# UPLINK AND DOWNLINK MC-DS-CDMA SENSITIVITY TO STATIC CLOCK FREQUENCY OFFSETS

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Abstract - We study the effect of fixed clock frequency offsets on the performance of multicarrier direct-sequence CDMA (MC-DS-CDMA) for both uplink and downlink communication, assuming orthogonal spreading sequences. We show that for both uplink and downlink MC-DS-CDMA, the performance in the presence of a clock frequency offset rapidly degrades with an increasing number of carriers. It turns out that this degradation is larger in the uplink than in the downlink, because the former suffers from a higher level of multiuser interference. For a given maximum clock frequency offset, enlarging the spreading factor in a fully loaded system does not affect the downlink degradation, but strongly increases the uplink degradation.

KEY WORDS: synchronization, MC-DS-CDMA, clock frequency offset

# I. INTRODUCTION

Because of the enormous growth of wireless services (cellular telephones, wireless LAN's, ...) during the last decade, the need of a modulation technique that can reliably transmit high data rates at a high bandwidth efficiency arises. As multicarrier (MC) systems have good bandwidth efficiency and can offer an immunity to channel dispersion, these techniques are excellent candidates for high data rate transmission over multipath channels [1]-[3]. Recently, the multicarrier modulation technique has been investigated in combination with the code-division multiple access (CDMA) technique [4]. One of these combinations is the multicarrier direct-sequence CDMA (MC-DS-CDMA) technique, which has been proposed for mobile radio communications [5]-[8]. In MC-DS-CDMA, the serial-to-parallel converted data stream is multiplied with the spreading sequence and then the chips belonging to the same symbol modulate the same carrier: the spreading is done in the time-domain.

The use of a large number of carriers makes multicarrier systems very sensitive to clock frequency offsets between the transmitter and the receiver clock. The effect of clock frequency offsets on multicarrier systems has been reported in [9]-[10] for orthogonal frequency-division multiplexing (OFDM) and in [11] for multicarrier CDMA (MC-CDMA) (which combines multicarrier modulation with frequency-domain spreading).

## II. SYSTEM DESCRIPTION

## II.1. Uplink MC-DS-CDMA

The conceptual block diagram of the transmitter of an MC-DS-CDMA system for a single user is shown in figure 1. In MC-DS-CDMA, the complex data symbols to be transmitted at rate  $R_s$ , are first split into  $N_c$  symbol sequences at rate  $R_s/N_c$ . Each of these lower rate symbol sequences modulates a different carrier of the orthogonal multicarrier

system. We denote by  $a_{i,k,\ell}$  the data symbol transmitted by user  $\ell$  on carrier k during the *i*<sup>th</sup> symbol interval; k belongs to a set  $I_c$  of  $N_c$  carrier indices. The data symbol  $a_{i,k,\ell}$  is then multiplied with a higher rate spreading sequence  $\{c_{i,n,\ell}|n=0,...,N_s-1\}$  with spreading factor  $N_s$ , where  $c_{i,n,\ell}$  denotes the  $n^{\text{th}}$  chip of the sequence that spreads the data symbols from user  $\hat{\ell}$  during the  $\hat{i}^{th}$  symbol interval. Note that the spreading sequence does not depend on the carrier index k: all  $N_c$  data symbols from user  $\ell$  that are transmitted during the same symbol interval of duration  $N_c/R_s$  are spread with the same spreading sequence. It is assumed that  $|c_{i,n,\ell}| = 1$ . We denote by  $\{b_{i,n,k,\ell} | n=0,...,N_s-1\}$  the  $N_s$  components of the spread data symbol  $a_{i,k,\ell}$ , i.e.  $b_{i,n,k,\ell} = a_{i,k,\ell}c_{i,n,\ell}/sqrt(N_s)$ . The components  $b_{i,n,k,\ell}$  are serially transmitted on the  $k^{\text{th}}$  carrier of an orthogonal multicarrier system, i.e., the spreading is done in the time-domain. Each component  $b_{i,n,k,\ell}$  has a duration  $(N_c/N_s)/R_s$ . To modulate the spread data symbols on the orthogonal carriers, an NF-point inverse fast Fourier transform (inverse FFT) is used. To avoid that the multipath channel causes interference between the data symbols at the receiver, each FFT block at the inverse FFT output is cyclically extended with a prefix of  $N_p$  samples. This results in the sequence of samples  $\{s_{i,n,m,\ell} | m = -N_p, ..., N_F - I\}$ , given by

$$s_{i,n,m,\ell} = \frac{1}{\sqrt{N_F + N_p}} \sum_{k \in I_c} b_{i,n,k,\ell} e^{j2\pi \frac{km}{N_F}}$$
(1)

The sequence  $\{s_{i,n,m,\ell} | m=-N_{p},...,N_{F}-I\}$  is fed to a square-root raised-cosine filter P(f) with rolloff  $\alpha$  and unit-energy impulse response p(t). The resulting continuous-time transmitted complex baseband signal  $s_{\ell}(t)$  is given by

$$s_{\ell}(t) = \sum_{i=-\infty}^{+\infty} \sum_{m=-N_{p}}^{N_{F}-1} \sum_{n=0}^{N_{s}-1} s_{i,n,m,\ell}.$$

$$p(t - (m + (n + iN_{s})(N_{F} + N_{p})) T - \tau_{i,n,m,\ell})$$
(2)

where  $1/T = (N_F + N_p)N_sR_s/N_c$  is the network reference clock frequency and  $\tau_{i,n,m,\ell}$  is a time-varying delay representing the transmit clock phase of user  $\ell$ . Because of the normalization factors introduced in (1) and (2), the transmitted energy per symbol on the  $k^{\text{th}}$  carrier from user  $\ell$  is given by  $E_{sk,\ell} = E[|a_{i,k,\ell}|^2]$ . In the following, it is assumed that carriers inside the rolloff area of the transmit filter are not modulated, i.e., they have zero amplitude. Hence, of the  $N_F$  available carriers, only  $N_c$  carriers are actually used  $(N_c \leq (1-\alpha)N_F)$ . Assuming  $N_c$  to be odd, the set  $I_c$  of carriers actually used is given by  $I_c = \{0, ..., N_c/2-1\} \cup \{N_F - (N_c/2-1), ..., N_F - 1\}$ . The corresponding carrier spacing  $\Delta f$  and system bandwidth B are given by  $\Delta f = 1/(N_FT) = (N_s/N_c)R_s(1+N_p/N_F) \approx (N_s/N_c)R_s$  and  $B = N_c \Delta f = N_c / (N_F T) = N_s R_s (1 + N_p / N_F) \approx N_s R_s$ . The above approximations are valid for  $N_p << N_F$ .

In a multiuser scenario, each user transmits to the basestation a similar signal  $s_{\ell}(t)$ . To separate the different user signals at the receiver, each user is assigned a unique spreading sequence  $\{c_{in,\ell}\}$ , with  $\ell$  denoting the user index. In this contribution, we consider orthogonal sequences, consisting of user-dependent Walsh-Hadamard (WH) sequences of length  $N_s$ , multiplied with a complex-valued random scrambling sequence that is common to all  $N_u$  active users. Hence, the maximum number of users that can be accommodated equals  $N_s$ , i.e. the number of WH sequences of length  $N_s$ . Note that the number of carriers  $N_c$  can be chosen independently of the spreading factor  $N_s$ , which in turn equals the maximum number of users. Without loss of generality, we focus on the detection of the data symbols transmitted by the reference user ( $\ell=0$ ).

The signal  $s_{\ell}(t)$  transmitted by user  $\ell$  reaches the basestation through a multipath channel with transfer function  $H_{ch\ell}(f)$  that depends on the user index  $\ell$ . The basestation receives the sum of the resulting user signals and an additive white Gaussian noise (AWGN) process w(t). The real and imaginary parts of w(t) are uncorrelated, and each have a power spectral density of  $N_0/2$ . The resulting signal is applied to the receiver filter, which is matched to the transmit filter, and sampled at the instants  $t_{i,n,m} = (m + (n + iN_s)(N_F + N_p))T$  (see figure 2). Only the  $N_F$  samples with  $m = 0, ..., N_F - I$  are kept for further processing.

In uplink communication, a timing misalignment of the FFT blocks transmitted by the different users is present. In the following, it is assumed that the transmitter of each user adapts its transmit clock phase  $\tau_{i,n,m,\ell}$ , such that the  $N_F$ samples kept for further processing at the basestation be free from interference from neighboring blocks. This adaptation introduces a timing offset  $\varepsilon_{i,n,m,\ell}T$  as compared to the sampling instants  $t_{i,n,m}$  of the basestation. The contribution of each user is affected by a different timing offset  $\varepsilon_{i,n,m,\ell}T$ , as each user signal is generated with a different transmit clock and is transmitted over a different multipath channel. This implies that the length of the cyclic prefix must be sufficiently longer than the maximum duration  $T_{ch}$  of the impulse responses of the composite channels with transfer functions  $H_{\ell}(f) = |P(f)|^2 H_{ch\ell}(f)$  ( $\ell = 0, ..., N_u - 1$ ). The  $N_F$  selected samples are applied to an N<sub>F</sub>-point FFT, followed by one-tap equalizers  $g_{i,n,k}$  that scale and rotate the FFT outputs. We denote by  $g_{i,n,k}$  the coefficient of the equalizer, operating on the  $k^{\text{th}}$  FFT output during the  $n^{\text{th}}$  FFT block of the  $i^{\text{th}}$  symbol interval. Each equalizer output is multiplied with the corresponding chip of the reference user's spreading sequence, and the  $N_s$  consecutive values are summed to yield the samples  $z_{i,k}$  at the input of the decision device.

$$z_{i,k} = \sqrt{\frac{N_F}{N_F + N_p}} \sum_{\ell=0}^{N_u - 1} \sum_{k' \in I_c} a_{i,k',\ell} I_{i,k,k',\ell} + W_{i,k}$$
(3)

where

$$I_{i,k,k',\ell} = \frac{1}{N_s} \sum_{n=0}^{N_s - 1} c_{i,n,0}^* c_{i,n,\ell} g_{i,k,n} A_{i,n,k,k',\ell}$$
(4)

Figure 1: MC-DS-CDMA transmitter for one user

$$A_{i,n,k,k',\ell} = \frac{1}{N_F} \sum_{m=0}^{N_F - 1} e^{-j2\pi \frac{m(k-k')}{N_F}} H_{k',\ell} \Big(\varepsilon_{i,n,m,\ell}\Big)$$
(5)

$$H_{k,\ell}\left(\varepsilon_{i,n,m,\ell}\right) = \frac{1}{T} \sum_{m'=-\infty}^{+\infty} H_{\ell}\left(\frac{k}{N_F T} + \frac{m'}{T}\right) e^{j2\pi \left(\frac{k}{N_F} + m'\right)\varepsilon_{i,n,m,\ell}}$$
(6)

and  $W_{i,k}$  is the additive noise contribution, with

$$E\left[W_{i,k}W_{i,k}^{*}\right] = N_{0}\delta_{i-i'}\delta_{k-k'}\frac{1}{N_{s}}\sum_{n=0}^{N_{s}-1}\left|g_{i,n,k}\right|^{2}$$
(7)

The quantity  $I_{i,k,k',\ell}$  denotes the contribution from the data symbol  $a_{i,k',\ell}$  to the sample  $z_{i,k}$  at the input of the decision device. The sample  $z_{i,k}$  from (3) contains a useful component with coefficient  $I_{i,k,k,0}$ . The quantities  $I_{i,k,k',0}$  ( $k' \neq k$ ) correspond to intercarrier interference (ICI), i.e., the contribution from data symbols transmitted by the reference user on other carriers. For  $\ell \neq 0$ , the quantities  $I_{i,k,k',\ell}$  correspond to multi-user interference (MUI), i.e., the contribution from data symbols transmitted by other users. The equalizer coefficients are selected such that the coefficients  $I_{i,k,k,0}$  of the useful component equal I, for  $k \in I_c$ . This yields

$$\mathbf{g}_{i,k,n} = \left(\frac{1}{N_F} \sum_{m=0}^{N_F - 1} H_{k,0}(\varepsilon_{i,n,m,0})\right)^{-1}$$
(8)

The performance of the MC-DS-CDMA system is measured by the signal-to-noise ratio (SNR), which is defined as the ratio of the power of the useful component ( $P_U$ ) to the sum of the powers of the intercarrier interference ( $P_{ICI}$ ), the multi-user interference ( $P_{MUI}$ ) and the noise ( $P_N$ ) at the input of the decision device. Note that these quantities depend on the index k of the considered carrier. This yields

$$SNR_{k}(\boldsymbol{\varepsilon}) = \frac{\frac{N_{F}}{N_{F} + N_{P}} P_{U_{k}}}{P_{N_{k}} + \frac{N_{F}}{N_{F} + N_{P}} \left( P_{ICI_{k}} + P_{MUI_{k}} \right)}$$
(9)

In (9), the powers of the useful component, ICI, MUI and noise are given by

$$P_{U_{k}} = E_{s_{k,0}}$$

$$P_{ICI_{k}} = \sum_{\substack{k' \in I_{c} \\ k' \neq k}} E_{s_{k',0}} E\left[\left|I_{i,k,k',0}\right|^{2}\right]$$

$$P_{MUI_{k}} = \frac{1}{N_{s} - 1} \sum_{\ell=1}^{N_{u} - 1} \sum_{k' \in I_{c}} E_{s_{k',\ell}} E\left[\left|I_{i,k,k',\ell}\right|^{2}\right]$$

$$P_{N_{k}} = N_{0} \frac{1}{N_{s}} \sum_{n=0}^{N_{s} - 1} \left|g_{i,n,k}\right|^{2}$$
(10)



In (10),  $E_{sk,\ell}=E[|a_{i,k,\ell}|^2]$  denotes the symbol energy transmitted on carrier *k* by user  $\ell$ . The MUI power from (10) represents an average over all possible assignments of the orthogonal spreading sequences to the users. In the absence of clock frequency offsets  $(\varepsilon_{i,n,m,\ell}=0 \text{ for } \ell=0,...,N_u-1)$ , the SNR (9) reduces to  $SNR_k(0) = (N_F/(N_F+N_p))|H_{k,0}|^2 (E_{sk,0}/N_0)$ . The degradation (in dB), caused by the timing offsets is defined as  $Deg_k = 10log(SNR_k(0)/SNR_k(\varepsilon))$ .

## II.2. Downlink MC-DS-CDMA

basestation In downlink MC-DS-CDMA, the synchronizes the  $N_u$  user signals  $(\tau_{i,n,m,\ell}=0 \text{ for } \ell=0,...,N_u-1)$ and broadcasts the sum of the  $N_{\mu}$  user signals  $s_{\ell}(t)$  from (2) to the different users. This broadcast signal reaches the receiver of the reference user through a multipath channel with transfer function  $H_{ch}(f)$ . The output of the channel is disturbed by AWGN w(t) with uncorrelated real and imaginary parts, each having a power spectral density of  $N_0/2$ . The resulting signal is applied to the receiver filter of figure 2 in order to detect the data symbols transmitted to the reference user ( $\ell=0$ ). The sampling instants are denoted  $t_{i,n,m} + \varepsilon_{i,n,m}T$ , where  $t_{i,n,m} = (m + (n + iN_s)(N_F + N_p))T$  and  $\varepsilon_{i,n,m}T$  is the deviation from  $t_{i,n,m}$ ; 1/T is the network reference clock frequency. Only the samples with indices  $m=0,...,N_F-1$  are kept for further processing. The receiver adjusts its sampling clock phase such that the  $N_F$  samples to be processed are free from interference between successive blocks. This implies that the length of the cyclic prefix is sufficiently longer than the duration  $T_{ch}$  of the composite channel with transfer function  $H(f) = |P(f)|^2 H_{ch}(f)$ . The sample  $z_{i,k}$  at the input of the decision device is represented by (3), which contains a useful component, intercarrier interference (ICI), multiuser interference (MUI) and noise. The quantities  $I_{ikk'\ell}$  are given by (4), with  $\varepsilon_{i,n,m,\ell}$  and  $H_{\ell}(f)$  substituted by  $\varepsilon_{i,n,m}$  and H(f), respectively. The equalizer coefficients  $g_{ink}$  that are selected such that the coefficients  $I_{i,k,k,0}$  of the useful component equal one, are given by (8), with  $\varepsilon_{i,n,m,\ell}$  and  $H_{\ell}(f)$  substituted by  $\varepsilon_{i,n,m}$ and H(f), respectively.

The performance is measured by the SNR, defined in (9). The powers of the useful component, the intercarrier interference, the multiuser interference and the noise are given by (10), with  $\varepsilon_{i,n,m,\ell}$  and  $H_{\ell}(f)$  substituted by  $\varepsilon_{i,n,m}$  and H(f), respectively for  $\ell=0,...,N_u-1$ .

# **III. CLOCK FREQUENCY OFFSET**

# III.1. Uplink MC-DS-CDMA

Assuming that the transmitter of each user has a freerunning clock with a relative clock frequency offset  $\Delta T_{\ell}/T$  as compared to the frequency l/T of the basestation clock, the timing deviation linearly increases with time:  $\varepsilon_{i,n,m,\ell} = \varepsilon_{0,\ell} +$   $(m+(n+iN_s)(N_F+N_p))\Delta T_\ell/T.$ Hence, an increasing misalignment in time between the transmitted and the received samples is introduced. To compensate for this increasing misalignment, a coarse synchronization is performed. In uplink MC-DS-CDMA, this coarse synchronization is done at the transmitter of each user, based upon timing information received from the basestation. At the transmitter of each user, the number of samples in the prefix is increased ( $\Delta T_{\ell} < 0$ ) or reduced ( $\Delta T_{\ell} > 0$ ), such that the  $N_F$ successive samples selected by the basestation for further processing are not affected by interference from neighboring blocks. After coarse synchronization, the resulting timing deviation can be written as  $\varepsilon_{i,n,m,\ell} = \varepsilon_{i,n,\ell} + m\Delta T_{\ell}/T$ , where  $\varepsilon_{i,n,\ell}$ denotes the timing deviation of the first of the  $N_F$  samples of the considered block that are processed by the basestation. For carriers outside the rolloff area, (4) reduces to

$$I_{i,k,k',\ell} = U_{k,k',\ell} C_{k,k',\ell} R_{i,\ell} (k,k')$$
(11)

In (11),  $U_{k,k',\ell} = H_{k',\ell}/H_{k,0}$ ;  $H_{k,\ell} = (1/T)$   $H_{\ell}(mod(k;N_F)/(N_FT))$ ;  $C_{k,k',\ell} = B_{k,k',\ell'}/B_{k,k,0}$ ;  $B_{k,k',\ell} = D((k'-k)/N_F + (mod(k';N_F)/N_F))$  $\Delta T_{\ell'}/T)$ ;  $mod(x;N_F)$  is the modulo- $N_F$  reduction of x, yielding a result in the interval  $[-N_F/2;N_F/2]$ , and

$$D(x) = \frac{1}{N_F} \sum_{m=0}^{N_F - 1} e^{-j2\pi mx} = e^{-j\pi (N_F - 1)x} \frac{\sin(\pi N_F x)}{N_F \sin(\pi x)} \quad (12)$$

$$R_{i,\ell}(k,k') = \frac{1}{N_s} \sum_{n=0}^{N_s - 1} \widetilde{c}_{i,n,k',\ell} \widetilde{c}_{i,n,k,0}$$
(13)

where  $\tilde{c}_{i,n,k,\ell} = c_{i,n,\ell} exp(j2\pi(mod(k;N_F)/N_F)\varepsilon_{i,n,\ell})$ . The quantity  $R_{i,\ell}(k,k')$  from (13) is the correlation between the sequences  $\{\tilde{c}_{i,n,k,\ell}\}$  and  $\{\tilde{c}_{i,n,k,\ell}\}$ . Note that  $C_{k,k,\ell}=1$  and  $R_{i,\ell}(k,k)=1$ . For  $\ell \neq 0$ , we obtain  $C_{k,k',\ell'} \neq 0$  and  $R_{i,\ell}(k,k') \neq 0$ : the clock frequency offset gives rise to multiuser interference. Further, for  $\ell=0$  and  $k' \neq k$ , we have  $C_{k,k',\ell'} \neq 0$  and  $R_{i,\ell}(k,k') \neq 0$ : the clock frequency offset also introduces intercarrier interference. In general, the one-tap equalizer can eliminate neither the ICI nor the MUI, i.e.,  $I_{i,k,k',\ell} \neq 0$  for  $k' \neq k$  or for  $\ell \neq 0$ . The quantity  $C_{k,k',\ell}$  in (11) is a function of  $\Delta T_{\ell}/T$ . Further, the quantity  $R_{i,\ell}(k,k')$  is a function of  $\Delta T_{\ell}/T$ .

In the following, we assume the maximum load  $(N_u=N_s)$ and all users having the same energy per symbol on each carrier, i.e.,  $E_{s_{k,\ell}}=E_s$   $(k \in I_c, \ell=0,...,N_s-1)$ . Let us consider the case where all clock frequency offsets are within the interval  $[-(\Delta T/T)_{max}, (\Delta T/T)_{max}]$ , with  $(\Delta T/T)_{max} < 2/N_F$ . We assume that the clock frequency offsets  $\Delta T_{\ell}/T$  for  $\ell > 0$  are uniformly distributed in the interval  $[-(\Delta T/T)_{max}, (\Delta T/T)_{max}]$ . Let  $[x_{min}, x_{max}]$  denote the interval to which the timing offsets  $\varepsilon_{i,0,\ell}$  ( $\ell = 0,...,N_s-1$ ) are uniformly distributed in the interval  $[x_{min}, x_{max}]$ . To clearly isolate the effect of the clock frequency offset, we consider the case of an ideal channel, i.e.  $H_{k,\ell}=1$ ,  $k \in I_c$ ,  $\ell=0,...,N_s-1$ . In this case, the performance degradation is given by

$$Deg_{k} = 10 \log \left( \frac{|B_{k,k,0}|^{-2}}{SNR(0) \left( \sum_{k' \in I_{c}, k' \neq k} |C_{k,k',0}|^{2} \widetilde{X}_{k,k',0} + \sum_{k' \in I_{c}} V_{k,k'} \right)} \right)$$
(14)

where

$$\widetilde{X}_{k,k',\ell} = \left(\frac{1}{x_{max} - x_{min}}\right)^{N_s x_{max}} \cdots \int_{x_{min}}^{x_{max}} X_{i,k,k',\ell} d\varepsilon_{i,0,0} \cdots d\varepsilon_{i,0,N_s-1}$$

$$V_{k,k'} = \frac{1}{2(\Delta T/T)_{max}} \int_{-(\Delta T/T)_{max}}^{(\Delta T/T)_{max}} |C_{k,k',\ell}|^2 (1 - \widetilde{X}_{k,k',\ell}) d\frac{\Delta T_{\ell}}{T}$$

$$X_{i,k,k',\ell} = \left|\frac{1}{N_s} \sum_{n=0}^{N_s-1} e^{-j2\pi} \frac{(mod(k;N_F)\varepsilon_{i,n,0} - mod(k';N_F)\varepsilon_{i,n,\ell})}{N_F}\right|^2$$
(16)

and  $SNR(0) = (N_F/(N_F + N_p))E_s/N_0$  is the SNR in the absence of clock frequency offsets. In (14), the total interference power is averaged over the initial timing offset  $\mathcal{E}_{i,0,\ell}$ . The degradation (14) still depends on the carrier index k and  $\Delta T_0/T$ . It turns out that the maximum performance degradation occurs for  $|\Delta T_0/T| = (\Delta T/T)_{max}$ . The maximum degradation occurs for carriers close to the edge of the rolloff area, i.e. for  $k \approx (N_c - 1)/2$  and  $k \approx N_F - (N_c - 1)/2$ . For given k, the degradation depends on the number  $N_c$  of modulated carriers, as in (14), the summation over k' ranges over the set  $I_c$  of  $N_c$  modulated carriers. An upper bound on this degradation is obtained by extending in (14) this summation interval over all  $N_F$  available carriers, i.e.,  $k'=0,...,N_F-1$ . This yields

$$Deg_{k} \leq 10 \log \left( \left| B_{k,k,0} \right|^{-2} + SNR(0) \left( \sum_{k=0,k'\neq k}^{N_{F}-1} |C_{k,k',0}|^{2} \widetilde{X}_{k,k',0} + \sum_{k=0}^{N_{F}-1} V_{k,k'} \right) \right)$$
(17)

This bound is also shown in figure 3, for  $k=0,...,N_F-1$ . For given k, the upper bound (17) on the degradation is independent of the number  $N_c$  of modulated carriers, and becomes maximum for the carrier  $k=N_F/2$ . The upper bound is reached when all carriers are modulated  $(N_c = N_F; \alpha = 0)$ . When  $\alpha > 0$ , the upper bound (17) yields an accurate approximation of the actual degradation. For small x, the approximation  $sin(\pi x) \approx \pi x$ holds, such that  $|D(x)| \approx \sin(\pi N_F x)/(\pi N_F x)$  (see (12)). Hence, for  $N_F >> N_p$ , the degradations (14) and (17) are essentially a function of  $N_F(\Delta T/T)_{max}$  when  $mod(k;N_F)/N_F$  is a fixed value. In addition, the degradation is a function of the spreading factor  $N_s$  as the degradation depends on  $N_s$  through  $X_{i,k,k',\ell}$ , given by (16). Figure 3 shows the bound (17) for  $k = (N_c - 1)/2$  and the maximum (over k) of the actual degradation (14), for several values of the spreading factor. Note that, for given  $N_F(\Delta T/T)_{max}$ , the degradation increases for an increasing spreading factor. For  $N_s N_F (\Delta T/T)_{max} >> 1$ , the maximum uplink degradation is essentially independent of  $N_s$ . For  $N_s N_F (\Delta T/T)_{max} << 1$ , the performance degradation is proportional to  $(N_s)^2$ . Further, the bound (17) virtually coincides with the actual maximum degradation: this illustrates the importance of the bound (17). For given  $(\Delta T/T)_{max}$ , the degradation increases with the number of carriers. Hence, in order to obtain small degradations, it is required that  $|N_s N_F(\Delta T/T)_{max}| \le 1$ , in which case the degradation is proportional to  $(N_s N_F \Delta T/T)^2$ .

# III.2. Downlink MC-DS-CDMA

When the receiver of the reference user has a free-running clock with a relative clock frequency offset  $\Delta T/T$  as compared to the frequency 1/T of the basestation clock, the timing deviation linearly increases with time:  $\varepsilon_{i,n,m} = \varepsilon_0 + (m+(n+iN_s)(N_F+N_p))\Delta T/T$ . However, in contrast with the uplink, where the increasing misalignment in time is compensated at the transmitters of the different users, the coarse synchronization in the downlink is performed at the receiver of each user. This implies that the number of samples in the prefix at the receiver is reduced  $(\Delta T/T>0)$  or increased  $(\Delta T/T<0)$ , such that the  $N_F$  successive samples kept for further processing remain in the region where interference from other blocks is absent. After coarse synchronization, the timing deviation is given by  $\varepsilon_{i,n,m} = \varepsilon_{i,n} + m\Delta T/T$ , where  $\varepsilon_{i,n}$ denotes the timing deviation of the first of the  $N_F$  samples of the considered block that are processed by the receiver.

The quantities  $I_{i,k,k',\ell}$  for the carriers outside the rolloff area are given by (4) with  $\varepsilon_{i,n,\ell}$ ,  $\Delta T_{\ell}/T$  and  $H_{\ell}(f)$  substituted by  $\varepsilon_{i,n}$ ,  $\Delta T/T$  and H(f), respectively, for  $\ell = 0, ..., N_u$ -1. It follows that  $C_{k,k',\ell}$  in (11) is independent of the user index  $\ell$ . In the following, we use the notation  $C_{k,k'}$ . The quantity  $R_{i,\ell}(k,k')$ from (13) is the correlation between the sequences  $\{\tilde{c}_{i,n,k',\ell}\}$ and  $\{\widetilde{c}_{i,n,k,0}\}$ . However, in contrast with the uplink,  $\varepsilon_{i,n}$  is independent of the user index  $\ell$ . Hence, it follows with k'=k, that  $C_{k,k} = l$  and  $R_{i,\ell}(k,k) = \delta_{\ell}$ : in downlink transmission, contributions from different users that are transmitted on carrier k do not give rise to interference on the k<sup>th</sup> FFT output. This also can be observed in (13): for k'=k, the chips  $c_{i,n,\ell}$  and  $c_{i,n,0}$  are rotated over the same chip-dependent angle, such that the orthogonality between the different users on the same carrier is not affected. For  $k' \neq k$ , we have  $C_{k,k'} \neq 0$  and  $R_{i,\ell}(k,k') \neq 0$ , i.e., user signals on other carriers do give rise to interference on the  $k^{\text{th}}$  carrier: the orthogonality between the user signals on different carriers is affected by the clock frequency offset. Hence, a clock frequency offset causes intercarrier and multiuser interference.

Assuming the maximum load  $(N_u=N_s)$  and all users having the same energy per symbol on each carrier  $(E_{s_{k,\ell}}=E_s, k \in I_c, \ell=0,...,N_s-1)$ , the degradation is given by (14), with  $\varepsilon_{i,n,\ell}$ ,  $\Delta T_{\ell}/T$  and  $H_{\ell}(f)$  substituted by  $\varepsilon_{i,n}$ ,  $\Delta T/T$  and H(f), respectively. It follows from (16) that  $X_{i,k,k}=1$ ,  $k \in I_c$ . The sum of the powers of the ICI and the MUI is independent of  $X_{i,k,k}$ :

$$P_{I_{k}} = P_{ICI_{k}} + P_{MUI_{k}} = E_{s} \sum_{\substack{k' \in I_{c}, k' \neq k}} \left| C_{k,k'} \right|^{2}$$
(18)

Assuming that the clock frequency offset is in the interval  $[-(\Delta T/T)_{max}, (\Delta T/T)_{max}]$ , we look for the value of  $\Delta T/T$  that maximizes the performance degradation. It turns out degradation is maximum for  $|\Delta T/T| = (\Delta T/T)_{max}$ . We consider the case of an ideal channel  $(H_k=I)$  to clearly isolate the effect of the clock frequency offset. In this case, the degradation as compared to the case of a zero clock frequency offset is given by

$$Deg_{k} = 10 \log \left( Y_{k} + SNR(0) \sum_{k' \in I_{c}, k' \neq k} |C_{k,k'}|^{2} \right)$$
(19)

This degradation is a function of the carrier index k, that becomes maximum for carriers close to the edge of the rolloff area, i.e. for  $k \approx (N_c - 1)/2$  and  $k \approx N_F - (N_c - 1)/2$ . For given k, the degradation depends on the number  $N_c$  of modulated carriers, as in (20), the summation over k' ranges over the set  $I_c$  of  $N_c$ modulated carriers. An upper bound on this degradation is obtained by extending in (20) this summation interval over all  $N_F$  available carriers, i.e.,  $k'=0,...,N_F-1$ . This yields

$$Deg_{k} \leq 10 \log \left( Y_{k} + SNR(0) \sum_{k=0,k' \neq k}^{N_{F}-1} |C_{k,k'}|^{2} \right)$$
 (20)

which, for given k, is independent of the number  $N_c$  of modulated carriers and becomes maximum for  $k=N_F/2$ . The bound (21) is reached when all carriers are modulated  $(N_c = N_F; \alpha = 0)$ . As for the uplink, the degradations (20) and (21) are a function of  $N_F(\Delta T/T)_{max}$  when  $mod(k;N_F)/N_F$  is a fixed value. Further, in (20) and (21) we observe that the performance degradation is independent of the spreading factor. In figure 3, the bound (21) for  $k=(N_c-1)/2$ , and the maximum (over k) of the actual degradation are shown. Note that the bound (21) is close to the actual maximum degradation (20). To obtain small degradations, it is required that  $|N_F(\Delta T/T)_{max}| \le 1$ , in which case the degradation is proportional to  $(N_F(\Delta T/T)_{max})^2$ .

We observe that the maximum degradation is much larger in the uplink than in the downlink. It can be verified that for  $\Delta T_0/T = \Delta T/T$ , the powers of the useful component, the intercarrier interference and the noise are the same in the uplink and the downlink. This indicates that the uplink degradation is dominated by the MUI, which is considerably larger than in the downlink. When  $N_s N_F (\Delta T/T)_{max} >> 1$ , the curves for the uplink degradation in figure 11 coincide for different values of  $N_s$ . A small degradation can be obtained only when  $N_s N_F (\Delta T/T)_{max} << 1$ . As compared to the downlink, this condition on  $(\Delta T/T)_{max}$  is more stringent by a factor of  $N_s$ . For  $N_s N_F (\Delta T/T)_{max} << 1$ , the uplink degradation is essentially proportional to  $(N_s N_F(\Delta T/T)_{max})^2$ ; in this region, the ratio between uplink and downlink degradation is proportional to  $(N_s)^2$ .

## **IV.** CONCLUSIONS

In this contribution, we have investigated the effect of fixed timing offsets and clock frequency offsets on uplink and downlink MC-DS-CDMA with orthogonal spreading sequences on multipath channels. Our conclusions can be summarized as follows:

• Clock frequency offsets give rise to a reduction of the useful component and the occurrence of ICI and MUI.

• For both the uplink and the downlink, the degradation caused by a clock frequency offset strongly increases with  $N_F(\Delta T/T)_{max}$ .

• For given  $N_F(\Delta T/T)_{max}$ , the degradation in the downlink does not depend on the spreading factor. The degradation in the uplink increases with an increasing spreading factor. The degradation in the uplink is larger than in the downlink, because the uplink is affected by a larger amount of MUI. It can be verified from [10]-[11] that the degradation (33) for downlink MC-DS-CDMA is exactly the same as the corresponding degradation for OFDM, and essentially the same as the degradation for downlink MC-CDMA, assuming the three multicarrier systems have the same carrier spacing.

### ACKNOWLEDGEMENT

The first author gratefully acknowledges the financial support from the Belgian National Fund for Scientific Research (FWO Flanders). Further, the authors acknowledge the financial support from the Federal Office for Scientific, Technical and Cultural Affairs (OSTC).



Figure 3: Maximum degradation in presence of clock frequency offset ( $N_F=64$ ,  $N_c=57$ ,  $SNR(0)=10 \ dB$ )

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