Sensitivity of Multicarrier Systems to Synchronization Errors

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ABSTRACT

In this contribution, we give an overview of the sensitivity of a number of multicarrier systems to carrier and symbol synchronization errors. A comparison is made between orthogonal frequency-division multiplexing/multiple access (OFDM(A)) and two combinations of the orthogonal multicarrier (MC) transmission technique with the code-division multiple access technique (CDMA), i.e. multicarrier CDMA (MC-CDMA), where the spreading is accomplished in the frequency domain, and multicarrier direct-sequence CDMA (MC-DS-CDMA), where the spreading is done in the time domain.

1. INTRODUCTION

During the last decade, we have witnessed a widespread deployment of digital communication services requiring an exchange of digital information at constantly increasing data rates (e.g., audio and video conferencing, internet applications, digital television,...). To satisfy this increasing demand for higher data rates, the data rates over the existing transmission media must be enhanced. Particularly multicarrier (MC) systems have received considerable attention in the context of high data rate communications, as they combine a high spectral efficiency with an immunity to channel dispersion [1]. One of the MC systems is the well-studied orthogonal frequency-division multiplexing (OFDM) system. The conventional OFDM system has been proposed and/or accepted for various applications such as transmission over twisted pair cables (xDSL) [2], broadcasting of digital audio (DAB) and digital television (DTTB) [3], or mobile radio [4]. Orthogonal frequency-division multiple access (OFDMA) is closely related to OFDM. However, in contrast with OFDM, where all carriers are modulated by the same user, in OFDMA the different carriers are modulated by different users. OFDMA has been proposed for the return path of the CATV (cable area TV) network [5]. Recently, different combinations of the MC technique and the code-division multiple access technique (CDMA) have been proposed [6]. Two of these combinations that make use of carriers satisfying the orthogonality constraint with minimum frequency separation are multicarrier CDMA (MC-CDMA) and multicarrier direct-sequence CDMA (MC-DS-CDMA). In the MC-CDMA technique, the spreading is done in the frequency domain [7], whereas in the MC-DS-CDMA technique, the spreading is accomplished in the time domain [8]. Both MC-CDMA and MC-DS-CDMA have been considered for mobile radio communications.

The transmitter of a digital communication system contains a clock, that indicates the timing instants at which the data symbols must be transmitted. Furthermore, the transmitter contains a carrier oscillator, necessary for the upconversion of the data carrying baseband signal to the bandpass signal to be transmitted. At the receiver, the received bandpass signal is downconverted using a local carrier oscillator. The resulting signal is sampled at timing instants determined by the receiver clock. Based on the resulting samples, a decision is taken about the transmitted data symbols. Hence, the receiver must make an estimation of the frequencies and phases of the carrier oscillator and the clock used at the transmitter, based upon the received signal. Because of interference, noise and other disturbances, these estimations are not perfect, resulting in carrier and symbol synchronization errors.

In the literature, it has been reported that multicarrier systems are very sensitive to some types of carrier and symbol synchronization errors when a large number of carriers is used. The effect of different types of carrier synchronization errors has been investigated in [9]-[17] for OFDM(A), MC-CDMA and MC-DS-CDMA, respectively. Especially a carrier frequency offset is shown to be detrimental for multicarrier systems [9]-[13]. The effect of carrier phase jitter, which results from a synchronization algorithm that is used to eliminate this carrier frequency offset, is investigated in [10]-[11], [14]-[15]. The impact of symbol synchronization errors is studied in [10]-[11], [16]-[17]. It is shown in [10]-[11], [16] that multicarrier systems are very sensitive to a clock frequency offset. The effect of timing jitter, which results from a synchronization algorithm that is used to eliminate this clock frequency offset, is studied in [10]-[11], [17].

In this contribution, we give an overview of the sensitivity of the different multicarrier systems on synchronization errors. To allow for a fair comparison between the different multicarrier systems, we assume that all considered systems are able to accommodate up to N users, each user operating at a data rate R_s.

2. MULTICARRIER SYSTEMS

In an orthogonal multicarrier system, the modulation of the (spread) data on the carriers is accomplished by an inverse fast Fourier transform (inverse FFT) of length N_c. In order to avoid that channel dispersion causes interference between data symbols at the receiver, each FFT block is cyclically extended with a prefix of N_p samples. Hence, each MC block consists of N_c+N_p samples, that are transmitted at a rate 1/T. At the receiver, the MC signal is sampled at a rate 1/T. The receiver removes from each block of N_c+N_p samples the N_p samples corresponding with the cyclic prefix, and demodulates the remaining N_c samples with an N_c-points FFT.

2.1 OFDM

In OFDM, the data symbols to be transmitted at a rate R_s are organized into blocks of N_c data symbols; $a_{i,n}$ denotes the n^{th} data symbol of the i^{th} block. The N_c data symbols $a_{i,n}$ are transmitted in parallel on the N_c carriers of the MC system. Hence, all carriers of the OFDM system are modulated with data from the same user. To support multiple users, we combine the OFDM technique with the time-division multiple access (TDMA) scheme. In this case, the time axis is partitioned into a number of non-overlapping time slots, as shown in figure 1. The time slots are grouped into frames of N time slots. During each frame, a user is assigned a time slot. Each time slot consists of a burst of OFDM blocks, during which N_c data symbols per OFDM block are transmitted. This indicates that in the OFDM system, the number of carriers N_c can be selected independently of the maximum number of users N.

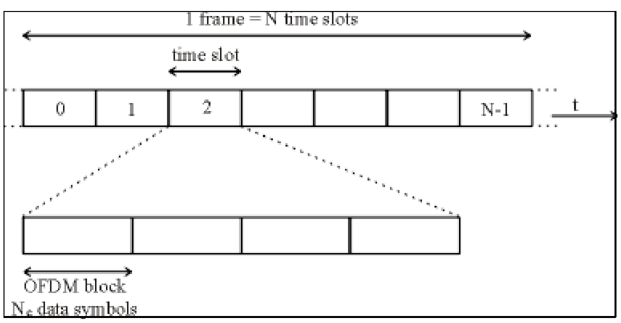

Figure 1. OFDM system.

2.2 OFDMA

OFDMA is closely related to OFDM. However, in OFDMA, the data streams that are transmitted on the different carriers belong to different users. Denote $a_{i,n}$ as the i^{th} data symbol transmitted at a rate R_s by the n^{th} user. The data symbols $a_{i,n}$ belonging to the different users are transmitted in parallel on the N_c carriers of the MC system. Hence, during one OFDM block, one data symbol per user is transmitted. This indicates that, as each user is assigned a different carrier, the number of carriers N_c equals the maximum number of users N.

2.3 MC-CDMA

In MC-CDMA, the symbol a_k to be transmitted is multiplied with a chip sequence $\{c_{kN_s+n}|n=0,...,N_s-1\}$ with spreading factor N_s. The resulting N_s components $\{a_k c_{kN_s+n}|n=0,...,N_s-1\}$ are transmitted in parallel on the $N_s(=N_c)$ carriers of the MC system. Hence, spreading is accomplished in the frequency domain. The N_s components corresponding to the same symbol index k are located in the same timeslot on different carriers; each component $a_k c_{kN_s+n}$ has a duration of $1/R_s$. In a multi-user scenario, each user transmits a similar signal, but with the data symbols and the chip sequences depending on the user index. Assuming orthogonal chip sequences, the maximum number of sequences of length N_s, hence the maximum number of users, equals N_s. This indicates that the number of carriers equals the spreading factor, which in turn equals the maximum number of users.

2.4 MC-DS-CDMA

In MC-DS-CDMA, the symbol sequence to be transmitted is split into N_c lower rate symbol sequences $\{a_{k,m}\}$, with m = 0, ..., N_c-1 denoting the carrier index. The symbol $a_{k,m}$ is multiplied with a chip sequence $\{c_{kN_s+n}|n=0,...,N_s-1\}$ with spreading factor N_s. The resulting N_s components $\{a_{k,m} c_{kN_s+n}|n=0,...,N_s-1\}$ are serially transmitted on the m^{th} carrier of the MC system. Hence, spreading is accomplished in the time domain. The N_s components corresponding to the same symbol index k are located in successive timeslots on the m^{th} carrier; each component $a_{k,m} c_{kN_s+n}$ has a duration of $(N_c/N_s)/R_s$. In a multi-user scenario, each user transmits a similar signal, but with the data symbols and the chip sequences depending on the user index. Assuming orthogonal chip sequences, the maximum number of users that do not create interference equals N_s. Note that the number of carriers N_c can be selected independently of the spreading factor N_s, which in turn equals the maximum number of users.

Table 1 shows an overview of the system parameters of OFDM(A), MC-CDMA and MC-DS-CDMA. For given N_c, N_p, N and R_s, the system bandwidth B is equal for all considered multicarrier systems. For $N_p<<N_c$, the system bandwidth can be approximated by $B\approx NR_s$. Hence, the system bandwidth only depends on the number of users N and the data rate R_s. For all considered MC systems, the sampling rate 1/T depends on the ratio N/N_c. For OFDMA and MC-CDMA, the number of carriers N_c equals the number of users N, whereas for OFDM and MC-DS-CDMA, the number of carriers N_c can be chosen independently of the number of users N. Hence, depending on the ratio N/N_c, the sampling rate for OFDM and MC-DS-

CDMA is larger (smaller) than that for OFDM and MC-DS-CDMA, when $N/N_c>1$ ($N/N_c<1$).

Table 1. Overview of system parameters.

	Bandwidth B	Sampling rate 1/T
OFDM	$(1+N_p/N_c)NR_s$	$(N_c+N_p)NR_s/N_c$
OFDMA	$(1+N_p/N_c)NR_s$	$(N_c+N_p)R_s$
MC-CDMA	$(1+N_p/N_c)NR_s$	$(N_c+N_p)R_s$
MC-DS-CDMA	$(1+N_p/N_c)NR_s$	$(N_c+N_p)NR_s/N_c$

3. SENSITIVITY TO SYNCHRONIZATION ERRORS

In this section, we consider the effect of carrier and symbol synchronization errors on the performance of the considered multicarrier systems. For all systems, we compare the degradation, caused by the synchronization errors, of the signal-to-noise ratio (SNR) at the input of the decision device, under the assumption of the full load (i.e. the number of users equals N) and an ideal channel. Further, we consider the case of downlink transmission, i.e. all users and all carriers exhibit the same synchronization errors. We separately consider the effect of a constant carrier phase and timing offset, a carrier and clock frequency offset, and carrier phase and timing jitter.

- A constant carrier phase offset ϕ between the carrier oscillators at the transmitter and the receiver introduces, for all multicarrier systems, a phase rotation over an angle ϕ of the samples at the input of the decision device, when no equalizer is used. This systematic phase rotation of the samples at the input of the decision device can be compensated without enhancing the noise power level. Hence, for any of the multicarrier systems, a constant carrier phase offset introduces no performance degradation.

- A carrier frequency offset ΔF causes, for all considered multicarrier systems, a rotation at a constant speed of $2\pi\Delta F$ rad/s of the samples at the input of the FFT. This rotation of the FFT input samples gives rise to a reduction of the useful component and severe interference at the FFT outputs. Further, the FFT outputs are rotating at a constant speed of $2\pi(N_c+N_p)\Delta FT$ rad/block. Hence, the multicarrier systems are degraded in the presence of a carrier frequency offset. This degradation, for all MC systems, is a strongly increasing function of $(N_c/N)(\Delta F/R_s)$. When $N_c=N$, the degradations for all multicarrier systems are the same. Hence, the ratio N_c/N determines which of the considered systems is more sensitive to a carrier frequency offset: when this ratio is larger (smaller) than 1, OFDM and MC-DS-CDMA are more (less) sensitive than OFDMA and MC-CDMA (for which yields $N_c=N$). Figure 2 shows the degradation for the different multicarrier systems caused by a carrier frequency offset.

- Carrier phase jitter causes, for all considered multicarrier systems, a degradation that is independent of the spectral contents of the jitter, the number of carriers and the number of users. The degradation only depends on the jitter variance σ_ϕ^2 and is the same for all multicarrier systems. The performance degradation caused by carrier phase jitter is shown in figure 3.

- A constant timing offset εT between the optimum timing instants and the estimated timing instants introduces, for all multicarrier systems, only a carrier dependent phase rotation of the FFT outputs at the receiver. As this phase rotation can be compensated without enhancement of the noise power level, a constant timing offset causes no performance degradation.

- A clock frequency offset $\Delta T/T$ between the transmitter and the receiver sampling clocks causes gives rise to a timing error that linearly increases in time. This clock frequency offset causes a reduction of the useful component and interference at the FFT outputs. Hence, the multicarrier systems are degraded by a clock frequency offset. In OFDM, OFDMA and MC-DS-CDMA, the degradation depends on the carrier index. For all multicarrier systems, the degradation strongly increases with $N_c\Delta T/T$. For $N_c=N$, the average (over the FFT outputs) degradation for OFDM(A) and MC-DS-CDMA is the same as the degradation for MC-CDMA. For the systems where the number of carriers N_c can be chosen independently of the number of users N (i.e. OFDM and MC-DS-CDMA), the average degradation is larger (smaller) than for OFDMA and MC-CDMA (where $N_c=N$) when the number of carriers is larger (smaller) than the number of users. The average degradation caused by a clock frequency offset is shown in figure 4.

- For all considered multicarrier systems, timing jitter gives rise to a degradation that is independent of the spectral contents of the jitter, the number of carriers and the number of users. In OFDM(A) and MC-DS-CDMA, the timing jitter introduces a degradation that depends on the carrier index. The average (over the FFT outputs) degradation only depends on the jitter variance σ_ε^2 and is the same for all multicarrier systems. Figure 5 shows the average degradation caused by timing jitter.

4. CONCLUSIONS

In this contribution, we have compared a number of multicarrier systems regarding their sensitivity to synchronization errors. The results can be summarized as follows:

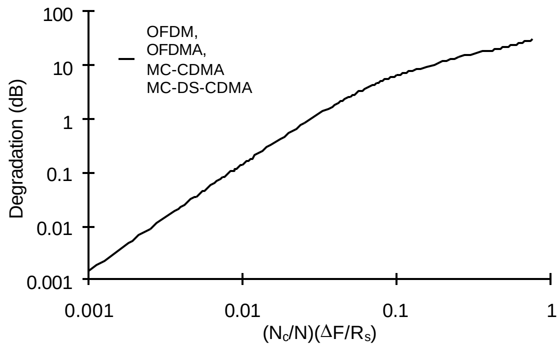

Figure 2. Degradation caused by carrier frequency offset, N=64.

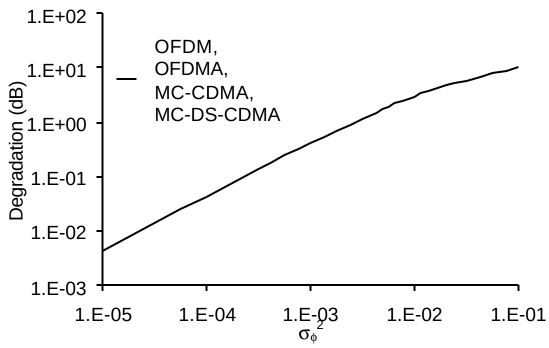

Figure 3. Degradation caused by carrier phase jitter.

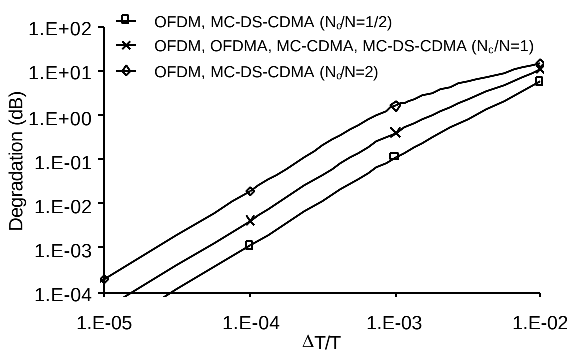

Figure 4. Degradation caused by clock frequency offset, N=64.

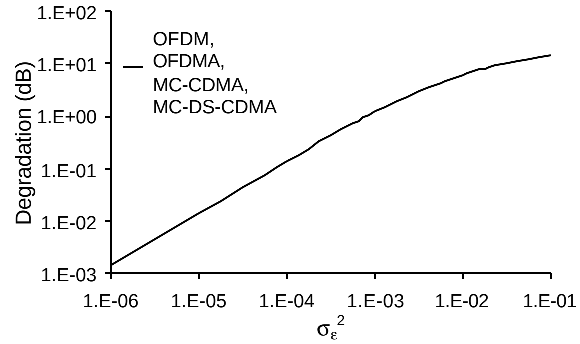

Figure 5. Degradation caused by timing jitter.

- None of the multicarrier systems is degraded by a constant phase offset or a constant timing offset
- All MC systems exhibit the same sensitivity to a carrier frequency offset. The degradation is an increasing function of $(N_c/N)(\Delta F/R_s)$.
- Multicarrier systems are very sensitive to a clock frequency offset. The degradations of OFDM(A) and MC-DS-CDMA depend on the carrier index. For OFDM and MC-DS-CDMA, the degradation is an increasing function of N_c/N. For $N_c/N=1$, the average (over the FFT outputs) degradation of OFDM(A) and MC-DS-CDMA is the same as the degradation of MC-CDMA.
- All MC systems exhibit the same sensitivity to carrier phase jitter. The degradation is independent of the spectral contents of the jitter, the number of carriers and the number of users, but only depends on the jitter variance.
- Timing jitter introduces a degradation that is essentially the same for all MC systems. The degradation is independent of the spectral contents of the jitter, the number of carriers and the number of users, but only depends on the jitter variance.

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