The Sensitivity of a Flexible Form of MC-CDMA to Synchronisation Errors

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ABSTRACT - In this contribution, we propose a variant of the MC-CDMA system that offers more flexibility than the traditional MC-CDMA system. We investigate the sensitivity of this flexible MC-CDMA system to some types of synchronisation errors. This flexible MC-CDMA system is not affected by a constant phase offset or a constant timing offset, as in the proposed flexible MC-CDMA system the carriers inside the rolloff area are not used. Furthermore, the flexible MC-CDMA system is very sensitive to a carrier frequency offset or a clock frequency offset, while the system is less sensitive to carrier phase jitter or timing jitter.

I. INTRODUCTION

The enormous growth of interest for multicarrier (MC) systems can be ascribed to its high bandwidth efficiency and its immunity to channel dispersion. Recently, MC systems have been investigated in combination with code-division multiple-access (CDMA) [1-7], in order to cope with the high bit error rates caused by the strong attenuation of some carriers: by combining a MC system with CDMA, frequency diversity can be achieved. The MC-CDMA system has been proposed for downlink communication in mobile radio [5-7].

The use of a large number of users makes the MC system highly sensitive to some types of synchronisation errors [8-11]. The synchronisation errors can be classified in two classes: the carrier phase errors, caused by the error between the carrier used for upconverting the baseband signal to an RF signal at the transmitter and the local oscillator used for downconverting the RF signal at the receiver, and the timing errors, which are the errors made in the process of extracting the sampling instants. In [11], it was shown that the traditional MC-CDMA system is very sensitive to a carrier frequency offset and a clock frequency offset, while the system is less sensitive to carrier phase jitter or timing jitter. Furthermore, the traditional MC-CDMA system is not affected by a constant phase offset and if the carriers inside the rolloff area are not used, a constant timing offset also introduces no degradation. In addition, it is shown that the carriers inside the rolloff area introduce a severe performance degradation in the presence of timing errors.

In the proposed system, the carriers inside the rolloff area are not used. However, as in practical situations, the FFT length N_{fft} and the number of chips per symbol N_{chip} are both a power of two, the available carriers $N_{carr}<N_{fft}$ can not be used efficiently in the traditional MC-CDMA system, as the number of used carriers is chosen equal to the number of chips per symbols. Therefore, we propose a variant of the MC-CDMA system that offers more flexibility than the traditional MC-CDMA system: in the proposed, flexible MC-CDMA system, the number of chips per symbol N_{chip}, the number of carriers N_{carr} and the FFT length N_{fft} can be chosen independently, so the available resources can be used more efficiently.

In this contribution, we investigate the sensitivity of this flexible MC-CDMA system in the presence of some types of synchronisation errors, and compare the results with the traditional MC-CDMA system.

II. SYSTEM DESCRIPTION

The conceptual block diagram of the considered MC-CDMA system (for one user) is given in figure 1. The data symbols $\{a_{i,m}\}$ transmitted at a rate R_s, where the symbol $a_{i,m}$ corresponds to the user m during the ith symbol interval, are multiplied by a higher rate chip sequence $\{c_{n+iN_{chip},m}|n=0,...,N_{chip}-1\}$, $c_{n+iN_{chip},m}$ denoting the nth chip of the sequence corresponding to user m during the ith symbol interval, resulting in the samples $b_{i,n}$. The complex chip sequence corresponding to user m consists of the product of a real-valued orthogonal sequence of length N_{chip} (e.g. Walsh-Hadamard sequences), corresponding to the considered user, and a complex-valued random

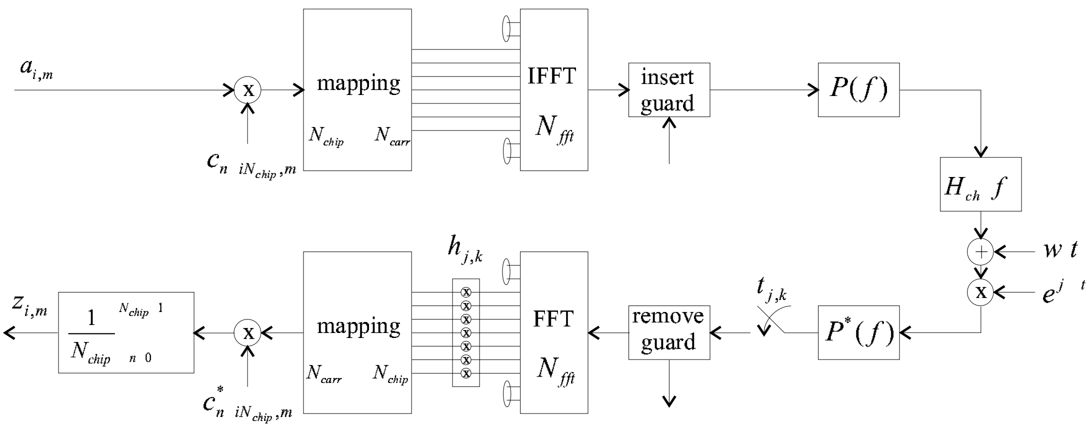

Figure 1 : Conceptual block diagram of the flexible MC-CDMA system for one user

sequence (e.g. a complex-valued pseudo-noise sequence of length $L>>N_{chip}$) which is equal for all users and has the same rate as the Walsh-Hadamard sequence. These hybrid sequences have better correlation properties than the pure Walsh-Hadamard sequences.

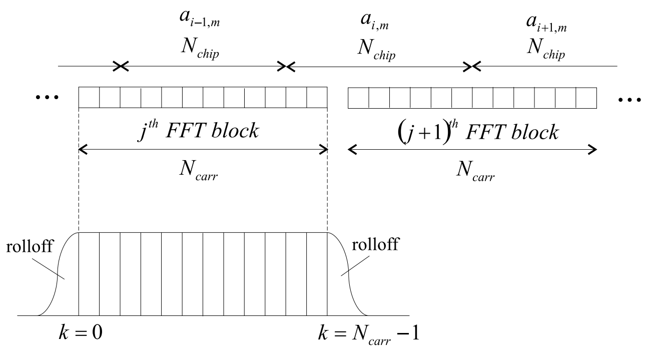

Figure 2 : Mapping of the chips on the carriers ($N_{chip}=8$, $N_{carr}=12$)

As carriers inside the rolloff area give rise to a severe performance degradation, we only use the carriers outside the rolloff area. The number of used carriers N_{carr} is not necessarily equal to the number of chips per symbol N_{chip}, so we can select N_{chip} independently of N_{carr}. The samples $b_{i,n}$ are mapped on the N_{carr} transmitted carriers (see figure 2) and modulated using an IFFT of length N_{fft} resulting in the time-domain samples $s_{j,k}$, the kth sample of the jth FFT block. As in traditional MC-CDMA, we insert a guard interval νT by cyclically extending the transmitted signal to avoid interference between successive FFT blocks. The duration of the transmitted block equals $(N_{fft}+\nu)T=N_{carr}/(R_s N_{chip})$. The resulting time-domain samples $\{s_{j,k}|k=-\nu,...,N_{carr}-1\}$ are fed to a transmit filter $P(f)$, a unit-energy square-root Nyquist filter, and applied to the dispersive channel with channel transfer function $H_{ch}(f)$. The signal is disturbed by additive white Gaussian noise with psd N_0 and a carrier phase error $\phi(t)$. At the receiver, the signal is applied to the receiver filter, which is matched to the transmit filter and sampled at the instants $t_{j,k}=kT+j(N_{fft}+\nu)T+\varepsilon_{j,k}T$, where $\varepsilon_{j,k}$ is the normalised timing error at the kth instant of the jth FFT block $(|\varepsilon_{j,k}|<1/2)$.

It can be verified that, when the carrier phase error is slowly varying as compared to T, the samples $r_{j,k}$ are given by

$$r_{j,k} = \sum_{\ell} h_{eq}\left(kT-\ell T; t_{j,k}\right) s_{j,\ell} + w_{j,k} \qquad (1)$$

where $w_{j,k}$ is the matched filter output noise at the instant $t_{j,k}$ and $h_{eq}(t;t_{j,k})$ is the impulse response of an equivalent time-varying filter with Fourier transform $H_{eq}(f;t_{i,k})=H(f)e^{j\phi(t_{i,k})}e^{j2\pi f\varepsilon_{i,k}T}$. The filter $H(f)$ consists of the cascade of the transmit filter, the channel transfer function and the receiver filter : $H(f)=|P(f)|^2 H_{ch}(f)$. The receiver selects the N_{fft} samples outside the guard interval and disregards the other ν samples. The selected samples are demodulated using the FFT; the kth output of the FFT is applied to a one-tap equaliser with coefficient $h_{j,k}$, scaling and rotating the corresponding FFT output. The receiver maps the outputs of the equalisers corresponding to the N_{carr} transmitted carriers into blocks of N_{chip} samples and correlates each block of N_{chip} samples with the chip of the considered user, resulting in the samples

$$z_{i,m} = \sum_{i'=-\infty}^{+\infty} \sum_{m'} \sqrt{E_{sm'}} a_{i',m'} I_{i,i',m,m'} + W_{i,m} \qquad (2)$$

where

$$I_{i,i',m,m'} = \frac{1}{N_{chip}} \sum_{n,n'=0}^{N_{chip}-1} c^*_{n+iN_{chip},m} B_{i,i',n,n'} c_{n'+i'N_{chip},m'} , \qquad (3)$$

E_{sm} is the energy per symbol corresponding to user m, $W_{i,m}$ is a zero-mean complex-valued Gaussian noise term and

$$B_{i,i',n,n'} = \delta_{div(n+iN_{chip},N_{carr})-div(n'+i'N_{chip},N_{carr})} A_{div(n+iN_{chip},N_{carr}),\mathrm{mod}(n+iN_{chip},N_{carr}),\mathrm{mod}(n'+i'N_{chip},N_{carr})} \quad (4)$$

$$A_{j',k,k'} = h_{j',k} \frac{1}{N_{fft}} \sum_{\ell=0}^{N_{fft}-1} H_{k'}(t_{j',\ell}) e^{-j2\pi\frac{\ell(k-k')}{N_{fft}}} \quad (5)$$

where $mod(x,M)$ is the modulo-M reduction of x, yielding a result in $(0,M-1)$ and $div(x,M)=floor(x/M)$. As we only consider the carriers outside the rolloff area and assuming the duration of $h_{eq}(t;t_{j,k})$ does not exceed the guard time duration, the coefficients $H_k(t_{j,\ell})$ are given by

$$H_k(t_{j,\ell}) = \frac{1}{T} H_{eq}\left(\frac{k}{N_{fft}T} - \frac{N_{carr}}{2N_{fft}T}; t_{j,\ell}\right) \quad (6)$$

For $m' \neq m$ and $i'=i$, the quantity (3) denotes the multi-user interference (MUI) at the decision device input of user m, originating from the user m' during the ith symbol interval. For $i' \neq i$, the quantity (3) denotes the intersymbol interference (ISI).

As we use random sequences, the quantities (3) are random variables. Furthermore, when the carrier phase error or the timing error can be modelled by random processes, the quantities (4) are random variables as well. The useful component can be decomposed as

$$I_{i,i,m,m} = E_{e,c}[I_{i,i,m,m}] + (I_{i,i,m,m} - E_{e,c}[I_{i,i,m,m}]) \quad (7)$$

where $E_{e,c}[.]$ denotes the average with respect to the random synchronisation errors and to the random chip sequences. The first component of (7) is the average useful component and the second component is the zero-mean fluctuation of the useful component. In order to eliminate the dependency of the symbol interval i, we consider the time average $AVG_i[.]$ of the powers. Defining the signal-to-noise ratio (SNR) at the input of the decision device as the time average of the power of the average useful component to the time average of the power of the remaining contributions, the SNR corresponding to user m is given by

$$SNR_m = E_{sm} AVG_i\left[\left|E_{e,c}[I_{i,i,m,m}]\right|^2\right] \Bigg\{ AVG_i\Bigg[E_{e,c}\left[|W_{i,m}|^2\right] + E_{sm} E_{e,c}\left[\left|I_{i,i,m,m} - E_{e,c}[I_{i,i,m,m}]\right|^2\right] \sum_{m' \neq m} E_{sm'} E_{e,c}\left[|I_{i,i,m,m'}|^2\right] + \sum_{i' \neq i} \sum_{m'} E_{sm'} E_{e,c}\left[|I_{i,i',m,m'}|^2\right] \Bigg] \Bigg\}^{-1} \quad (8)$$

In the case of an ideal channel and in the absence of synchronisation errors, the SNR equals $SNR_m=E_{sm}/N_0$. Hence, the degradation caused by a non-ideal channel and/or in the presence of synchronisation errors is given by $D_m=10log((E_{sm}/N_0)/SNR_m)$.

In order to clearly isolate the effect of the synchronisation errors, we consider the case of an ideal channel. We also consider the case of downstream communication, such that all transmitted carriers exhibit the same carrier phase error as they are up-converted by the same oscillator and the signals sent to the users exhibit the same timing error as they are synchronised at the base station. We assume each user exhibits the same energy per symbol $E_{sm}=E_s$ and a maximum load, i.e. the number of users equals N_{chip}. The transmit filter and the receiver filter are square-root raised-cosine filters with rolloff α. We compare the results of the flexible MC-CDMA system with the traditional MC-CDMA system where the number of chips per symbol equals $N=N_{fft}$.

III. Carrier Phase Errors

In this section, we investigate the influence of the carrier phase errors on the performance of the flexible MC-CDMA system in the absence of timing errors.

IIIa. Constant Phase Offset

In the case of a constant carrier phase offset $\phi(t)=\phi$, the quantities (5) reduce to $A_{i,k,k'}=h_{i,k}e^{j\phi}\delta_{k,k'}$. As in the traditional MC-CDMA system, it can be verified that this phase rotation yields only a phase rotation of the useful component but introduces neither MUI nor ISI. As a rotation of the FFT outputs has no influence on the noise power, a constant phase offset can be compensated without loss of performance, by rotating the FFT outputs over an angle $-\phi$, i.e. $h_{i,k}=e^{-j\phi}$.

IIIb. Carrier Frequency Offset

When the transmitter and receiver carrier oscillators exhibit a frequency offset ΔF, the carrier phase error is given by $\phi(t)=2\pi\Delta Ft$. Assuming a slowly varying phase error as compared to T, i.e. $\Delta FT<<1$, the quantities (5) yield

$$A_{i,k,k'} = h_{i,k} e^{j2\pi i N_{fft}\Delta FT} G\left(\frac{k'-k}{N_{fft}} + \Delta FT\right) \quad (9)$$

where

$$G(x) = \frac{1}{N_{fft}} \sum_{k=0}^{N_{fft}-1} e^{j2\pi kx} \tag{10}$$

Assuming the receiver can estimate ΔF, the equaliser multiplies the FFT outputs with $h_{i,k}=exp(-j2\pi iN_{fft}\Delta FT)/G(\Delta FT)$: the equaliser compensates for the phase rotation and the attenuation of the useful component, however it is not able to eliminate the interference. It can be verified that the degradation becomes independent of N_{chip} when assuming a maximum load. The degradation caused by the presence of a carrier frequency offset is shown in figure 3. The degradations of the flexible MC-CDMA system and the traditional MC-CDMA system are essentially the same. We observe a high sensitivity of the MC-CDMA system to this carrier frequency offset. To obtain small degradations, the frequency offset must be limited, i.e. $\Delta FT<<1/N_{fft}$.

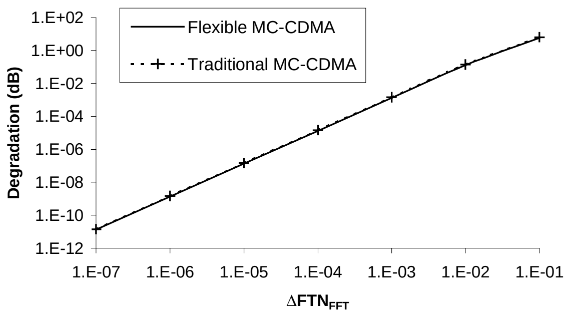

Figure 3 : Carrier frequency offset, $\alpha=0.1$, $E_s/N_o=20dB$

IIIc. Carrier Phase Jitter

In order to avoid this strong degradation caused by the carrier frequency offset, we can use a phase-locked local oscillator for RF to baseband conversion. The phase error resulting from the PLL can be modelled as a zero-mean stationary process with jitter spectrum $S_\phi(f)$ and jitter variance σ_ϕ^2. Assuming small jitter variances $\sigma_\phi^2<<1$, the quantities (5) can be approximated by

$$A_{i,k,k'} = h_{i,k}\left(\delta_{k,k'} + \frac{1}{N_{fft}}\sum_{\ell=0}^{N_{fft}-1} j\phi(t_{i,\ell})e^{-j2\pi\frac{\ell(k-k')}{N_{fft}}}\right) \tag{11}$$

For small jitter variances, the equaliser coefficients are essentially the same as in the absence of phase jitter, i.e. $h_{i,k}=1$. It can be verified that the degradation is essentially independent of the FFT length and becomes independent of N_{chip} when assuming a maximum load. The degradation caused by the phase jitter is shown in figure 4. The degradations of the flexible MC-CDMA system and the traditional MC-CDMA system are essentially the same.

IV. Timing Errors

In this section, we investigate the influence of timing errors on the performance of the flexible MC-CDMA system in the absence of carrier phase errors.

IVa. Constant Timing Offset

In the case of a constant timing offset, i.e. $\varepsilon_{i,k}=\varepsilon$, the quantities (5) yield

$$A_{i,k,k'} = \delta_{k,k'} h_{i,k} e^{j2\pi\left(k-\frac{N_{carr}}{2}\right)\frac{\varepsilon}{N_{fft}}} \tag{12}$$

Assuming the receiver can estimate ε, the equaliser multiplies the FFT outputs with $h_{i,k}=exp(-j2\pi(k-N_{carr}/2)\varepsilon/N_{fft})$. The equaliser compensates for the phase rotation of the FFT and avoids interference, i.e. a constant timing offset can be compensated without loss of performance.

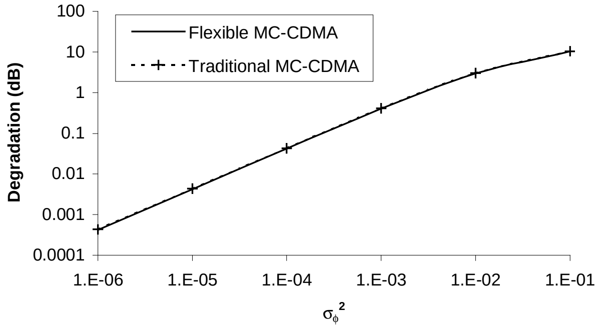

Figure 4 : Carrier phase jitter, $\alpha=0.1$, $E_s/N_o=20dB$, $N_{fft}=32$

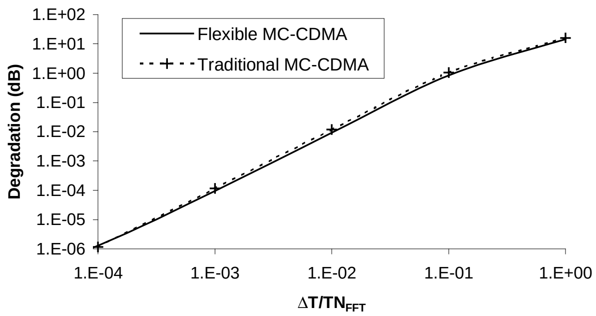

Figure 5 : Clock frequency offset, $\alpha=0.1$, $E_s/N_o=20dB$

IVb. Clock Frequency Offset

Sampling by means of a free-running clock with a relative clock frequency offset $\Delta T/T$ results in a timing error $\varepsilon_{i,k}=(k+iN_{fft})\Delta T/T$. For a small relative clock frequency offset $\Delta T/T<<1$, the quantities (5) are given by

$$A_{i,k,k'} = h_{i,k} e^{j2\pi\left(k'-\frac{N_{carr}}{2}\right)i\frac{\Delta T}{T}} G\left(\frac{k'-k}{N_{fft}} - \left(\frac{k'}{N_{fft}} - \frac{N_{carr}}{2N_{fft}}\right)\frac{\Delta T}{T}\right) \quad (13)$$

The equaliser multiplies the FFT outputs with $h_{i,k}=exp(-j2\pi(k-N_{carr}/2)i\Delta T/T)/G((k-N_{carr}/2)/N_{fft}\Delta T/T)$. The equaliser compensates for the attenuation and phase rotation of the useful component, but is not able to eliminate the interference. When assuming a maximum load, the degradation becomes independent of N_{chip}. In figure 5, we observe that the flexible MC-CDMA system slightly outperforms the traditional MC-CDMA system. The degradation strongly depends on the clock frequency offset. To obtain small degradations, the clock frequency offset must be small, i.e. $\Delta T/T<<1/N_{fft}$.

IVc. Timing Jitter

In order to get rid of the clock frequency offset, synchronised sampling can be used, e.g. by means of a PLL. The resulting normalised timing error can be modelled as a zero-mean stationary process with jitter spectrum $S_\varepsilon(f)$ and jitter variance σ_ε^2. Assuming small jitter variances $\sigma_\varepsilon^2<<1$, the quantities (5) can be approximated by

$$A_{i,k,k'} = h_{i,k}\left(\delta_{k,k'} + j2\pi T\left(\frac{k'}{N_{fft}} - \frac{N_{carr}}{2N_{fft}}\right)\frac{1}{N_{fft}}\sum_{\ell=0}^{N_{fft}-1} \varepsilon\left(t_{i,\ell}\right) e^{-j2\pi\frac{\ell(k-k')}{N_{fft}}}\right) \quad (14)$$

For small jitter variances, the equaliser coefficients are essentially the same as in the absence of timing jitter, i.e. $h_{i,k}=1$. It can be verified that the degradation caused by the timing jitter is essentially independent of the FFT length. Furthermore, for a maximum load, the degradation becomes independent of N_{chip}. The degradation caused by the timing jitter is shown in figure 6. The degradation of the flexible MC-CDMA system is slightly smaller than the degradation of the traditional MC-CDMA system.

V. CONCLUSIONS

We have investigated the sensitivity of the flexible MC-CDMA system to synchronisation errors. No degradation was introduced by a constant phase error or a constant timing error. A carrier frequency offset and a clock frequency offset both introduce a degradation that strongly depends on the FFT length, while the degradations caused by carrier phase jitter and timing jitter are both essentially independent of the FFT length. Furthermore, all degradations are independent of the number of chips per symbol when assuming a maximum load. Comparing flexible MC-CDMA with traditional MC-CDMA, we observe that the results are essentially the same in the case of carrier phase errors, while the flexible MC-CDMA slightly outperforms the traditional MC-CDMA system, as in the flexible MC-CDMA the carriers inside the rolloff area are not used.

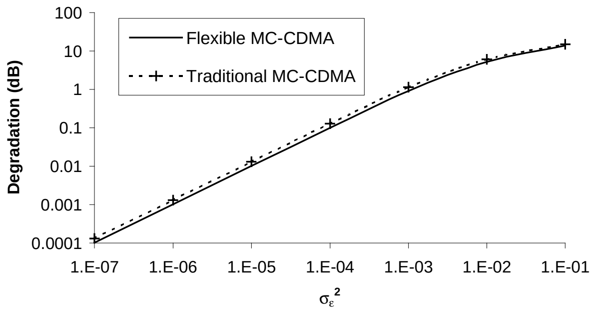

Figure 6 : Timing jitter, $\alpha=0.1$, $E_s/N_o=20dB$, $N_{fft}=32$

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