An RSS Approximation for Visible Light Positioning in the Presence of Orientation Uncertainty

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***Abstract*—In practice, the orientation of a VLP receiver is often estimated with an external orientation estimation device. However, these devices generally suffer from drift and misalignment, causing an uncertainty in the measured orientation. In this paper, we evaluate the effect of the orientation uncertainty. We consider the first-order Taylor series expansion of the received signal strength (RSS) to cope with the non-linear relationship between the RSS and the orientation uncertainty, which results in a closed-form expression for the PDF of the RSS.**

***Index Terms*—Visible light positioning, Orientation uncertainty, Approximation.**

I. Introduction

During the last decade, visible light positioning (VLP) received an increasing amount of attention. Due to the directionality of the transmitter and the confine field-of-view (FOV) of the receiver, VLP systems are able to accurately estimate the receiver's position. However, the RSS not only depends on the position of the receiver, but also on its orientation. Many works, e.g. [1]–[3], consider the performance of the VLP system when the orientation of the receiver is known. They assume that the orientation is provided by an external orientation estimation device. However, the accuracy of the orientation is limited as 1) the external device typically experiences severe biases and drift problems, and 2) the movement of the receiver degrades the orientation estimation performance. As a conclusion, the orientation is not perfectly known but is subject to noise. As this orientation uncertainty will affect the positioning performance of state-of-the-art algorithms, the uncertainty should be investigated.

In this paper, we evaluate the effect of the orientation uncertainty. As the relationship between the RSS and the orientation of the receiver is highly non-linear, finding the PDF of the RSS, required in designing efficient estimators, is hard. To find a closed-form approximation for this PDF, we model the orientation uncertainty using the concepts from the Lie algebra [4] and approximate the non-linear relationship between the RSS and orientation uncertainty using the Taylor series expansion of the RSS.

II. System model

Assuming the LED can be modeled as a Lambertian radiator, the channel gain for the system can be written as [5]

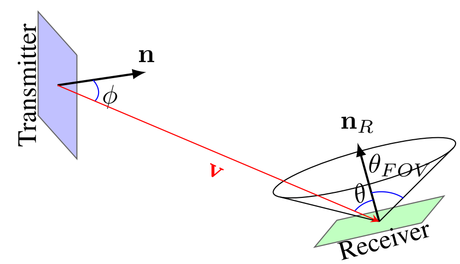

Fig. 1: System model

$$h = \frac{(\gamma + 1)\, A_R}{2\pi v^2} \cos^{\gamma}(\phi) \cos(\theta)\, \Pi\left(\frac{\theta}{\theta_{FOV}}\right), \tag{1}$$

where v is the distance between the LED and the receiver, ϕ is the radiation angle at the LED, θ is the incidence angle at the PD, A_R is the area of the PD, θ_{FOV} is the FOV of the PD, γ is the Lambertian order of the LED, and $\Pi(x) \triangleq \begin{cases} 1, & |x| \le 1, \\ 0, & |x| > 1. \end{cases}$ is the rectangular function. The received signal strength yields

$$P = R_p P_t h + w, \tag{2}$$

where R_p is the responsivity of the PD, P_t the power transmitted by the LED, and w is the shot noise, which is assumed to be zero-mean Gaussian distributed.

It is assumed that the LED has coordinates $\mathbf{r} \in \mathbb{R}^{3\times1}$ and normal $\mathbf{n} \in \mathbb{R}^{3\times1}$, and the PD has coordinates $\mathbf{r}_R \in \mathbb{R}^{3\times1}$ and normal $\mathbf{n}_R \in \mathbb{R}^{3\times1}$. The orientation of the PD is expressed in terms of a rotation matrix $\mathbf{R} \in SO(3)$ with respect to a reference orientation $\mathbf{n}_{R,0}$, i.e. $\mathbf{n}_R = \mathbf{R}\mathbf{n}_{R,0}$. As the reference orientation, we select the case of the receiver pointing straight upwards, i.e. $\mathbf{n}_{R,0} = [0\,0\,1]^{\mathrm{T}}$. The rotation matrix $\mathbf{R}$ is decomposed into a deterministic rotation $\tilde{\mathbf{R}} \in SO(3)$, corresponding to the estimate of the orientation, and a random rotation $\mathbf{R}_{\boldsymbol{\epsilon}} \in SO(3)$, containing the orientation uncertainty, with $\mathbf{R} = \mathbf{R}_{\boldsymbol{\epsilon}} \cdot \tilde{\mathbf{R}}$. Using the concept of Lie groups, the random rotation can be expressed as $\mathbf{R}_{\boldsymbol{\epsilon}} = \exp(\boldsymbol{\epsilon}_{\times})$ where $\boldsymbol{\epsilon} = [\epsilon_x\ \epsilon_y\ \epsilon_z]^{\mathrm{T}}$ is a random rotation vector and the operator $(\cdot)_{\times}$ converts the vector $\boldsymbol{\epsilon}$ into an element of the Lie algebra $\mathfrak{so}(3)$ of the 3D rotation group $SO(3)$. We further define the receiver normal without orientation uncertainty as $\tilde{\mathbf{n}}_R = \tilde{\mathbf{R}}\mathbf{n}_{R,0}$. The distribution of the rotation matrix $\mathbf{R}_{\boldsymbol{\epsilon}}$ is determined by the distribution of the rotation vector $\boldsymbol{\epsilon}$. This rotation vector is assumed to be zero-mean Gaussian distributed.

The channel gain depends on the orientation uncertainty through the incidence angle θ only. Let us define the incidence vector $\mathbf{v}$ as the vector between the LED and the PD, i.e. $\mathbf{v} = \mathbf{r}_R - \mathbf{r}$. Hence, the channel gain (1) can be rewritten as

$$h = K(\mathbf{n}, \mathbf{v}) \cos(\theta), \tag{3}$$

where $K(\mathbf{n}, \mathbf{v}) = \frac{(\gamma+1)A_R}{2\pi v^2} \cos^\gamma(\phi) \Pi(\theta/\theta_{FOV})$ is a function of $\mathbf{n}$ and $\mathbf{v}$, but independent of the orientation uncertainty $\boldsymbol{\epsilon}$. Further, using this definition, the incidence angle θ between $\mathbf{v}$ and the normal $\mathbf{n}_R$ of the receiver can be expressed as

$$\cos(\theta) = -\frac{\mathbf{n}_R^{\mathrm{T}}\mathbf{v}}{\|\mathbf{v}\|} = -(\exp(\boldsymbol{\epsilon}_\times)\tilde{\mathbf{n}}_R)^{\mathrm{T}} \bar{\mathbf{v}}, \tag{4}$$

where $\bar{\mathbf{v}} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$. Similarly, the incidence angle $\tilde{\theta}$ between $\mathbf{v}$ and $\tilde{\mathbf{n}}_R$, i.e. without orientation uncertainty, equals $\cos(\tilde{\theta}) = -\tilde{\mathbf{n}}_R^{\mathrm{T}}\bar{\mathbf{v}}$.

III. Approximation of the RSS

To be able to evaluate the effect of orientation uncertainty and compensate its effect, we need the PDF of the RSS in the presence of orientation uncertainty. Due to the non-linear behavior, obtaining a closed-form expression of the PDF is hard. Therefore, we consider an approximation to the PDF based on the Taylor series expansion: $\exp(\boldsymbol{\epsilon}_\times) = \mathbf{I}_{3\times3} + \boldsymbol{\epsilon}_\times + \frac{1}{2!}\boldsymbol{\epsilon}_\times^2 + \frac{1}{3!}\boldsymbol{\epsilon}_\times^3 + \cdots$. Discarding the second and higher order terms, the cosine of the incidence angle (4) can be approximated by

$$\begin{aligned} \cos(\theta) &\approx -((\mathbf{I} + \boldsymbol{\epsilon}_\times)\tilde{\mathbf{n}}_R)^{\mathrm{T}}\bar{\mathbf{v}} \\ &= -\tilde{\mathbf{n}}_R^{\mathrm{T}}\bar{\mathbf{v}} - (\tilde{\mathbf{n}}_R \times \bar{\mathbf{v}})^{\mathrm{T}}\boldsymbol{\epsilon}, \end{aligned} \tag{5}$$

where $\mathbf{a} \times \mathbf{b}$ is the cross product of the vectors $\mathbf{a}$ and $\mathbf{b}$. As a result, the channel gain (3) can be approximated by

$$h^{(1)} = -K(\mathbf{n}, \mathbf{v})\left(\tilde{\mathbf{n}}_R^{\mathrm{T}}\bar{\mathbf{v}} + (\tilde{\mathbf{n}}_R \times \bar{\mathbf{v}})^{\mathrm{T}}\boldsymbol{\epsilon}\right). \tag{6}$$

Using this first-order approximation, the observation yields

$$P^{(1)} = \mu + \mathbf{s}^{\mathrm{T}}\boldsymbol{\epsilon} + w, \tag{7}$$

where $\mu = -\breve{K}\tilde{\mathbf{n}}_R^{\mathrm{T}}\bar{\mathbf{v}}$, $\mathbf{s} = -\breve{K}(\tilde{\mathbf{n}}_R \times \bar{\mathbf{v}})$, and $\breve{K} = R_p P_t K(\mathbf{n}, \mathbf{v})$. Taking into account that both $\boldsymbol{\epsilon}$ and $\mathbf{w}$ are zero-mean Gaussian distributed, $P^{(1)}$ is Gaussian and $p(P; \mathbf{r}_R)$ is approximated by

$$p(P; \mathbf{r}_R) = \frac{1}{\sqrt{2\pi\sigma_P^2}} \exp\left(-\frac{1}{2\sigma_P^2}\|P - \mu\|^2\right), \tag{8}$$

where $\sigma_P^2 = \sigma_s^2 + \sigma_w^2$ with $\sigma_s^2 = \mathbf{s}^{\mathrm{T}}\boldsymbol{\Sigma}_{\boldsymbol{\epsilon}}\mathbf{s}$.

IV. Evaluation of the Approximation

In this section, we evaluate the accuracy of the approximation discussed above through simulations. To this end, we consider the case where the position and the normal of the LED are given by $\mathbf{r} = [0, 0, 3]^{\mathrm{T}}$ and $\mathbf{n} = [0, 0, -1]^{\mathrm{T}}$, i.e. the LED points straight downwards. Further, the LED transmits a power $P_t = 1$ W and has Lambertian order $\gamma = 1$. For the receiver, we consider a photo diode with area $A_R = 1$ cm^2 and FOV $\theta_{FOV} = 85°$. The receiver is placed below the LED, i.e. $\mathbf{r}_R = [0, 0, 1.5]^{\mathrm{T}}$. The covariance matrix $\boldsymbol{\Sigma}_\epsilon$ of the orientation uncertainty is assumed to be $\boldsymbol{\Sigma}_\epsilon = \sigma_\epsilon^2\mathbf{I}_{3\times3}$, where $\sigma_{\boldsymbol{\epsilon}}^2 = 2.0 \times 10^{-2}$ rad^2.

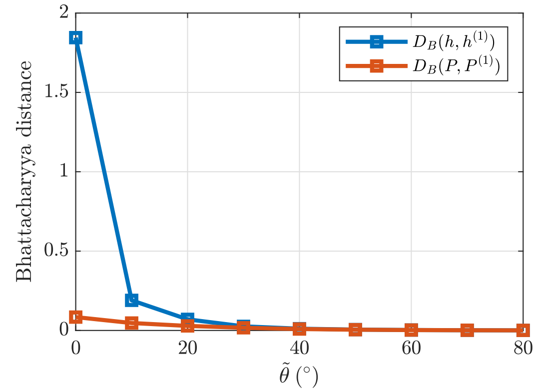

Fig. 2: Bhattacharyya distance between received power and approximated ones.

In order to numerically measure similarity of probability distributions, the Bhattacharyya distance is used, which is

$$D_B(p, q) = -\ln(BC(p, q)), \tag{9}$$

where $BC(p, q) = \sum_{i=1}^{n}\sqrt{p_i q_i}$ is the Bhattacharyya coefficient. We consider $D_B(P, P^{(1)})$ – the distance between the PDF of the RSSs P and $P^{(1)}$ – and $D_B(h, h^{(1)})$ – the distance between the PDF of the channel gains h and $h^{(1)}$ – as functions of the incidence angle $\tilde{\theta}$ of the VLP receiver. As shown in Fig. 2, both distances $D_B(P, P^{(1)})$ and $D_B(h, h^{(1)})$ decrease as $\tilde{\theta}$ increases, implying that the first-order approximation achieves a better accuracy for a larger $\tilde{\theta}$. In addition, contrary to $D_B(h, h^{(1)})$, even if $\tilde{\theta}$ is small, $D_B(P, P^{(1)})$ is closer to zero, which means that the distribution of $P^{(1)}$ approximates the true distribution of P with a higher accuracy, although $h^{(1)}$ only approximates h accurately when $\tilde{\theta}$ is large.

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References

[1] H. Steendam, T. Q. Wang, and J. Armstrong, "Theoretical lower bound for indoor visible light positioning using received signal strength measurements and an aperture-based receiver," *Journal of Lightwave Technology*, vol. 35, no. 2, pp. 309–319, Jan 2017.

[2] S. H. Yang, H. S. Kim, Y. H. Son, and S. K. Han, "Three-dimensional visible light indoor localization using AOA and RSS with multiple optical receivers," *Journal of Lightwave Technology*, vol. 32, no. 14, pp. 2480–2485, July 2014.

[3] Q.-L. Li, J.-Y. Wang, T. Huang, and Y. Wang, "Three-dimensional indoor visible light positioning system with a single transmitter and a single tilted receiver," *Optical Engineering*, vol. 55, no. 10, p. 106103, 2016.

[4] T. D. Barfoot and P. T. Furgale, "Associating uncertainty with three-dimensional poses for use in estimation problems," *IEEE Transactions on Robotics*, vol. 30, no. 3, pp. 679–693, June 2014.

[5] J. M. Kahn and J. R. Barry, "Wireless infrared communications," *Proceedings of the IEEE*, vol. 85, no. 2, pp. 265–298, Feb 1997.