

# An Overview of MC-CDMA Synchronisation Sensitivity

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Abstract: This paper presents an overview of the effect of synchronisation errors on MC-CDMA performance in downlink communications. We distinguish two types of synchronisation errors: carrier phase errors and timing errors. We show that the MC-CDMA system is very sensitive to a carrier frequency offset or a clock frequency offset. For a maximal load, carrier phase jitter and timing jitter give rise to a degradation that is independent of the spectral content of the jitter; moreover, the degradation caused by carrier phase jitter and timing jitter is (essentially) independent of the number of carriers. A constant carrier phase offset and a constant timing offset cause no degradation of the MC-CDMA system performance.

## 1. INTRODUCTION

The enormous growth of interest for multicarrier (MC) systems can be ascribed to its high bandwidth efficiency and its immunity to channel dispersion. Recently, different combinations of orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA) have been investigated in the context of high data rate communication over dispersive channels [1]-[9]. One of these systems is multicarrier CDMA (MC-CDMA), which has been proposed for downlink communication in mobile radio. In MC-CDMA the data symbols are multiplied with a higher rate chip sequence and then modulated on orthogonal carriers.

This paper presents an overview of the effect of synchronisation errors on MC-CDMA performance in downlink communications. We can distinguish mainly two levels of synchronisation: carrier synchronisation and timing recovery. In carrier synchronisation, a local reference carrier with a phase and frequency as closely matching to that of the carrier used for upconverting the transmitted signal, must be generated for the downconversion of the signal to a baseband signal. The effect of the carrier phase errors, caused by the error between the carrier used for the upconversion of the data signal and the local reference carrier has been investigated in [10]-[15]. The effect of a frequency offset was studied in [10], [12]-[14], while the sensitivity of MC-CDMA to carrier phase jitter was described in [11], [13]-[14].

The next problem is the recovery of the timing instants, as the sampling clock oscillator of the receiver has a phase and frequency drift against that of the transmitter. The influence of the timing errors, made in the process of extracting the sampling instants, was studied in [13]-[15]. The sensitivity of MC-CDMA to a clock frequency offset between the transmitter clock and the receiver sampling clock and the effect of timing jitter resulting from a phase-locked sampling clock have been studied in [13]-[14].

## 2. SYSTEM DESCRIPTION

In this paper, we consider the MC-CDMA system that is shown in figure 1 for one user. The data symbols  $\{a_{i,m}\}$ , transmitted at a rate  $R_s$ , where  $a_{i,m}$  denotes the  $i$ -th symbol belonging to user  $m$ , is first multiplied by a higher rate chip sequence of length  $N$ ,  $\{c_{n,m}|n=0,\dots,N-1\}$ ,  $c_{n,m}$  denoting the  $n$ -th chip of the sequence belonging to user  $m$ . Sequences belonging to different users are assumed to be orthogonal. The resulting samples are mapped on the orthogonal carriers and modulated using the inverse fast Fourier transform (IFFT). We insert a guard interval, consisting of a cyclic prefix of the transmitted samples, to avoid interframe interference. The resulting samples are applied to a transmit filter, which is a unit-energy square-root Nyquist filter (e.g. a cosine rolloff filter with rolloff  $\alpha$ ) and transmitted over a (possibly dispersive) channel. The channel output signal is disturbed by additive white Gaussian noise (AWGN) with power spectral density  $N_o$  and affected by a carrier phase error. The resulting signal is then fed to the receiver filter, which is matched to the transmit filter and sampled at the instants  $t_{i,k}=kT+i(N+v)T+\epsilon_{i,k}T$ , where  $\epsilon_{i,k}$  is the normalised timing error at the  $k$ -th instant of the  $i$ -th transmitted frame.

When the phase error is slowly varying as compared to  $T=1/((N+v)R_s)$ , it was shown in [13] that the synchronisation errors can be included in an

equivalent time-varying impulse response with Fourier transform  $H_{eq}(f; t_{i,k}) = H(f) e^{j\phi(t_{i,k})} e^{j2\pi f \epsilon_{i,k} T}$ , where  $\phi(t_{i,k})$  and  $\epsilon_{i,k}$  are respectively the carrier phase error and the timing error at the instant  $t_{i,k}$  and  $H(f)$  consists of the cascade of the transmit filter, the channel and the receiver filter. We assume that the duration of the equivalent time-varying impulse response does not exceed the duration of the guard interval. We select the  $N$  samples outside the guard interval for further processing and demodulate the signal using the FFT. Then each FFT output is multiplied with the corresponding chip of the considered user and applied to a one-tap MMSE equaliser. The outputs of the equalisers are summed to obtain the samples at the input of the decision device. The signal-to-noise ratio (SNR) is defined as the ratio of the useful power to the sum of the interference power and the noise power. In the case of an ideal channel and in the absence of synchronisation errors, the SNR yields  $E_s/N_0$ , where  $E_s$  is the energy per symbol transmitted to each user and  $N_0$  is the noise power spectral density. The SNR will degrade in the presence of synchronisation errors. We define the degradation as  $Deg = 10\log(E_s/N_0) - 10\log(SNR)$ .

In the following, we consider the case of downstream communication. As in downstream communication, the signals sent to the different users are synchronised at the basestation, the timing errors are the same. In addition, as all transmitted carriers are generated by the same oscillator, they exhibit the same carrier phase errors. To clearly isolate the effect of the synchronisation errors, we consider the case of an ideal channel.

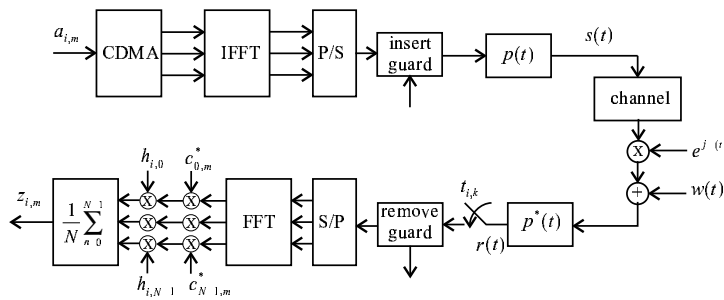


Figure 1. Conceptual block diagram of the MC-CDMA system for one user

### 3. CARRIER PHASE ERRORS

#### 3.1 Constant Phase Error

In the case of a constant phase offset, the orthogonality between the different users is not affected, i.e. no multiuser interference (MUI) is introduced, as a constant phase offset only causes a carrier independent phase rotation of the FFT outputs. This means that, when no correction was applied by the equaliser, the samples at the input of the decision device are rotated over an angle  $\phi$ . To avoid the reduction of the noise margins, this phase rotation of the useful component is corrected by the equaliser: the equaliser rotates the FFT outputs over (an estimate of) the angle  $-\phi$ , i.e.  $h_{i,k}=e^{-j\phi}$ . As a phase rotation of the FFT outputs has no influence on the noise power level, this constant phase offset is compensated by the equaliser without loss of performance.

#### 3.2 Carrier Frequency Offset

If the downconversion of the signal is performed by means of a free running local oscillator, a carrier frequency offset can occur. The carrier frequency offset  $\Delta F$  causes a shift of the frequency band of the transmitted signal. When we focus on the  $n$ -th transmitted carrier, we observe in figure 2 that the frequency shift of this transmitted carrier gives rise to an attenuation of the  $n$ -th observed carrier, thus an attenuation of the useful component. Furthermore, all other observed carriers are disturbed by a non-zero interference caused by the  $n$ -th transmitted carrier. In [11] it is shown that a one-tap equaliser is not able to eliminate this MUI, i.e. a carrier frequency offset will introduce a performance degradation that depends on the product of the number of carriers  $N$  and the frequency offset  $\Delta F$ . In figure 3, this degradation is shown for the maximum load (i.e. the number of users equals  $N$ ). We observe a high sensitivity of the MC-CDMA system to the carrier frequency offset. From figure 2 it follows that a frequency offset equal to the carrier spacing ( $\Delta F=1/NT$ ) gives rise to a severe performance degradation, as the spread data  $\{a_0c_n\}$  is shifted over one carrier and is not correlated with the corresponding chips. Therefore, the frequency offset must be limited, i.e.  $\Delta F \ll 1/N$ . The carrier frequency offset therefore must be compensated before demodulation, i.e. in front of the FFT.

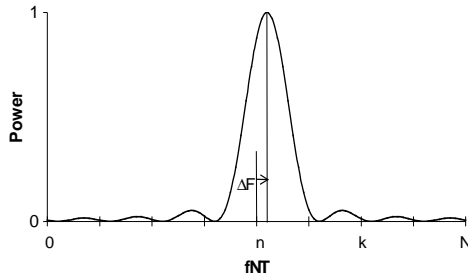


Figure 2. Contribution of the n-th transmitted carrier to the k-th FFT output ( $N=8$ ,  $\Delta FT=0.1$ )

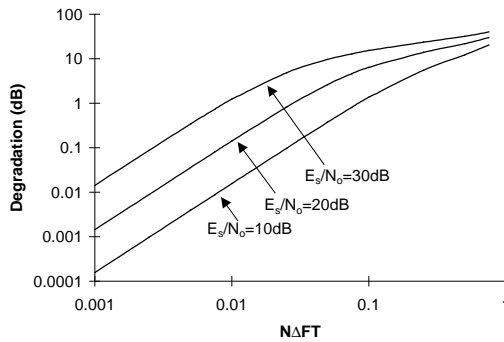


Figure 3. Influence of carrier frequency offset

### 3.3 Carrier Phase Jitter

To avoid the strong degradation caused by the carrier frequency offset, a phase-locked local oscillator can be used for downconverting the RF signal. The phase error resulting from this PLL can be modelled as a zero-mean stationary random process with jitter spectrum  $S_{\phi}(f)$  and jitter variance  $\sigma_{\phi}^2$ . Assuming slowly varying phase errors and small jitter variances, the equaliser coefficients are essentially the same as in the absence of carrier phase jitter. In [12] it is shown that the fluctuation of the useful component, caused by the random character of the jitter, mainly consists of the low frequency components ( $<1/NT$ ) of the jitter, while the MUI is mainly determined by the high frequency components ( $>1/NT$ ) of the jitter. For the maximum load, the degradation becomes independent of the spectral contents of the jitter and of the number of carriers. The degradation, which in

this case only depends on the jitter variance, is shown in figure 4. The scatter diagrams however, will differ considerably depending of the spectral contents of the jitter. Jitter with mainly low frequency components ( $<1/NT$ ) gives rise to a random rotation of the useful component and will show an angular displacement in the scatter diagram (figure 5a). Phase jitter with mainly high frequency components ( $>1/NT$ ) will introduce MUI which causes a circular cloud (figure 5b), as the term of the MUI, which consists of a large number of statistically independent contributions, has uncorrelated real and imaginary parts.

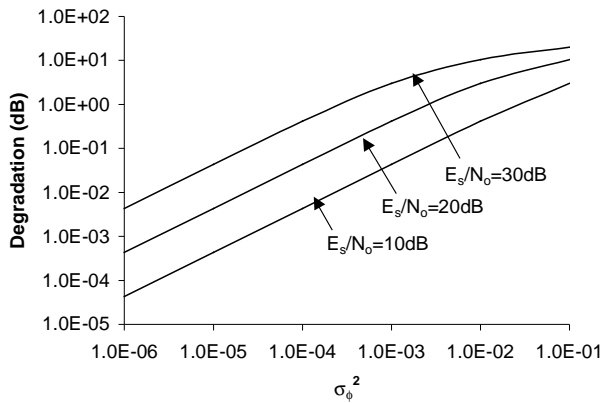


Figure 4. Influence of carrier phase jitter

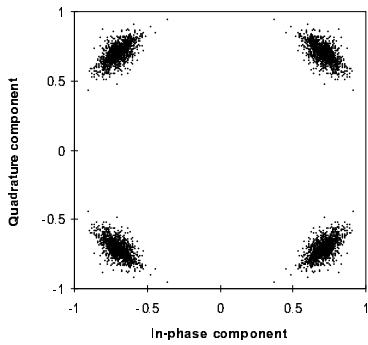


Figure 5a. Scatter diagram for jitter with mainly low frequency components, N=128

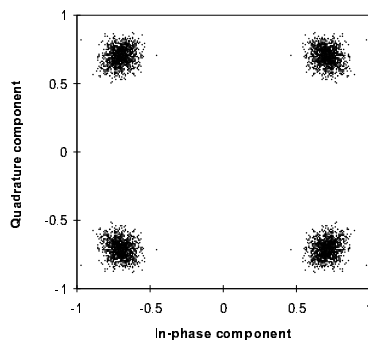


Figure 5b. Scatter diagram for jitter with mainly high frequency components, N=128

## 4. TIMING ERRORS

### 4.1 Constant Timing Offset

In the case of a constant timing offset, the coefficients of the spread data at the outputs of the FFT are affected as shown in figure 6. For carriers outside the rolloff area, the constant timing offset has no influence on the amplitude of the coefficient, but only introduces a phase rotation proportional to the carrier index. For carriers inside the rolloff area, the coefficients are rotated over some angle and attenuated as compared to the coefficients of the carriers outside the rolloff area. The equaliser attempts to compensate for the attenuation, caused by the carriers inside the rolloff area and the rotation. However, scaling the FFT outputs affects the noise power level. The MMSE filter therefore makes a compromise between the MUI caused by the carriers inside the rolloff area and the increase of the noise power caused by the scaling. It is clear that the sensitivity of MC-CDMA to the constant timing offset can be eliminated by not using the carriers inside the rolloff area.

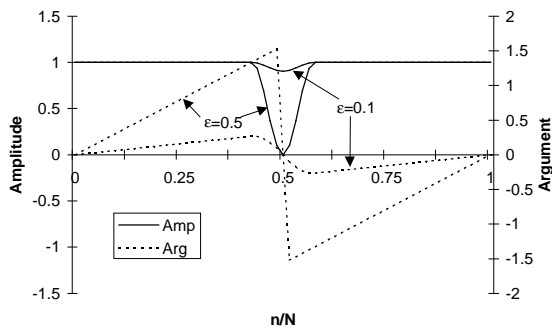


Figure 6. Influence of constant timing error on outputs FFT

### 4.2 Clock Frequency Offset

When sampling is performed by means of a free-running local oscillator, a clock frequency offset  $\Delta T/T$  can occur. This clock frequency offset engenders compression ( $\Delta T/T > 0$ ) or expansion ( $\Delta T/T < 0$ ) of the observed frequencies at the output of the FFT, giving rise to a carrier dependent

frequency shift as compared to the transmitted carriers (figure 7). When we focus on the  $n$ -th transmitted carrier, we observe an attenuation of the amplitude of the  $n$ -th observed carrier, caused by the carrier dependent frequency shift. All other observed carriers are disturbed by non-zero interference caused by the  $n$ -th transmitted carrier. Therefore, a clock frequency offset results in an attenuation of the useful component and MUI. It was shown in [13] that a one-tap equaliser is not able to eliminate this MUI, so the clock frequency offset gives rise to performance degradation. This degradation depends on the product of the number of carriers  $N$  and the clock frequency offset  $\Delta T/T$ . From figure 8, where the degradation is shown for the maximum load and  $\alpha=0$ , we observe that the MC-CDMA system is very sensitive to a clock frequency offset. To obtain small degradations, the clock frequency offset must be limited, i.e.  $\Delta T/T \ll 1/N$ .

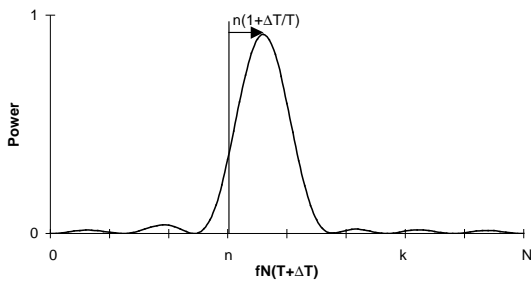


Figure 7. Contribution of the  $n$ -th transmitted carrier to the  $k$ -th FFT output ( $N=8$ ,  $\Delta T/T=0.2$ )

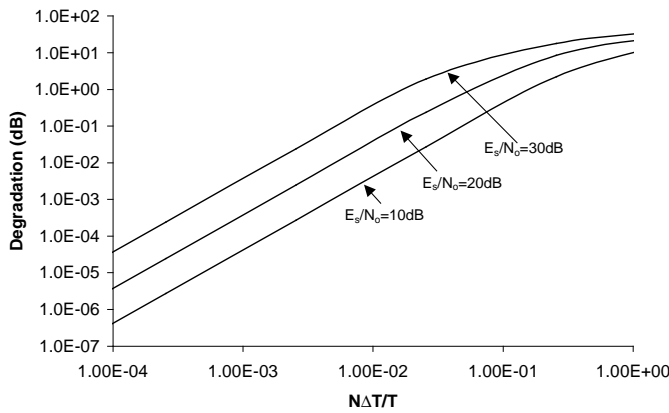


Figure 8. Influence of clock frequency offset,  $\alpha=0$



### 4.3 Timing Jitter

In order to get rid of the constant timing error and the clock frequency offset we can perform synchronised sampling, e.g. by means of a phase-locked sampling clock. The timing error resulting from this PLL can be modelled as a zero-mean stationary process with jitter spectrum  $S_e(f)$  and jitter variance  $\sigma_e^2$ . Assuming slowly varying timing errors and small jitter variances, the equaliser coefficients are essentially the same as in the absence of timing jitter. In [13] it is shown that, for the maximum load,  $\alpha=0$  and for large  $N$  ( $N \rightarrow \infty$ ), the sum of the powers of the fluctuation of the useful component, caused by the random character of the jitter, and the MUI is essentially independent of the number of carriers. Furthermore, this degradation, which is mainly caused by the MUI [13], is independent of the spectral contents of the jitter but only depends on the jitter variance. In figure 9, this degradation is shown as function of the jitter variance.

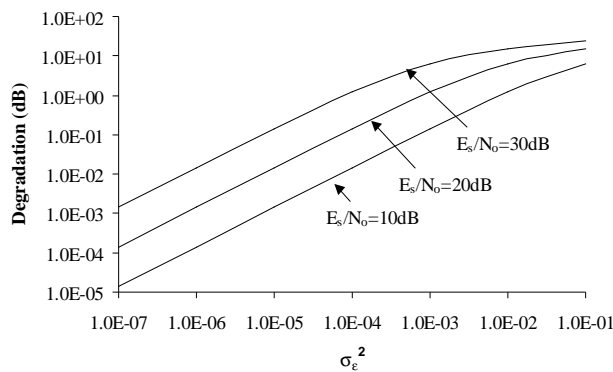


Figure 9. Influence of timing jitter,  $\alpha=0$

## 5. CONCLUSIONS

In this contribution, we have presented an overview of the effect of synchronisation errors on the performance of a MC-CDMA system. A constant phase offset and a constant timing offset can be compensated without loss of performance while the MC-CDMA performance degrades for time-varying timing and carrier phase errors. In the case of a carrier frequency offset or a clock frequency offset, the MC-CDMA performance rapidly degrades and strongly depends on the number of carriers. Stationary carrier phase jitter and timing jitter introduce a degradation that, for the

maximum load, is independent of the number of carriers and the spectral contents of the jitter, but only depends on the jitter variance.

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