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UPLINK AND DOWNLINK MC-DS-CDMA SYNCHRONIZATION SENSITIVITY

Abstract. In this paper, we consider the effect of synchronization errors on the performance of the multicarrier direct-sequence code-division multiple access (MC-DS-CDMA) system and compare the results for downlink and uplink transmission. To evaluate the effect of small synchronization errors on the BER performance of the MC-DS-CDMA system, we derive simple analytical expressions for the BER degradation that are based upon truncated Taylor series expansions. We point out that a constant carrier phase offset or a constant timing offset do not give rise to performance degradation, for neither uplink nor downlink MC-DS-CDMA. The MC-DS-CDMA system is strongly degraded in the presence of a carrier frequency offset or a clock frequency offset. This degradation is proportional to the squares of the frequency offset and the number of carriers. Further, the degradation in the uplink is a factor N_s^2 (N_s is the spreading factor) larger than in the downlink, because the former suffers from a higher level of multi-user interference. The degradation caused by carrier phase jitter or timing jitter is the same in the uplink and the downlink, when the spectrum of the jitter is the same for all users. Further, the degradation is independent of the spectral contents of the jitter, the spreading factor and the number of carriers, but only depends on the jitter variance.

1. THE MC-DS-CDMA SYSTEM

The MC-DS-CDMA system is a combination of the MC transmission technique and the CDMA multiple access (CDMA) technique. In MC-DS-CDMA, the serial-to-parallel converted data stream is multiplied with the spreading sequence and then the chips belonging to the same symbol modulate the same carrier: the spreading is done in the time domain. MC-DS-CDMA has been proposed for mobile communications [1]-[3].

Without loss of generality, we use the terminology for the uplink. In MC-DS-CDMA, the symbol sequence to be transmitted at rate R_s is split into N_c lower rate symbol sequences $\{a_{i,k,\ell}\}$, where $a_{i,k,\ell}$ denotes the ith data symbol transmitted by user ℓ on the kth carrier of the multicarrier system; k belongs to a set I_c of N_c carrier indices. The symbol $a_{i,k,\ell}$ is multiplied with a spreading sequence $\{c_{i,n,\ell}|n=0,\ldots,N_s-1\}$ with spreading factor N_s. Each user is assigned a unique orthogonal spreading sequence. The resulting N_s components of the spread data symbol $a_{i,k,\ell}$, i.e. $\{a_{i,k,\ell}c_{i,n,\ell}| n=0,\ldots,N_s-1\}$ are then serially transmitted on the kth carrier of the multicarrier system. Hence, the spreading is accomplished in the time domain. To modulate the spread data symbols on the orthogonal carriers, an N_F-point inverse fast Fourier transform (inverse FFT) is used. To avoid that the multipath channel causes interference between the data symbols at the receiver, each FFT block is cyclically extended with a prefix of N_p samples. The resulting sequence $\{s_{i,n,m,\ell}\}$ is fed to a square root raised cosine filter $P(f)$ with roll off α and unit-energy impulse response

$p(t)$ at a nominal rate $1/T=(N_F+N_p)N_sR_s/N_c$, resulting in the signal $s_\ell(t)$. We assume the carriers inside the roll off area are not modulated. Hence, of the N_F available carriers, only $N_c \leq N_F$ carriers are actually used. Without loss of generality, we focus on the detection of the data symbols transmitted by the reference user ($\ell=0$).

The transmitted signal $s_\ell(t)$ reaches the receiver through a slowly multipath fading channel. Assuming the path gains are constant over the duration N_c/R_s of N_s FFT blocks, the corresponding channel transfer function experienced by the ith symbol from user ℓ can be denoted by $H_{ch\ell}(f,i)$. Restricting our attention to wide-sense stationary uncorrelated scattering (WSSUS), the second-order moment $E[|H_{ch\ell}(f,i)|^2]$ is independent of both f and i. The output of the channel is disturbed by additive white Gaussian noise (AWGN) $w(t)$ with uncorrelated real and imaginary parts, each having a power spectral density of $N_0/2$. Further, the signal of user ℓ is affected by a carrier phase error $\phi_\ell(t)$. The sum of the different user signals is applied to the receiver filter, which is matched to the transmit filter, and sampled at nominal rate $1/T$. The contribution of user ℓ is disturbed by a timing offset $\varepsilon_{i,n,m,\ell}T$. In the uplink, where the contribution of each user is generated with a different transmit clock, upconverted by a different carrier oscillator and transmitted over a different multipath channel, the carrier phase error $\phi_\ell(t)$ and timing offset $\varepsilon_{i,n,m,\ell}T$ generally depend on the user index ℓ. In the downlink, on the other hand, the base station synchronizes the different user signals, and upconverts the sum of the different user signals with the same carrier oscillator. Further, as the different user signals reach the receiver of the reference user through the same multipath channel, the carrier phase error $\phi(t)$ and timing offset $\varepsilon_{i,n,m}T$ are the same for all users.

In the following, we assume that the transmitter (uplink) or receiver (downlink) of each user adapts its transmit clock phase such that N_F samples can be found outside the cyclic prefix that are free from interference from neighbouring FFT blocks. The resulting N_F samples are kept for further processing. As the removal of the cyclic prefix eliminates the interference between neighbouring blocks, the data symbols $a_{i,k,\ell}$ transmitted during symbol interval i are not affected by intersymbol interference from other symbol intervals. Hence, we omit the symbol index i in the sequel.

The N_F selected samples are applied to an N_F-point FFT, followed by one-tap equalizers $g_{n,k}$ that scale and rotate the FFT outputs. We denote by $g_{n,k}$ the coefficient of the equalizer, operating on the kth FFT output during the nth FFT block. We consider the case of the maximum ratio combiner (MRC). Each equalizer output is multiplied with the corresponding chip of the reference user's spreading sequence, and the N_s consecutive values are summed to yield the samples z_k at the input of the decision device. Based on the sample z_k, a decision is made about the data symbol $a_{k,0}$.

To measure the performance, we use the signal to interference and noise ratio (SINR), defined by $SINR_k(\phi,\varepsilon)=\beta P_{U,k}/(P_{N,k}+\beta P_{I,k})$, where $\beta=N_F/(N_F+N_p)$ and $P_{U,k}$, $P_{I,k}$ and $P_{N,k}$ are the powers of the average useful component, the interference and the noise, respectively. Note that in general these powers depend on the carrier index k. In the absence of synchronization errors, the SINR reduces to

$SINR_k(0)=\beta|H_{k,0}|^2E_{s,k,0}/N_0$, where $E_{s,k,\ell}$ is the symbol energy transmitted on carrier k by user ℓ, $H_{k,\ell}=H_\ell(\mathrm{mod}(k;N_F)/\,(N_FT))/T$, $\mathrm{mod}(x;N_F)$ is the modulo-N_F reduction of x, yielding a result in the interval $[-N_F/2,N_F/2]$, and $H_\ell(f)=|P(f)|^2H_{ch,\ell}(f)$. The quantity $SINR_k$ still depends on the particular realization of the transfer functions $H_{k,\ell}$ ($k\in I_c$, $\ell=0,\ldots,N_u$-1) and the spreading sequences during the considered sequence of N_s FFT blocks. Hence, a more convenient performance indicator is $\overline{SINR}_k$, which is obtained by replacing $P_{X,k}$ ($X=U,\ I,\ N$) by their averages $\overline{P}_{X,k}$ over the fading characteristics and over all possible assignments of spreading sequences to the users. Because of the WSSUS assumption, $E[|H_{k,\ell}|^2]$ does not depend on the carrier index. We assume perfect power control: $E_s=E_{s,k,\ell}E[|H_{k,\ell}|^2]$. In this case, $\overline{SINR}_k(0)$ reduces to $\beta E_s/N_0$, which is independent of the carrier index k. The degradation (in dB) caused by the synchronization errors is defined by $Deg_k=(\overline{SINR}(0)/\,\overline{SINR}_k(\phi,\varepsilon))$.

2. CARRIER PHASE ERRORS

In this section, we investigate the sensitivity of MC-DS-CDMA to carrier phase errors in the absence of timing errors ($\varepsilon_{n,m,\ell}=0$).

2.1 Carrier Frequency Offset

In the case of small carrier frequency offsets ΔF_ℓ ($\ell=0,\ldots,N_u$-1), the carrier phase error linearly increases in time [4]: $\phi_\ell(t)=\phi_\ell(0)+2\pi\Delta F_\ell t$. For small carrier frequency offsets ($|N_F\Delta F_\ell T|<<1$), the useful power and noise power can be approximated by $\overline{P}_{U,k}=E_s$ and $\overline{P}_{N,k}=N_0$. The contribution of user ℓ to the interference power is proportional to R_ℓ, which is the correlation between the sequences $\{\tilde{c}_{n,\ell}\}$ and $\{\tilde{c}_{n,0}\}$, where $\tilde{c}_{n,\ell}=c_{n,\ell}\exp(j(2\pi n(N_F+N_p)\Delta F_\ell T+\phi_\ell(0)))$. In the uplink, for $\ell\neq0$, the chips $c_{n,\ell}$ of user ℓ and the chips $c_{n,0}$ of the reference user are rotated over different angles, so that the orthogonality between the different user signals is destroyed: $R_\ell\neq0$ for $\ell\neq0$. Hence, the carrier frequency offsets give rise to intercarrier and multi-user interference. In the downlink, however, all chips are rotated over the *same* angle, so that the orthogonality between the different user signals is maintained: $R_\ell=0$. Hence, in the downlink, multi-user interference is absent, and the carrier frequency offset only introduces intercarrier interference. In the following, we approximate the interference power by a truncated Taylor series (keeping up to quadratic terms) around $\Delta F_\ell=0$.

We assume that the carrier frequency offsets are within the interval $[-F_{max},F_{max}]$, where F_{max} is smaller than the carrier spacing: $N_FF_{max}T<<1$. In the downlink, where multi-user interference is absent, the total interference power yields

$$\overline{P}_{I,k} \approx E_s \left(\pi N_F \Delta F_0 T\right)^2 / 3 \tag{2}$$

Note that the degradation becomes independent of the carrier index k. The maximum degradation occurs for $|\Delta F_0|=F_{max}$.

In uplink MC-DS-CDMA, multi-user interference is present. For $N_s(N_F+N_p)F_{max}T <<1$, the orthogonality between the rotated chip sequences $\{\tilde{c}_{n,\ell}\}$, $\ell=0,\ldots,N_u-1$ is only slightly affected. Assuming the carrier frequency offsets ΔF_ℓ of the interfering users are uniformly distributed in the interval $[-F_{max},F_{max}]$, one obtains

$$\overline{P}_{I,k} \approx E_s \frac{N_u-1}{N_s-1} B\left(\left(\Delta F_0 T\right)^2 + \frac{1}{3}\left(F_{max}T\right)^2\right) \tag{3}$$

where N_u is the number of active users and $B=(\pi N_s(N_F+N_p))^2/3$. When $N_s(N_F+N_p)F_{max}T<<1$ is no longer valid, the orthogonality between the rotated chip sequences $\{\tilde{c}_{n,\ell}\}$, $\ell=0,\ldots,N_u-1$ is strongly affected. In this case, the interference power can be approximated by

$$\overline{P}_{I,k} \approx E_s\left(N_u-1\right)/\left(N_s-1\right) \tag{4}$$

From (4) and (5) it follows that the degradation in the uplink is independent of the carrier index k. From (10), we observe that $|\Delta F_0|=F_{max}$ yields the maximum degradation. For $N_s(N_F+N_p)F_{max}T<<1$, the maximum degradation is proportional to $(N_s(N_F+N_p)F_{max}T)^2$, whereas for $N_s(N_F+N_p)F_{max}T>1$ the maximum degradation becomes essentially independent of N_s, N_F and $F_{max}T$. When $N_s(N_F+N_p)F_{max}T<<1$, the maximum degradation in the uplink is a factor N_s^2 larger than in the downlink. This can also be observed in figure 1, where the maximum degradation, obtained with the approximations (2) and the minimum of (3) and (4), is shown along with the actual degradation, for the maximum load ($N_u=N_s$). As we observe, the approximation is close to the actual degradation. Hence, the truncated Taylor series expansions can be used to compute the actual degradation caused by carrier frequency offsets in uplink and downlink MC-DS-CDMA.

When $\Delta F_\ell=0$, $\ell=0,\ldots,N_u-1$, the carrier phase error reduces to a constant phase offset $\phi_\ell(t)=\phi_\ell(0)$. The only effect of a constant phase offset is a rotation over an angle $\phi_\ell(0)$ of the contribution of user ℓ at the FFT outputs. As this rotation can be compensated for by the equalizer, a constant phase does not introduce a performance degradation.

2.2 Carrier Phase Jitter

To get rid of the strong degradation caused by carrier frequency offsets, a phase-locked loop (PLL) can be used in the upconversion and downconversion of the signals. The residual carrier phase error $\phi_\ell(t)$ can be modelled as a zero-mean stationary process with jitter spectrum $S_{\phi,\ell}(f)$ and jitter variance $\sigma^2_{\phi,\ell}$ [5]. For small jitter variances, i.e. $\sigma^2_{\phi,\ell} \ll 1$, the phase rotation $\exp\{j\phi_\ell(t)\}$ of the data symbols of user ℓ at the FFT outputs can be approximated by a truncated Taylor series: $\exp\{j\phi_\ell(t)\} \approx 1 + j\phi_\ell(t)$.

When all jitter processes in the uplink have the same jitter spectrum $S_{\phi,\ell}(f) = S_\phi(f)$, thus jitter variances $\sigma^2_{\phi,\ell} = \sigma^2_\phi$, and the load is maximum ($N_u = N_s$) [5], the degradation in the uplink and the downlink is the same, and given by

$$Deg \approx 10\log\left(1 + SINR(0)\sigma^2_\phi\right) \tag{5}$$

This degradation is independent of the carrier index k, the number of carriers, the spreading factor and the spectral contents of the jitter, but only depends on the jitter variance.

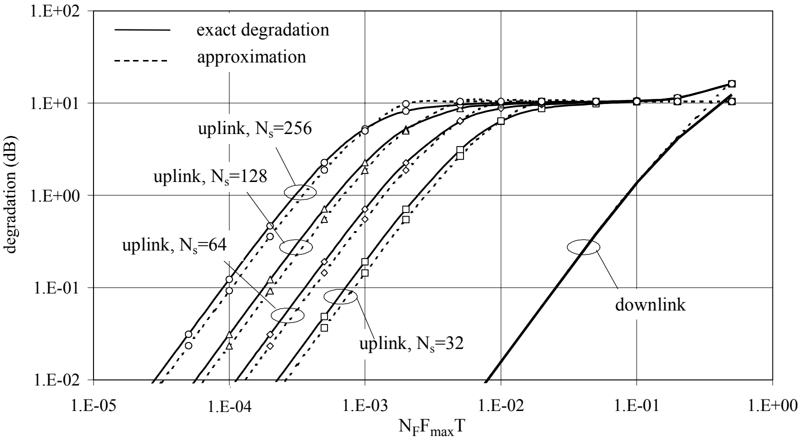

Figure 1. Carrier frequency offset, $N_u = N_s$, $N_p = 0$, $SINR(0) = 10$ *dB*

3. TIMING ERRORS

In this section, we investigate the sensitivity of MC-DS-CDMA to carrier phase errors in the absence of carrier phase errors ($\phi_\ell(t) = 0$).

3.1 Clock Frequency Offset

Assuming the transmitter (uplink) and receiver (downlink) of each user has a free-running clock with a relative clock frequency offset $\Delta T_\ell/T$ as compared to the frequency $1/T$ of the base station clock, the timing deviation linearly increases in time: $\varepsilon_{n,m,\ell}=\varepsilon_{n,\ell}+m\Delta T_\ell/T$, m=0,…,$N_F$-1, where $\varepsilon_{n,\ell}$ is the timing deviation of the first of the N_F samples of the considered FFT block that are processed at the receiver [6]. For small clock frequency offsets ($|N_F\Delta T_\ell/T|<<1$), the useful power and noise power can be approximated by $\overline{P}_{U,k}=E_s$ and $\overline{P}_{N,k}=N_0$. The contribution of the data symbol $a_{k',\ell}$ to the interference power on the kth carrier is proportional to $R_\ell(k,k')$, which is the correlation between the sequences $\{\tilde{c}_{n,k',\ell}\}$ and $\{\tilde{c}_{n,k,0}\}$. In the sequence $\tilde{c}_{n,k,\ell}$, the chips $c_{n,\ell}$ are rotated over an angle $\exp(j2\pi\,\varepsilon_{n,\ell}\mathrm{mod}(k;N_F)\Delta T_\ell/T)$. As the chip sequences belonging to different users or transmitted on different carriers are rotated over different angles, the orthogonality between the signals transmitted by different users or on different carriers is lost: the clock frequency offset introduces intercarrier and multi-user interference, for both uplink and downlink MC-DS-CDMA. Note however that in the downlink, $R_\ell(k,k)=\delta_\ell$: symbols transmitted on carrier k to non-reference users do not give rise to interference at the kth output at the receiver of the reference user. As for given k the largest interference in the uplink comes from the multi-user interference from the contributions transmitted on carrier k, it follows that the multi-user interference in the uplink is substantially larger than in the downlink. In the following, we approximate the interference power by a truncated Taylor series (keeping up to the quadratic terms) around $\Delta T_\ell/T=0$.

We assume that the clock frequency offsets $\Delta T_\ell/T$, $\ell=0,\ldots,N_F-1$ are restricted to the interval $[-(\Delta T/T)_{max},(\Delta T/T)_{max}]$, where $|N_F(\Delta T/T)_{max}|<<1$ to keep the degradation sufficiently small. In downlink MC-DS-CDMA, the interference power can be approximated by

$$\overline{P}_{I,k} \approx \frac{E_s}{3}\left(\pi\mathrm{mod}(k;N_F)\frac{\Delta T_0}{T}\right)^2 \tag{6}$$

The degradation depends on the carrier index k, and becomes maximum for carriers at the edge of the roll off area. Further, $|\Delta T_0/T|=(\Delta T/T)_{max}$ maximizes the degradation.

In the uplink, we can distinguish two cases. First, when $N_s(N_F+N_p)(\Delta T/T)_{max}<<1$, the chip sequences $\tilde{c}_{n,k,\ell}$ are rotated over nearly the same angle, so that the orthogonality between the different carriers and the different users is only slightly affected. Assuming that the clock frequency offsets $\Delta T_\ell/T$ of the interfering users are uniformly distributed in the interval $[-(\Delta T/T)_{max},(\Delta T/T)_{max}]$, the interference power is approximated by

$$\overline{P}_{I,k} \approx E_s \frac{N_u - 1}{N_s - 1} B\big(\mathrm{mod}(k;N_F)\big)^2 \left(\left(\frac{\Delta T_0}{T}\right)^2 + \frac{1}{3}\left(\frac{\Delta T}{T}\right)^2_{2\,max} \right) \tag{7}$$

where $B=(\pi N_s)^2/3$. The degradation depends on the carrier index k and becomes maximum for carriers at the edge of the roll off area. Further, the degradation is maximum when $|\Delta T_0/T|= (\Delta T/T)_{max}$. From (6) and (7), it follows that the maximum degradation in the uplink is a factor N_s^2 larger than in the downlink, when $N_s(N_F+N_p)(\Delta T/T)_{max}<<1$. When $N_s(N_F+N_p)(\Delta T/T)_{max}<<1$ is no longer valid, the orthogonality between the signals from the different users and on different carriers is strongly affected. In this case, one obtains

$$\overline{P}_{I,k} \approx E_s\left(N_u - 1\right)/\left(N_s - 1\right) \tag{8}$$

which is essentially independent of N_s, N_F and $(\Delta T/T)_{max}$. In figure 2, the maximum degradation obtained with the approximations (6) and the minimum of (7) and (8) is shown, along with the actual degradation. It follows that (6) and the minimum of (7) and (8) yield accurate approximations for the actual degradations.

When $\Delta T_\ell/T=0$, the timing error reduces to a constant timing offset $\varepsilon_{n,m,\ell}=\varepsilon_{0,\ell}$. As only carriers outside the roll off area are used, the only effect of the constant timing offset is a rotation over an angle $2\pi\ \varepsilon_{0,\ell}\mathrm{mod}(k;N_F)/N_F$ of the contribution of user ℓ at the kth FFT output. As the equalizer can compensate for this rotation, a constant timing offset does not yield performance degradation.

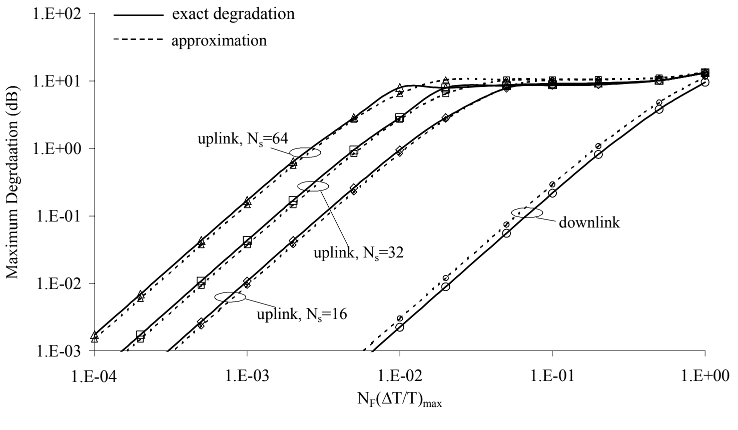

Figure 2. Clock frequency offset, N_u=N_s, N_p=5, SINR(0)=10 dB

3.2 Timing Jitter

When the strong degradation caused by clock frequency offsets cannot be tolerated, the transmitter (uplink) and receiver (downlink) clock phase can be adjusted using a PLL. The timing jitter $\varepsilon_{i,n,m,\ell}T$ introduced by this adaptation process can be modelled as a zero-mean stationary process with jitter spectrum $S_{\varepsilon,\ell}(f)$ and jitter variance $\sigma^2_{\varepsilon,\ell}$ [7]. For small jitter variances ($\sigma^2_{\varepsilon,\ell}$ <<1), the phase rotation $\exp\{j\text{mod}(k;N_F)/N_F\,\varepsilon_{i,n,m,\ell}T\}$ at the FFT outputs can be approximated by a truncated Taylor series: $\exp\{j\text{mod}(k;N_F)/N_F\,\varepsilon_{i,n,m,\ell}T\}\approx 1+j\text{mod}(k;N_F)/N_F\,\varepsilon_{i,n,m,\ell}T$.

When the load is maximum ($N_u=N_s$) and the jitter spectrum is the same for all users: $S_{\varepsilon,\ell}(f)= S_\varepsilon(f)$, (hence $\sigma^2_{\varepsilon,\ell}= \sigma^2_\varepsilon$), the degradation in the uplink is the same as in the downlink. This degradation depends on the carrier index k. The average degradation, which is obtained by replacing in the SINR the powers of the useful component, the interference and the noise by their arithmetical average over all carriers is given by:

$$Deg_{av} \approx 10\log\left(1+SINR(0)\frac{\pi^2}{3}\sigma^2_\varepsilon\right) \tag{9}$$

which is independent of the number of carriers, the spreading factor and the spectral contents of the jitter, but only depends on the jitter variance.

4. CONCLUSIONS

To evaluate the effect of small synchronization errors on the BER performance of the MC-DS-CDMA system, we derive simple analytical expressions for the BER degradation that are based upon truncated Taylor series expansions. Computer simulations indicate that the degradation obtained from the Taylor series expansion yields a good approximation of the actual degradation. For both the uplink and the downlink, we compare the degradation caused by different types of synchronization errors. Assuming the load is maximum ($N_u=N_s$), and noting that the number of carriers N_c is proportional to the FFT length N_F, the results can be summarized as follows:

a) Constant phase offsets or constant timing offsets do not give rise to performance degradation, for neither uplink nor downlink MC-DS-CDMA, because these offsets can be compensated for at the FFT outputs.

b) The MC-DS-CDMA system is strongly degraded in the presence of carrier frequency offsets. This degradation is proportional to $(N_c\Delta FT)^2$, and the degradation in the uplink is N_s^2 times higher than in the downlink, as in the downlink the amount of multi-user interference is much higher than in the downlink.

c) For both the uplink and the downlink, the degradation caused by clock frequency offsets strongly increases with $(N_c\Delta T/T)^2$. The degradation in the uplink is a factor N_s^2 higher than in the downlink, as the uplink is affected by a larger amount of multi-user interference.

d) When the spectrum of the carrier phase jitter or timing jitter is the same for all users, the degradation caused by carrier phase jitter or timing jitter is the same in the uplink and the downlink. The corresponding degradation is independent of the spectral contents of the jitter, the spreading factor and the number of carriers, but only depends on the jitter variance.

5. ACKNOWLEDGMENT

This work has been supported by the Interuniversity Attraction Poles Program - Belgian State - Federal Office for Scientific, Technical and Cultural Affairs.

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7. AFFILIATIONS

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