# The Effect of Narrowband Interference on Timing Synchronization for OFDM Systems

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## Abstract

*Orthogonal frequency division multiplexing (OFDM) has been of major interest for both wire line and wireless applications due its high data rate transmission capability and its robustness to multipath delay spread. However, OFDM systems can be extremely sensitive and vulnerable to synchronization errors. This paper investigates the effect of narrowband interference on timing synchronization for OFDM systems. The performance of the Schmidl & Cox (S&C) symbol timing synchronizer [1] is evaluated in an analytical way in the presence of narrowband interferers. Further, simulations have been carried out to verify the validity of approximations in the analysis. It is shown that the S&C algorithm is robust to narrowband interference as long as the signal to interference ratio is not too low.*
***Keywords:*** *Orthogonal frequency division multiplexing, symbol timing synchronization, training symbol, and interference.*

## 1 System Description

The basic block diagram of the OFDM system and NBI signal is shown in Fig.1. In the OFDM transmitter, the data stream is grouped in blocks of $N_u$ data symbols. Next, an $N$- point inverse fast Fourier transform (IFFT) is performed on each data block, where

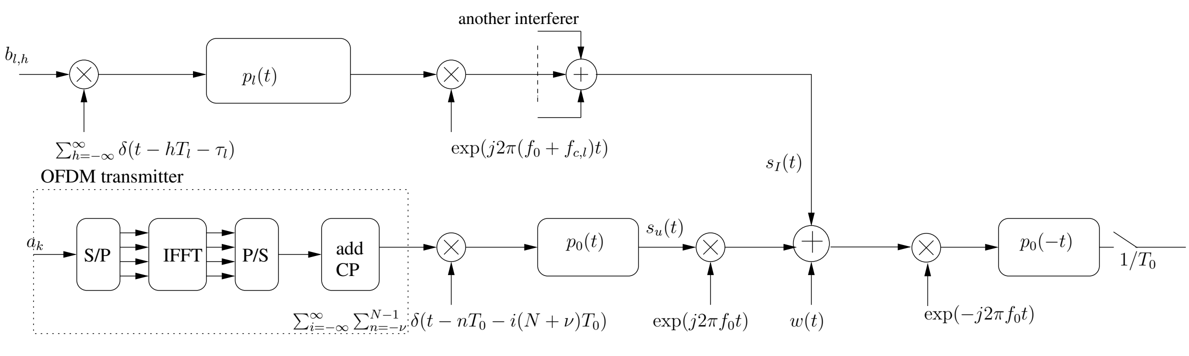


Figure 1: block diagram of OFDM system including one interfering signal

$N > N_u$, and a cyclic prefix of length $\nu$ is inserted [2,3]. The $n$th time domain sample of the $i$th OFDM block can be written as

$$s^i(n) = \sqrt{\frac{1}{N+\nu}} \sum_{k \in I_u} a_k^i \, e^{\frac{j2\pi kn}{N}} \qquad -\nu \leq n \leq N-1 \tag{1}$$

where $I_u$ is a set of $N_u$ carrier indices and $a_k^i$ is the $k$th data symbol of the $i$th OFDM block; data symbols are independently and identically distributed with zero mean and variance $E\left[|a_k^i|^2\right] = E_s$. The time domain signal of the baseband OFDM signal $s_u(t)$ consists of the concatenation of all time domains blocks $s^i(n)$:

$$s_u(t) = \sum_{i=-\infty}^{\infty} \sum_{n=-\nu}^{N-1} s^i(n) \, p_0(t - nT_0 - i(N+\nu)T_0) \tag{2}$$

where $p_0(t)$ is the transmit pulse of the OFDM system and $1/T_0$ is the sample rate. The transmit pulse $p_0(t)$ is assumed to be a square-root-raised cosine filter with rolloff $\alpha_0$. The baseband signal (2) is up-converted to the radio frequency $f_0$. At the receiver, the signal is first down-converted, then fed to the matched filter and finally sampled at rate $1/T_0$, resulting in the samples $r_u(mT_0)$. Note that, when the number of carriers $N$ is large, the sample $s^i(n)$ consists of a large number of contributions. Hence, taking into account the central limit theorem, the real and imaginary parts of $s^i(n)$ can be modeled as Gaussian random variables with zero mean and variance $\sigma_s^2$ equal to $\frac{E_s \cdot N_u/2}{N+\nu}$. The OFDM signal is disturbed by additive white Gaussian noise with uncorrelated real and imaginary parts, each having variance $\sigma_n^2$. The signal to noise ratio ($SNR$) at the output of the matched filter is defined as $\frac{\sigma_s^2}{\sigma_n^2}$.

Further, the signal is disturbed by narrowband interference residing within the same frequency band as the wideband OFDM signal as shown in Fig. 2. The interfering signal $s_I(t)$ may be modeled as the sum of $N_I$ narrowband interfering signals

$$s_I(t) = \sum_{l=1}^{N_I} s_l(t) \tag{3}$$

where $s_l$(t) is the $l$th NBI component:

$$s_l(t) = \sum_{h=-\infty}^{\infty} b_{l,h} p_l(t - hT_l - \tau_l) \cdot e^{j2\pi(f_0 + f_{c,l})t} \tag{4}$$

where $b_{l,h}$ is the $h$th interfering data symbol of the $l$th interferer, $p_l(t)$ is the transmit pulse of the $l$th interferer, $\tau_l$ is its delay, and $1/T_l$ its sample rate; $p_l(t)$ is assumed to be a square root-raised cosine filter with rolloff $\alpha_l$. The interfering signal is modulated to radio frequency $f_0 + f_{c,l}$, where $f_{c,l}$ is the carrier frequency deviation from $f_0$ for the $l$th interferer. The total NBI signal may be seen at the output of matched filter of OFDM receiver as

$$r_I(t) = \sum_{l=1}^{N_I} \sum_{h=-\infty}^{\infty} b_{l,h} \, e^{j2\pi f_{c,l} hT_l} \, g_l(t - hT_l) \tag{5}$$

where $g_l(t)$ is the convolution of $p_0(t)$ and $p_l(t-\tau_l)\exp(j2\pi f_{c,l} t)$. It is assumed that the interfering symbols are uncorrelated with each other, i.e. $E[b_{l,h} b^*_{l',h'}] = E'_l \delta_{ll'} \delta_{hh'}$, where

$E_l'$ is the energy per symbol of the $l$th interferer. Further, the interfering data symbols $b_{l,h}$ are statistically independent of the OFDM data symbols $a_k^i$. The signal to interference ratio ($SIR$) at the input of the receiver is defined as

$$SIR = \frac{2\sigma_s^2/T_0}{\sum_{l=1}^{N_I} \frac{E_l'}{T_l}} \tag{6}$$

## 2 Statistical Properties of the Timing Metric in the Presence of NBI

In the S&C algorithm [1], the training symbol contains two identical halves in the time domain and the timing delay estimator searches for the peak of correlation between the matched filter output samples that are separated by half an OFDM symbol. The sample $r(mT_0)$ at the output of the matched filter of the OFDM receiver consists of a useful signal $r_u(mT_0)$, an interfering $r_I(mT_0)$, and a noise $w(mT_0)$ component. It may be expressed as

$$r(mT_0) = r_u(mT_0) + r_I(mT_0) + w(mT_0) \tag{7}$$

The symbol timing estimator takes the instant $d$, where the timing metric

$$M(d) = \frac{|P(d)|^2}{R(d)^2} \tag{8}$$

is maximum, as the starting point of the frame. In (8), $P(d)$ and $R(d)$ are given by

$$P(d) = \sum_{m=0}^{N/2-1} r^*\left((d+m)T_0\right) \cdot r\left((d+m+N/2)T_0\right) \tag{9}$$

$$R(d) = \sum_{m=0}^{N/2-1} \left|r\left((d+m+N/2)T_0\right)\right|^2 \tag{10}$$

This timing metric is not only used to find the optimum timing instant, but also to determine whether or not a training sequence is received. To do this, we use a threshold: when the timing metric exceeds this threshold for at least one value of $d$, we decide that it is possible to detect a training sequence, whereas when the threshold is never exceeded, we decide that it is not possible to detect a training sequence. The value of the threshold must be selected such that the probability of missing a training symbol when there is one present, and the probability of falsely detecting a training sequence when there is none present, are as small as possible. To find these probabilities, and hence the threshold, we need to know the statistical properties of the timing metric. In [1], the statistical properties of the timing metric at the optimum timing point and a position outside the training sequence were investigated for a AWGN channel. In this section, we extend the results of [1] to obtain the statistical properties of the timing metric in the presence of NBI.

## 2.1 Optimum Timing Point

Let us define $q = \frac{|P(d_{opt})|}{R(d_{opt})}$ as the square root of the timing metric $M(d_{opt})$ at the optimum timing point $d_{opt}$. To simplify the notation, we drop the subscript 'opt'. Due to the symmetry of the pilot symbol, the signal component $\sum_{m=0}^{N/2-1} r_u^*((m+d)T_0) \cdot r_u((m+d+N/2)T_0)$ of $P(d)$ is real valued. It can easily be shown that the other components of $P(d)$ have zero mean and small variance when the $SIR$ and $SNR$ are sufficiently large. Therefore, at high values of $SIR$ and $SNR$, the imaginary part of $P(d)$ can be neglected. From (7), (9), and (10), it can be easily verified that $|P(d)|$ and $R(d)$ contain a common term $\alpha$. Hence, we can rewrite $|P(d)|$ and $R(d)$ as, $|P(d)| = \alpha + \beta$ and $R(d) = \alpha + \gamma$. It can easily be shown that the expressions for $\alpha$, $\beta$, and $\gamma$ consist of a large number of contributions when $N$ is large. Therefore, according to the central limit theorem, the variables $\alpha$, $\beta$, and $\gamma$ have approximate Gaussian distributions. Taking into account that for both the numerator and denominator of $q$, standard deviations are much smaller than the averages, $q$ can be approximated by a Gaussian variable [1] with mean and variance given by the following approximations:

$$\mu_q \approx \frac{\mu_\alpha + \mu_\beta}{\mu_\alpha + \mu_\gamma} \tag{11}$$

$$\sigma_q^2 \approx \mu_q^2 \left( \frac{\sigma_\beta^2}{(\mu_\alpha + \mu_\beta)^2} + \frac{\sigma_\gamma^2}{(\mu_\alpha + \mu_\gamma)^2} \right) \tag{12}$$

where $\mu_x$ and $\sigma_x^2$ is the mean and variance of $x$ respectively. In (12), we have used the approximation $a(1+b)/(1+c) \simeq a(1+b-c)$ as in [1]. Taking into account that $q$ is approximately a Gaussian random variable, and $M(d)$ is the square of $q$, it follows that $M(d)$ also is approximately a Gaussian variable [4]:

$$\begin{aligned} M(d) &= \left(\mu_q + N\left(0\,,\,\sigma_q^2\right)\right)^2 \\ &\approx \mu_q^2 + 2 \cdot \mu_q \cdot N\left(0\,,\,\sigma_q^2\right) \end{aligned} \tag{13}$$

where $N(x, y)$ is a Gaussian random variable with mean $x$ and variance $y$. The mean of $M(d)$, i.e. $\mu_M$, can be easily computed calculated by using (11):

$$\mu_M = \left( \frac{N + \frac{1}{\sigma_s^2} \sum_{l=1}^{N_I} E_l' \Upsilon_l}{N + \frac{N}{SNR} + \frac{1}{\sigma_s^2} \sum_{l=1}^{N_I} E_l' \phi_l} \right)^2 \tag{14}$$

where $\Upsilon_l$ and $\phi_l$ are given as :

$$\Upsilon_l = Re \sum_{m=0}^{N/2-1} \sum_{h=-\infty}^{\infty} g_l^*((m+d)T_0 - hT_l) \cdot g_l((m+d+N/2)T_1 - hT_l) \tag{15}$$

$$\phi_l = \sum_{m=0}^{N/2-1} \sum_{h=-\infty}^{\infty} |g_l(m+d+N/2)T_0 - hT_l)|^2 \tag{16}$$

It can be shown that $\Upsilon_l \ll \phi_l$. Hence, the effect of the interference on the numerator of (14) will be much smaller than its effect on the denominator. Therefore, decreasing the $SIR$ will result in a decrease of $\mu_M$. According to (13), the variance of $M(d)$ is given by

$$\sigma_M^2 = 4 \cdot \mu_q^2 \cdot \sigma_q^2 \tag{17}$$

where $\sigma_q^2$ can be easily computed by using the equations (11) and (12).

### 2.2 Timing position outside of training sequence

In this section, the statistical properties of a timing position outside of the training sequence are considered, when NBI is present. Using a similar analysis as in [1], where the analysis was done for AWGN channel and no NBI, it can easily be verified that for sufficient large $SNR$ and $SIR$, the timing metric is approximated by a chi-square distributed random variable :

$$M(d_{outside}) = \frac{D^2}{2G^2}\chi^2(2) \tag{18}$$

where $\chi^2(2)$ is a chi-square distributed random variable with two degrees of freedom and its mean and variance equal 2 and 4 respectively; $G$ and $D^2$ are given in (22) and (21) respectively. Therefore, the mean and the variance of $M(d_{outside})$ are given by:

$$\mu_{M(d_{outside})} = \frac{D^2}{G^2} \tag{19}$$

$$\sigma^2_{M(d_{outside})} = \frac{D^4}{G^4} \tag{20}$$

where $D^2$ and $G$ are given as:

$$D^2 = N/2\left(2\sigma_s^2 + 2\sigma_n^2\right)^2 + \sum_{m=0}^{N/2-1} A^{(m,m)}(0,0) \cdot A^{(m,m)}(N/2, N/2) \tag{21}$$

$$G = N\left(\sigma_s^2 + \sigma_n^2\right) + \sum_{m=0}^{N/2-1} A^{(m,m)}(N/2, N/2) \tag{22}$$

and $A^{(m,m)}(x,y)$ is defined as

$$A^{(m,m')}(x,y) = \sum_{l=1}^{N_I}\sum_{h=-\infty}^{\infty} E'_l\, g_l^*\left((m+d+x)T_0 - hT_l\right) \cdot g_l\left((m'+d+y)T_0 - hT_l\right) \tag{23}$$

Note that these results are the same as in [1] when $SIR$ tends to infinity.

# 3 Analytical and Simulation Results

The numerical and simulation results in this paper are obtained with the following OFDM and interference parameters: the total number of subcarriers is $N$ = 1024, the total number of active subcarriers is $N_u$ = 1000, the guard interval is set to about 10 % of useful part, $\nu = 102$, the bandwidth of OFDM spectrum, $B_0 = 1024$ kHz, QPSK modulation are used for the data symbols of the OFDM and the interferer signals, transmit filters are square-root raised-cosine filters with roll off factors $\alpha_0 = 0.25$ and $\alpha_l = 0.5$ for OFDM and interfering signals, respectively, and time delay of the interferers is $\tau_l = 0$.

Fig.3 compares the analytical results with the simulation for the mean and the variance of the timing metric at the optimum timing instant, as function of the $SIR$ at $NBW = .0244$, $\Gamma = 0$, and $N_I = 1$ . $NBW$ is defined as normalized interference bandwidth, i.e $NBW = \frac{B_0}{B_l}$, $\Gamma$ is defined as normalized interference frequency, i.e $\Gamma = \frac{f_{c,l}}{(1+\alpha_0)/2T_0}$, and $N_I$ is the number of interference signals. As can be observed, for a large range of $SIR$, the theoretical results and the simulation results agree well. At large $SIR$, the mean of the timing metric reaches an asymptote, corresponding to the case where no interference is present. This asymptote only depends only on the $SNR$ and is given by $(1 + 1/SNR)^{-2}$. At very low $SIR$, the mean of the timing metric becomes independent of the $SNR$. The theoretical results give rise to a maximum timing metric that is smaller than for the simulations when the $SIR$ is small. Hence, the theoretical results can be considered as a lower bound on the performance for small $SIR$. The variance of the timing metric decreases for increasing $SIR$. The variance at large $SIR$ decreases with increasing $SNR$. This can easily be explained as at high $SIR$ and $SNR$, the effect of the noise and interference is small, such that the symmetry of the pilot symbol is almost not affected, and the timing metric will be close to its average. At low $SIR$, the variance decreases with decreasing $SIR$. The explanation for this is that at low $SIR$, the effect of the interferer on the timing metric starts to to dominate. As the contribution of the interferer to the variance of the timing metric is smaller than the contribution of the data symbols and noise, the overall variance will reduce. We also observed that at low $SIR$, the variance obtained with the theoretical results diverges from the obtained with the simulations. As the standard deviation (i.e. the square root of the variance) in this case is of the order of the mean, it is clear that the performance of S&C synchronization estimator will be poor if the $SIR$ is too small.

Fig. 4 displays the analytical and simulation results of mean and variance of the metric at the optimum timing point as function of the interference bandwidth for different values of $SIR$. The results indicate that at given $SIR$, the mean is independent of the interference bandwidth. This can be explained as follows: it can easily be shown that $\phi_l$ increases linearly with the interferer bandwidth, and according to the $SIR$ definition (6) at given $SIR$ value, $E_l'$ decreases linearly with the $NBW$, so for given $SIR$, $E_l' \cdot \phi_l$ is a constant. Taking into account the fact that $\Upsilon_l \ll \phi_l$, it is clear from (14) that at given value of $SIR$, $\mu_M$ is essentially independent of interference bandwidth. Also it is observed from Fig. 4 that the dependency of the variance on the interference bandwidth increases when $SIR$ decreases. This is explained as at high $SIR$, the effect of interference diminishes and the variance mainly depends on noise. At low $SIR$, the variance decreases when the interference bandwidth increases.

Fig 5. shows the mean and the variance of the timing metric at the optimum synchronization point as function of the number of interfering signals, $N_I$, in two cases. In case 'A', we consider that the $SIR$ is fixed per interferer, so the total SIR decreases inversely proportional to $N_I$. In case 'B', we consider a fixed total $SIR$, i.e. $SIR$ per interferer decreases linearly as $N_I$ increases. As the mean and variance mainly depends on the total $SIR$, it follows that the mean and variance will be decreasing functions of the number $N_I$ of interferers in case 'A', whereas they are independent of $N_I$ in case 'B'.

Fig. 6 illustrates the analytical and simulation results of the mean and the variance of the timing metric at a timing instant outside the training symbol as function of $SIR$. At high $SIR$ values, the interference signal does not affect the statistical properties of timing metric outside training symbol. Note that the simulation results are slightly different from the analytical results. This is caused by correlations between the signal terms $r^*((m+d)T_0)$ and $r((m+d+N/2)T_0)$ outside the training symbol, while the theoretical analysis assumed that they were independent.

## 4 Conclusions

This paper evaluates the performance of the S&C symbol timing estimator [1] in the presence of narrowband interference for different interference signal parameters. The statistical properties of the estimator are evaluated analytically and by means of simulation at the optimum synchronization point and outside the training sequence. The agreement between the theoretical and simulation results proves the validity of our analysis. Generally, results indicate that the estimator works well for a wide range of signal to interference ratios. At high $SIR$ values ( > 10 dB), results show that noise is the dominant factor on the statistical properties of the estimator. Furthermore, at given value of $SIR$, the interference bandwidth does not influence the mean of the metric while the dependency of the variance of the metric on the interferer bandwidth increases when $SIR$ decreases. Results show that the statistical properties of the estimator at optimum timing point do not depend on the $SIR$ per interferer but only depends on the total $SIR$. Finally, at high values of $SIR$, results indicate that statistical properties of the estimator at outside training symbol are essentially independent of the interference signal.

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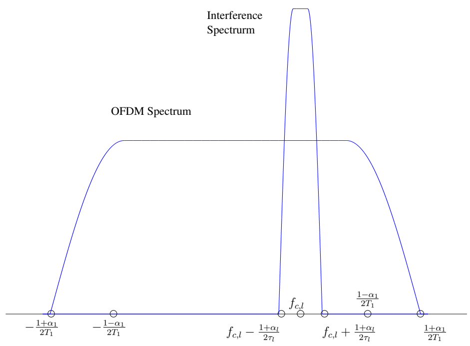


Figure 2: Baseband OFDM and one interfering signal spectrum

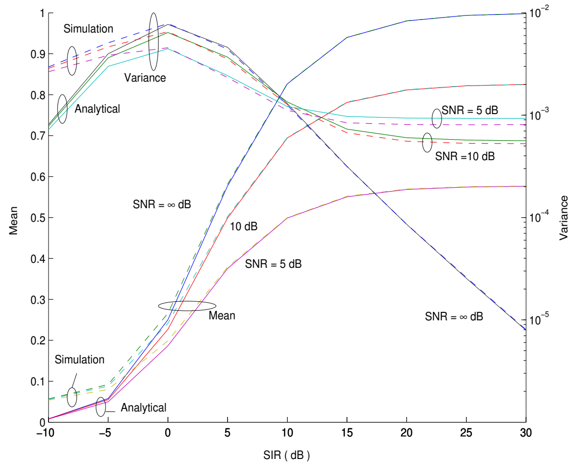


Figure 3: Mean and variance of timing metric at optimum point versus $SIR$ ($NBW = .0244,\ N_I = 1,\ \Gamma = 0$ )

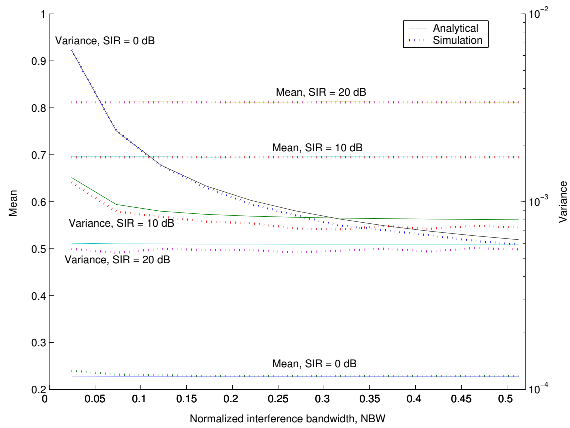


Figure 4: Mean and variance of timing metric at optimum point versus $NBW$ ($SNR = 10\,dB,\ \Gamma = 0,\ N_I = 1$)

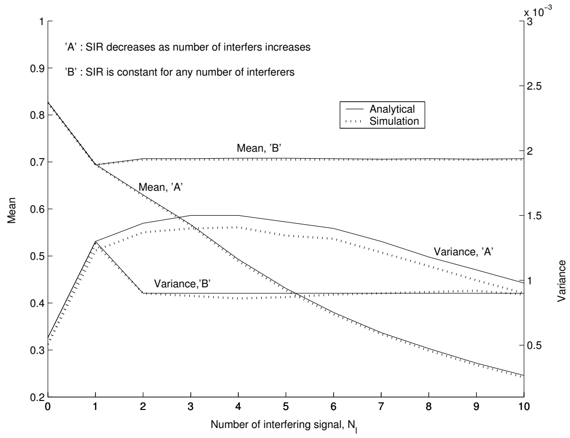


Figure 5: Mean and variance of timing metric at optimum point versus $N_I$, $SNR$ = 10 dB , $SIR$ = 10 dB, ($NBW = .0244,\ \Gamma = 0$)

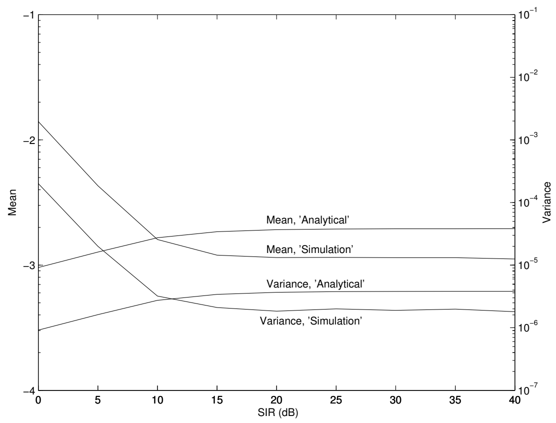


Figure 6: Mean and variance of timing metric outside training sequence versus $SIR$ at $SNR$ = 10 dB, ($NBW = .0244,\ \Gamma = 0,\ N_I = 1$)