 ISBN 9789090311722

Effect of TI-ADC Mismatches on OFDM BER

Vo-Trung-Dung Huynh, Nele Noels, Heidi Steendam
Department of Telecommunications and Information Processing, Ghent University
{votrungdung.huynh, nele.noels, heidi.steendam}@ugent.be

***Abstract*—For the extremely high sampling rate and high resolution required for Terahertz (THz) communication, time-interleaved analog-to-digital converters (TI-ADCs) are being considered. However, in practice, mismatches between the parallel sub-ADCs degrade the system performance. In this extended abstract, we provide an overview of the effects of TI-ADC offset, gain and timing mismatches on the bit error rate (BER) of orthogonal frequency division multiplexing (OFDM) system.**

***Index Terms*—TI-ADC, BER, OFDM, offset mismatch, gain mismatch, timing mismatch.**

I. Introduction

Known for its high spectral efficiency and tolerance against channel dispersion, orthogonal frequency division multiplexing (OFDM) is used in many wired and wireless broadband communication systems. The use of OFDM in Terahertz (THz) communication has recently attracted increasing interest. In such systems, the receiver requires analog-to-digital converters (ADCs) operating at an extremely high sampling rate. To overcome the physical constraints of a conventional ADC, which cannot meet that demand, a time-interleaved (TI) structure of L identical parallel ADCs is often used as a practical alternative. However, mismatches between the parallel sub-ADCs, such as offset, gain and timing mismatch, degrade the OFDM system performance. The derivation of analytical expressions for the bit error rate (BER) in OFDM systems affected by such mismatches is not trivial, but it is important because it can significantly contribute to a better understanding of the TI-ADC design trade-offs. In this extended abstract, we provide an overview of our related work.

II. System Model

Fig. 1 illustrates a block diagram of the considered OFDM system. The transmitted data vector $\mathbf{X}$ consists of N complex-valued symbols, i.e., $\mathbf{X} = (X_0, X_1, ..., X_{N-1})^T$, which are taken from a unit-energy M^2-square QAM constellation. At the transmitter, bit-to-symbol mapping is performed by mapping sequences of $2\log_2 M$ bits onto the constellation points using the binary reflected Gray code (BRGC) rule for both in-phase (I) and quadrature (Q) dimensions. The vector $\mathbf{X}$ is applied to an inverse discrete Fourier transform (IDFT) of size N. The resulting time-domain samples are extended with a cyclic prefix (CP), which protects the received OFDM symbol against inter-symbol interference (ISI). Before transmission over the channel, the samples s_k are passed through a digital-to-analog converter (DAC) and a transmit filter. At the receiver, we assume perfect timing synchronization and matched filtering. After passing through the receive filter, the receive waveform is sampled at the Nyquist rate $\frac{1}{T_s}$ by a TI-ADC with L sub-ADCs. As, in practice, the offset do_l, gain dg_l and timing dt_l mismatch values of the sub-ADCs in a TI-ADC vary only very slowly with time, we model them as constants over an OFDM symbol period. The receiver removes the CP and converts the remaining N samples to the frequency domain using a discrete Fourier transform (DFT). Before data detection, the receiver employs a zero-forcing (ZF) equalizer to compensate for the channel. The n-th output of the equalizer can be approximated by

$$R_n \approx \sqrt{E_s} DGT_{0,n} X_n + \sqrt{E_s} \frac{\Phi_n}{H_n}, \; n = 0, 1, ..., N-1. \tag{1}$$

In (1), $\Phi_n = \sum_{i=1}^{L-1} \sum_{a=0}^{N-1} DGT_{i,a} X_a H_a f(a - p_{i,n}) e^{-j\pi(a-p_{i,n})}$ $+ \sum_{i=0}^{L-1} DO_i \mathrm{sinc}\,(p_{i,n})\, e^{j\pi p_{i,n}} + W_n$, where X_n is the n-th transmitted data symbol, H_n is the channel frequency response, $p_{i,n} = \mathrm{mod}\big(n - i\frac{N}{L}, N\big)$ denotes the remainder after division of $n - i\frac{N}{L}$ by N, W_n denotes additive white Gaussian noise (AWGN) samples, and $f(z) = \frac{\sin(\pi z)}{\pi z}$ if N/L is non-integer and $f(z) = \delta(z)$ if N/L is integer with $\delta(z)$ the dirac function. Further, $DGT_{x,y} = \frac{1}{L} \sum_{l=0}^{L-1} (1 + dg_l) \cdot e^{-j2\pi \frac{xl}{L}} \cdot e^{-j2\pi \frac{y dt_l}{N}}$ and $DO_i = \frac{1}{L} \sum_{l=0}^{L-1} do_l \cdot e^{-j\frac{2\pi i}{L} l}$.

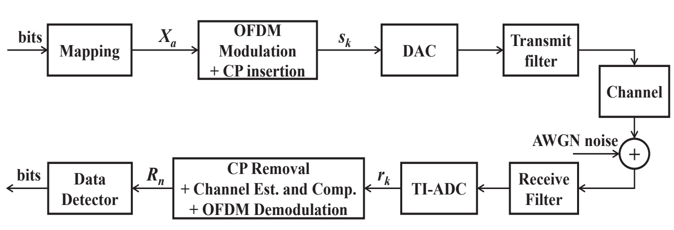

Fig. 1. Block diagram of an OFDM system with a TI-ADC at the receiver.

III. Main Results

Throughout this work, equation (1) has served as the basis to derive (semi-) analytical BER expressions. Having such expressions at our disposal allows for a much more efficient performance analysis than with purely simulation-based methods. Moreover, the expressions have shown their value in the derivation of practical design guidelines for TI-ADCs. Below, we briefly review our main accomplishments and conclusions. To simplify the analysis, we have considered the influence of each mismatch separately.

Offset Mismatch: In [1], exact BER expressions, as well as simplified BER expressions that hold for high signal-to-

ISBN 9789090311722

noise ratios (SNRs) and large offset mismatch values, have been derived in the presence of offset mismatch only. From the obtained exact BER expressions, an offset mismatch level threshold was established above which the BER performance shows an error floor at high SNRs. Numerical results showed that if we keep the offset mismatch level below 25% of this threshold, there is essentially no BER performance degradation as compared to the offset-mismatch free case. Our analysis further evinced that the tolerable level of the offset mismatch is proportional to the square-root of the number of sub-ADCs, indicating that as opposed to what might be expected, the offset mismatch level that can be tolerated actually increases with the number of sub-ADCs. Moreover, in [2], to estimate and compensate the offset mismatch, we have proposed two simple offset-mismatch calibration methods based on the least-squares (LS) and the linear minimum mean-square-error (LMMSE) principles. The effectiveness of our proposed methods has been confirmed by BER simulation results.

Gain Mismatch: In [3,4,5], we have focused solely on the effect of gain mismatch. We have derived an approximate BER expression, based on a Gaussian approximation of the induced inter-carrier interference (ICI) term. It was found that in the case of small L, the derived BER expression is less accurate for fixed channels. To avoid this issue, we have proposed an efficient semi-analytic approach to evaluate the BER exactly (without approximation). Further, from the approximate BER expressions, a threshold is established on the gain mismatch level at which an error floor caused by the gain mismatch is below a given BER value at high SNRs. Numerical results have shown that if and only if we keep the gain mismatch below 25% of this threshold, there is essentially no BER performance degradation compared with the mismatch-free case. We have also analytically evaluated BERs for OFDM systems that are impaired by both TI-ADC gain mismatch and channel estimation errors of a zero-forcing equalizer.

Timing Mismatch: The effect of timing mismatch in TI-ADCs has been studied in [6]. From (1), it follows that a TI-ADC with timing mismatch (i.e., $do_l = dg_l = 0$) causes a phase rotation of the desired symbols and generates ICI. Building on this result, we have derived an approximate closed-form BER expression assuming an AWGN channel (i.e., $H_n = 1$). Numerical results have been provided to demonstrate the accuracy of the presented formula. Currently, we are extending this work to multi-path fading channels.

As an example to illustrate the accuracy of the derived BER expressions in [1, 3-6], Fig. 2 shows the empirical and theoretical BER results for 16 QAM, $N = 2048$ and $L = 256$. As can be seen, the theoretical curves closely approach to the simulated curves.

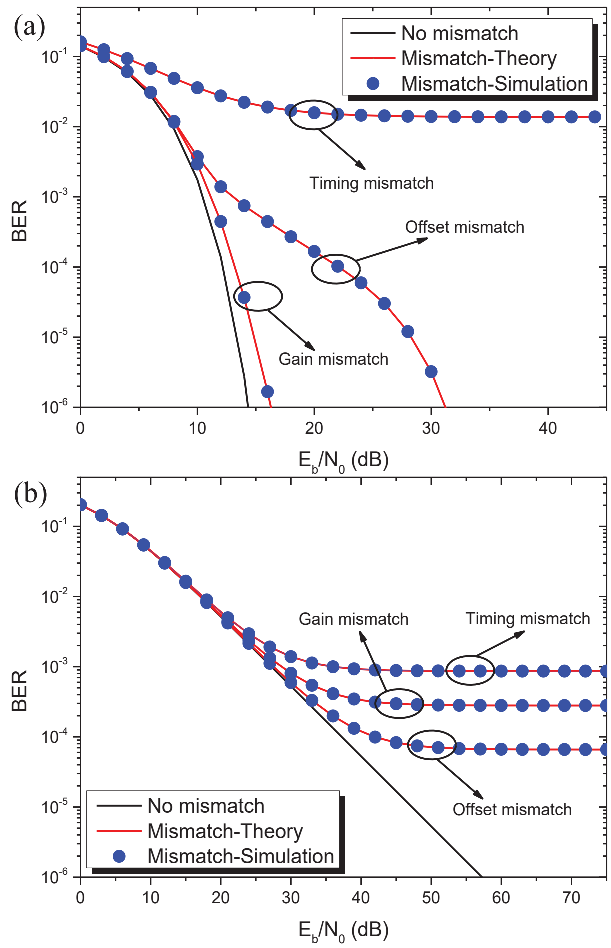

Fig. 2. BER curves with offset mismatch, gain mismatch and timing mismatch: (a) in an AWGN channel, (b) in a Rayleigh channel.

Acknowledgment

This research has been funded by the EOS programme of the Flemish Research Council (FWO) through the grant 30452698.

References

[1] V. T. D. Huynh, N. Noels, and H. Steendam, "Effect of offset mismatch in time-interleaved ADC circuits on OFDM-BER performance," *IEEE Trans. on Circuits and Systems I: Regular Papers*, vol. 64, no. 8, pp. 21952206, Aug 2017.

[2] V. T. D. Huynh, N. Noels, and H. Steendam, "Offset mismatch calibration for TI-ADCs in high-speed OFDM systems," *IEEE Symp. on Comm. and Vehicular Techn. in the Benelux*, Luxembourg City, Luxembourg, Jan. 2015.

[3] V. T. D. Huynh, N. Noels, and H. Steendam, "Approximate BER for OFDM systems impaired by a gain mismatch of a TI-ADC realization," *24th Intern. Conf. on Telecomm.*, Cyprus, May 2017, pp. 16, 2017.

[4] V. T. D. Huynh, N. Noels, and H. Steendam, "Accurate and Efficient BER Evaluation for High-Speed OFDM Systems Impaired by TI-ADC Circuits Gain Mismatch," *23rd Asia-Pacific Conf. Comm.*, Perth, Australia, Dec. 2017.

[5] V. T. D. Huynh, N. Noels, and H. Steendam, "BER Evaluation of OFDM Systems with Joint Effect of TI-ADC Circuits Gain Mismatch and Channel Estimation Error," *IEEE Trans. on Comm.* (submitted)

[6] V. T. D. Huynh, N. Noels, and H. Steendam, "Closed-Form BER Expression for OFDM with the Effect of TI-ADCs Timing Mismatch," *25th Intern. Conf. on Telecomm.*, Saint Malo, France, Jun. 2018.