbf{r}_R$ are the location vectors of respectively the transmitting LED and receiving PD.

$$\cos\phi_S = \mathbf{n}_S \cdot \frac{\mathbf{r}_R - \mathbf{r}_S}{d} \tag{4}$$

$$\cos\phi_R = \mathbf{n}_R \cdot \frac{\mathbf{r}_S - \mathbf{r}_R}{d} \tag{5}$$

In (3), the concentrator gain can be written as

$$g(\phi_R) = \begin{cases} \frac{n_c^2}{\cos^2\psi_c} & 0 \le \phi_R \le \psi_c \\ 0 & \phi_R > \psi_c \end{cases}, \tag{6}$$

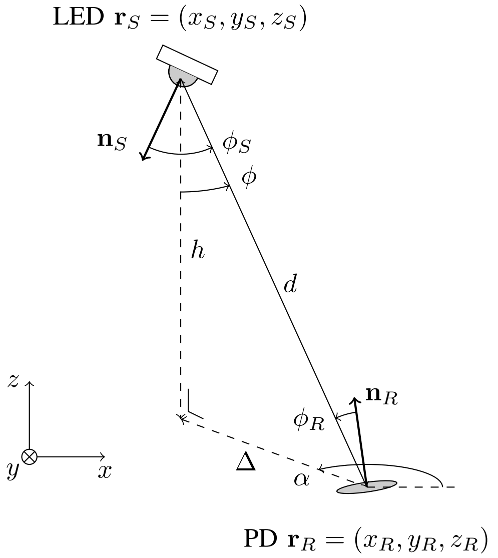

Fig. 1. The transmitting LED is situated at $\mathbf{r}_S = (x_S, y_S, z_S)$, while the receiving photodiode at $\mathbf{r}_R = (x_R, y_R, z_R)$.

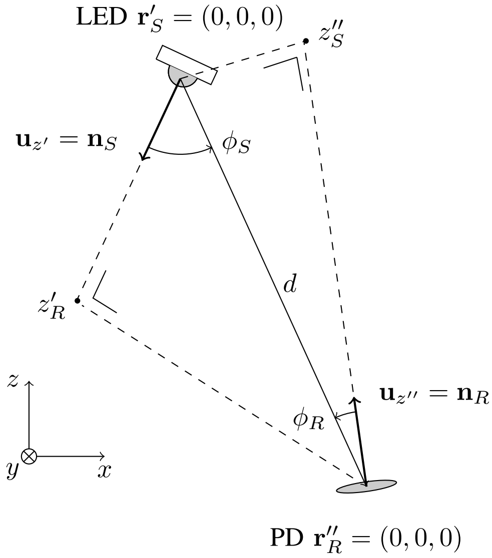

Fig. 2. Geometric interpretation of of z'_R and z''_S.

where n_c is the refractive index of the concentrator, and ψ_c the concentrator field-of-view (FOV) semi-angle. Note that within the FOV of the PD, the concentrator gain is constant. Further, also the signal transmission gain can be modeled as a constant. Hence, we can rewrite the channel gain as

$$h_c = \frac{\gamma}{d^2} \cos^m \phi_S \cos \phi_R, \qquad \text{for } \phi_R \leq \psi_c, \tag{7}$$

where the prefactor $\gamma = \frac{(m+1)A}{2\pi} T(\phi_R) g(\phi_R)$ is constant for $\phi_R \leq \psi_c$.

From (2) and (7), it follows that the RSS value depends on the distance d through the channel gain. This channel gain is in inverse proportion to d^2, but further depends on the distance and orientation through the angles ϕ_S and ϕ_R. In order to determine the relationship between ϕ_R, ϕ_S and d, we introduce two additional coordinate systems: i) the LED-centric coordinate system (LCS) (x', y', z'), and ii) the receiver-centric coordinate system (RCS) (x'', y'', z''). To obtain the relationship between the different coordinate systems, we define the orientations of the LED and PD through the Euler angles, i.e. for the LED, we have the Euler angles set $\boldsymbol{\psi}_S = (\psi_{S,1}, \psi_{S,2}, \psi_{S,3})$, and for the PD the set $\boldsymbol{\psi}_R = (\psi_{R,1}, \psi_{R,2}, \psi_{R,3})$, where the first and third angle correspond to a rotation around the z-axis, and the second angle to a rotation around the y-axis. The LCS coordinates are obtained by applying the rotations $\boldsymbol{\psi}_S$ to the GCS coordinate system, with as center the LED with GCS coordinates $\mathbf{r}_S = (x_S, y_S, z_S)$:

$$\mathbf{r}' = \mathbf{M}_{\psi_{S,3}} \mathbf{M}_{\psi_{S,2}} \mathbf{M}_{\psi_{S,1}} (\mathbf{r} - \mathbf{r}_S), \tag{8}$$

where $\mathbf{r} = (x, y, z)$ are the coordinates in the GCS, and $\mathbf{r}'$ the coordinates in the LCS. It is clear that based on (8), $\mathbf{r}'_S = \mathbf{0}$.

The rotation matrices are given by

$$\begin{aligned} \mathbf{M}_{\psi_{S,i}} &= \begin{pmatrix} \cos\psi_{S,i} & \sin\psi_{S,i} & 0 \\ -\sin\psi_{S,i} & \cos\psi_{S,i} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad i = 1, 3, \\ \mathbf{M}_{\psi_{S,2}} &= \begin{pmatrix} \cos\psi_{S,2} & 0 & -\sin\psi_{S,i} \\ 0 & 1 & 0 \\ \sin\psi_{S,i} & 0 & \cos\psi_{S,i} \end{pmatrix}. \end{aligned} \tag{9}$$

Similarly, the RCS coordinates are obtained by applying a rotation to the GCS, but now with as center the PD, having GCS coordinates $\mathbf{r}_R = (x_R, y_R, z_R)$:

$$\mathbf{r}'' = \mathbf{M}_{\psi_{R,3}} \mathbf{M}_{\psi_{R,2}} \mathbf{M}_{\psi_{R,1}} (\mathbf{r} - \mathbf{r}_R), \tag{10}$$

where $\mathbf{r}'' = (x'', y'', z'')$ are the coordinates in the RCS, and the transformation matrices $\mathbf{M}_{\psi_{R,i}}$ are defined similarly as (9). Here, $\mathbf{r}''_R = \mathbf{0}$ is valid. The angles $\psi_{S,i}$ and $\psi_{R,i}$ are chosen so that $\mathbf{u}_{z'} = \mathbf{n}_S$ and $\mathbf{u}_{z''} = \mathbf{n}_R$, where $\mathbf{u}_{z'}$ and $\mathbf{u}_{z''}$ are the unit vectors of the z' and the z'' axis, respectively. Let us denote z'_R as the z'-value of the receiver in the LCS coordinate system and z''_S as the z''-value of the LED in the RCS coordinate system. The geometric interpretation of z'_R and z''_S is shown in Fig. 2. Using these coordinate systems, we can obtain the expressions for $\cos\phi_S$ and $\cos\phi_R$, corresponding to the definition of these angles in Fig. 1:

$$\begin{aligned} \cos\phi_S &= \frac{z'_R}{d} \\ \cos\phi_R &= \frac{z''_S}{d}. \end{aligned} \tag{11}$$

where $d^2 = ||\mathbf{r}_S - \mathbf{r}_R||^2 = ||\mathbf{r}'_S - \mathbf{r}'_R||^2 = ||\mathbf{r}''_S - \mathbf{r}''_R||^2$. Note that, when $z'_R < 0$, the PD will not receive a signal as the LED does not radiate at angles ϕ_S outside the interval $[0°, 90°]$. Hence, $h_c = 0$ if $z'_R < 0$. Further, when $z''_S < 0$, the LED will be out of the FOV of the PD. Hence, $h_c = 0$ if $z''_S < 0$. Further, the concentrator gain is zero if $\phi_R > \psi_c$, which can be rewritten as $z''_S \geq d \cos\psi_c$. As the half-angle $\psi_c \in [0°, 90°]$, it follows that $d\cos\psi_c \geq 0$, which implies

$z''_S \geq \max(0, d\cos\psi_c) = d\cos\psi_c$, i.e. the concentrator gain limits z''_S. Taking this into account, we obtain

$$h_c = \gamma \frac{1}{d^{m+3}} (z'_R)^m (z''_S), \quad (12)$$

for $z'_R \geq 0$ and $z''_S \geq d\cos\psi_c$. The values z'_R and z''_S still depend on the distance d between the LED and the PD. Therefore, we take a closer look at these terms. First we recall that $\mathbf{r}'_S = \mathbf{0}$ and $\mathbf{r}''_R = \mathbf{0}$. Further, we denote $h = z_S - z_R$ as the vertical distance between the LED and the PD in the GCS, and $\Delta = h\tan\phi = \sqrt{d^2 - h^2}$ as the horizontal distance, and define the rotation angle α:

$$\begin{aligned} x_S - x_R &= \Delta\cos\alpha \\ y_S - y_R &= \Delta\sin\alpha, \end{aligned} \quad (13)$$

where ϕ and α are defined as shown in Fig. 1. This results in

$$\begin{aligned} z'_R &= -\Delta\sin\psi_{S,2}\cos(\psi_{S,1} - \alpha) - h\cos\psi_{S,2} \\ z''_S &= \Delta\sin\psi_{R,2}\cos(\psi_{R,1} - \alpha) + h\cos\psi_{R,2}. \end{aligned} \quad (14)$$

To show the symmetry between the effects of the transmitter and receiver orientation and to simplify the interpretation of the results, we introduce the supplementary angle $\overline{\psi}_{S,2}$ of $\psi_{S,2}$ (i.e. $\overline{\psi}_{S,2} + \psi_{S,2} = 180°$). We obtain the following expressions for z'_R and z''_S.

$$\begin{aligned} z'_R &= -\Delta\sin\overline{\psi}_{S,2}\cos(\psi_{S,1} - \alpha) + h\cos\overline{\psi}_{S,2} \\ z''_S &= \Delta\sin\psi_{R,2}\cos(\psi_{R,1} - \alpha) + h\cos\psi_{R,2}. \end{aligned} \quad (15)$$

Hence, the condition $z'_R \geq 0$ can be rewritten as

$$\frac{\Delta}{h} \leq \frac{1}{\tan\overline{\psi}_{S,2}\cos(\psi_{S,1} - \alpha)} \quad (16)$$

and $z''_S \geq d\cos\psi_c$ as

$$\frac{\Delta}{h}\cos(\psi_{R,1} - \alpha) \geq \frac{d}{h}\frac{\cos\psi_c}{\sin\psi_{R,2}} - \frac{1}{\tan\psi_{R_2}}, \quad (17)$$

which, depending on the sign of $\cos(\psi_{R,1} - \alpha)$, results in an upper or lower bound for Δ/h.

First, we notice that the channel gain, and thus the RSS value, is independent of the Euler angles $\psi_{S,3}$ and $\psi_{R,3}$. This is explained as follows. Recall that the distance d is independent of the considered coordinate system. Hence, the channel gain depends on the orientation of the LED and PD through their z-coordinates in the LCS and RCS only. As the last Euler angle corresponds to a rotation around the z-axis, it does not alter this z-coordinate. Therefore, without loss of generality, we can set $\psi_{S,3} = \psi_{R,3} = 0°$. Secondly, notice that the channel gain depends on $\psi_{S,1}$ and $\psi_{R,1}$ through $\cos(\psi_{S,1} - \alpha)$ and $\cos(\psi_{R,1} - \alpha)$ only. Taking into account that $\psi_{S,1}$ and $\psi_{R,1}$ can be selected in the interval $[0°, 180°]$, we can restrict our attention to the case $\alpha = 0°$ in our analysis. To evaluate the effect of the transmitter and receiver orientation on the RSS values, we define the normalized channel gain:

$$h_{c,n}(z'_R, z''_S, h) = \frac{h_c}{h_c^{(0)}} = \frac{(z'_R)^m (z''_S)}{h^{m+1}}, \quad (18)$$

where $h_c^{(0)}$ is the channel gain if the LED points straight downwards, i.e. $\psi_{S,2} = 0°$, and the PD straight upwards, i.e. $\psi_{R,2} = 0°$, implying $h_c^{(0)} = \gamma\frac{h^{m+1}}{d^{m+3}}$.

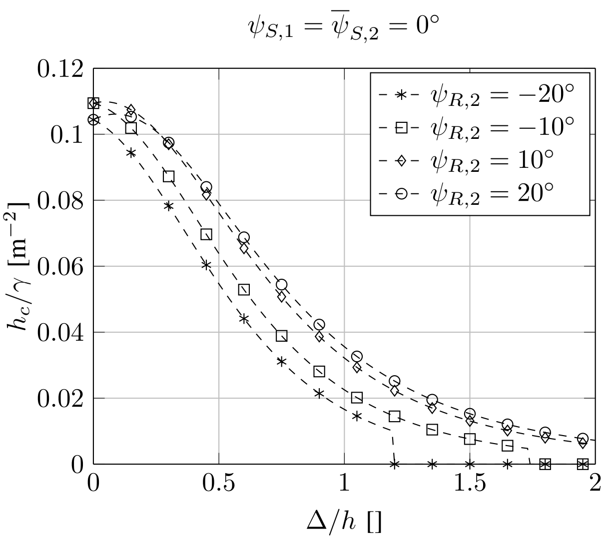

Fig. 3. h_c/γ as function of the normalized horizontal distance Δ/h when the LED is pointing straight downwards and the PD is tilted, for $\psi_{R,1} = 0°$ and various values for the tilt $\psi_{R,2}$.

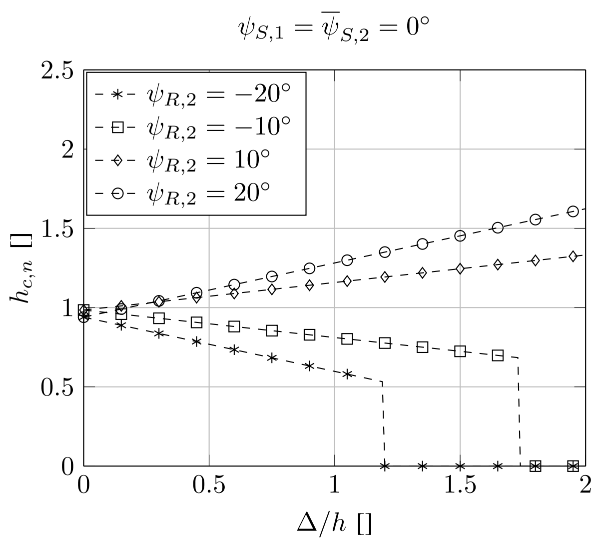

Fig. 4. Normalized channel gain $h_{c,n}$ as function of the normalized horizontal distance Δ/h when the LED is pointing straight downwards and the PD is tilted, for $\psi_{R,1} = 0°$ and various values for the tilt $\psi_{R,2}$.

III. Numerical Results

In this section, we will evaluate the effect of the transmitter and receiver orientation on the channel gain. Unless stated otherwise, we assume that the angle $\alpha = 0°$. This implies the vector $\mathbf{r}_S - \mathbf{r}_R$ is in a plane parallel to the xz-plane, with $x_R \leq x_S$. In our simulations, we consider a PD with FOV angle $\psi_c = 70°$, and the Lambertian order of the LED equals $m = 1$. The vertical distance between the LED and the PD is assumed $h = 3$ m.

In Fig. 3 and Fig. 4, we study the case where the LED is pointing straight downwards, i.e. $\psi_{S,1} = \overline{\psi}_{S,2} = 0°$ so $\phi_S = \phi$, and the PD is tilted over different angles $\psi_{R,2}$, with $\psi_{R,1} = 0°$. We also evaluated other rotation angles $\psi_{R,1}$. The results are similar to the case $\psi_{R,1} = 0°$. The

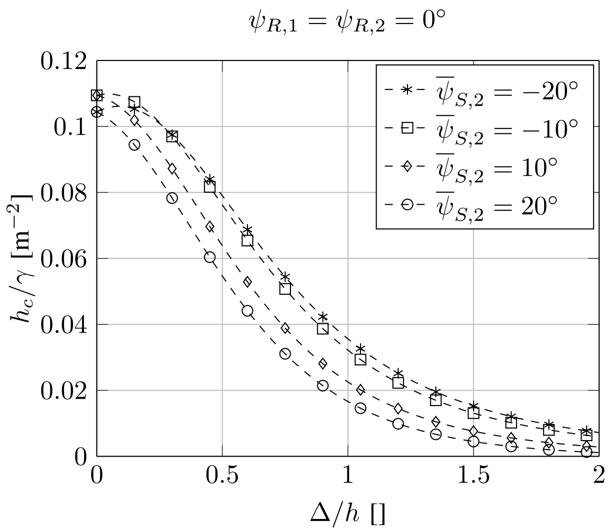

Fig. 5. h_c/γ as function of the normalized horizontal distance Δ/h when the PD is pointing straight upward and the LED is tilted, for $\psi_{S,1} = 0°$ and various values for the tilt $\overline{\psi}_{S,2}$.

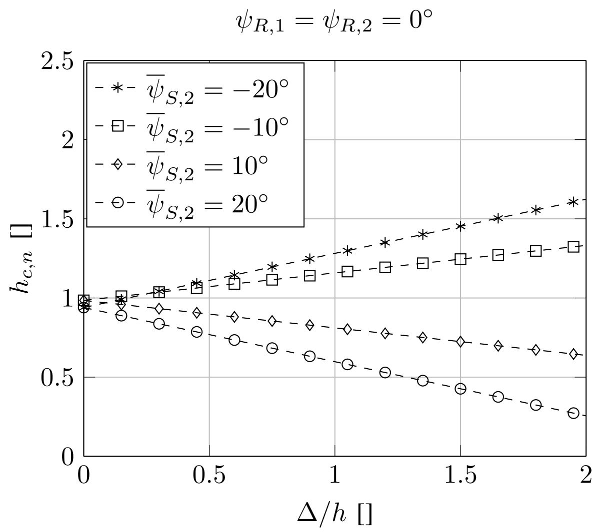

Fig. 6. Normalized channel gain $h_{c,n}$ as function of the normalized horizontal distance Δ/h when the PD is pointing straight upward and the LED is tilted, for $\psi_{S,1} = 0°$ and various values for the tilt $\overline{\psi}_{S,2}$.

effect of the tilt angle $\psi_{R,2}$ on the channel gain h_c can be found in Fig. 3. An angle $\psi_{R,2} < 0°$ indicates that the PD is tilted away from the LED, while $\psi_{R,2} > 0°$ implies the PD is tilted towards the LED. As expected, when the PD is tilted away from the LED, the channel gain monotonically decreases as function of the lateral displacement Δ, as not only the distance d between the LED and the PD increases, but also the angle ϕ_R. On the other hand, if the PD is tilted towards the LED, the channel gain first slightly increases as function of Δ until it reaches a maximum. Although in this region of Δ the resulting distance d increases when Δ increases, this increase of d is relatively small as Δ is small compared to the height h. At the same time, the angle ϕ_R noticeably reduces, causing the larger channel gain. When Δ increases further, not only its effect on the distance d will increase, but also the angle ϕ_R will start to increase, resulting in a similar reduction of the channel gain as when $\psi_{R,2} < 0°$.
In Fig. 4, we show the normalized channel gain $h_{c,n}$. This normalized channel gain allows us to compare the channel gain for the tilted PD with the channel gain for the parallel configuration, i.e. when $\psi_{R,2} = 0°$. As expected, we obtain a noticeable relative increase of the channel gain when the PD is pointing towards the LED ($\psi_{R,2} > 0°$) while when the PD looks away from the LED ($\psi_{R,2} < 0°$), the channel gain is reduced compared to the parallel case.
The effect of the orientation of the LED on the channel gain is shown in Fig. 5 and Fig. 6, for the case where the PD is pointing straight upwards ($\psi_{R,1} = \psi_{R,2} = 0°$ so $\phi_R = \phi$), for different tilt angles $\overline{\psi}_{S,2}$ with $\psi_{S,1} = 0°$. Other rotation angles $\psi_{S,1}$ were also evaluated and resulted in similar conclusions as for the case $\psi_{S,1} = 0°$. An angle $\overline{\psi}_{S,2} < 0°$ indicates that the LED is tilted towards the PD, while when $\overline{\psi}_{S,2} > 0°$, the LED is tilted away from the PD. The results are similar to the case where the PD is tilted and the LED is pointing straight downwards. The channel gain increases when the LED is pointing towards the PD.

For several applications such as the localization of robots and forklift trucks in large warehouses, one may suppose that by construction, the tilt of transmitter and receiver are small. Considering the expressions of z'_R and z''_S (15), the following first order approximation is valid:

$$\begin{aligned} z'_R &\simeq -\Delta\overline{\psi}_{S,2}\frac{\pi}{180°} + h \\ z''_S &\simeq \Delta\psi_{R,2}\frac{\pi}{180°} + h. \end{aligned} \tag{19}$$

Remark that we have corrected the angles by a factor $\pi/180°$ when the tilt is expressed in degrees. Using these small-tilt approximations, we obtain a compact expression for the normalized channel gain $h_{c,n}$:

$$h_{c,n} \simeq 1 + \frac{\Delta}{h}\left(\psi_{R,2} - m\overline{\psi}_{S,2}\right)\frac{\pi}{180°}. \tag{20}$$

One can see that the rotations around the y-axis dominate (taking into account previous remark on $\alpha = 0°$) and that the relative importance of the LED rotation with regard to the receiver tilt is weighted by the order m of the LED. Moreover, the tilt effect is least pronounced when the receiver is positioned just below the transmitter (i.e. $\Delta = 0$) and it increases linearly with the transversal distance Δ.
In the future, the above findings will be evaluated experimentally.

IV. Conclusions

In this work, we constructed general applicable expressions for the channel gain in VLP when receiver and/or transmitter tilt occur. Applying the transformation by means of Euler rotations, we obtained a compact equation that can easily be interpreted graphically by means of projections on the local coordinate system of transmitter and receiver. The resulting equations were used to evaluate a number of representative configurations, involving both receiver and transmitter tilt. For indoor positioning solutions where tilt values are typically small, a first order approximation of the normalized channel gain can be used. Based on this expression, the impact on the channel increases linearly with the lateral distance between the transmitter and receiver.

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