The Effect of Narrowband Interference on Frequency Ambiguity Resolution for OFDM

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***Abstract*— In orthogonal frequency division multiplexed (OFDM) systems affected by carrier frequency offsets, frequency ambiguity resolution, i.e. the estimation of the part of the frequency offset corresponding to an integer times the carrier spacing, is a crucial issue. The proper action of frequency ambiguity resolution algorithms can be strongly affected by the presence of disturbances, like narrowband interference (NBI). In this paper, the susceptibility of the blind and data aided ML frequency ambiguity estimators to NBI signals is investigated in an analytical way. The analytical results are verified by means of simulations. Although the estimators turn out to be essentially independent of the bandwidth of the interferers and the number of interferers, the performance of the estimators is very sensitive to the positions of the interferers.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is an efficient transmission technique for high speed data transmission. OFDM is already used or standardized in several wireline and wireless applications such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), universal mobile telecommunication system (UMTS), and wireless local area networks (WLAN's) [1]–[3]. The principle weakness of OFDM is its sensitivity to carrier frequency offset (CFO) caused by Dopper shifts and/or oscillator instabilities [4]. A CFO results in a shift of the received signal spectrum in the frequency domain. The CFO can be divided into an integer and a fractional part with respect to the OFDM subcarrier spacing δf. If the integer part of the CFO equals I and the fractional part is zero, then the received subcarriers are shifted by $I \cdot \delta f$ in the frequency domain: the subcarriers are still mutually orthogonal, but the received data symbols, which are mapped to the OFDM spectrum, are in the wrong positions in the demodulated spectrum, resulting in a BER of .5 [5]. In this paper, we concentrate on the estimation of the integer part of the CFO, i.e. the frequency ambiguity resolution. We assume that the fractional part of the CFO can be correctly estimated with another algorithm, and is corrected before the estimation of the integer CFO.

Narrowband interference (NBI) is a major impairment for broadband transmission over wired and wireless channels. The interfering signal may arise in communication systems where OFDM based systems coexist with narrowband systems. Wires, in particular the last meters to the subscriber as well as the in-house wiring, act as antennas which pick up radio signals from their environment. Similarly in unlicensed frequency band wireless systems, NBI may occur, e.g., the industrial-scientific-medical (ISM) band, where NBI signals interfere with OFDM-based wireless local area network (WLANs) such as Hiperlan II and IEEE 802.11a [3].

In some applications, the OFDM system must coexist with these NBI signals. The presence of the NBI signals can hamper the proper action of the synchronization algorithms used to synchronize the OFDM system [6]. In this paper, the effect of NBI on the performance of blind and data aided ML integer CFO estimators for OFDM system is investigated.

II. System Description

The basic block diagram of the OFDM system and NBI signal is shown in Fig.1. In the OFDM transmitter, the data stream is grouped in blocks of N_u data symbols. Next, an N-point inverse fast Fourier transform (IFFT) is performed on each data block, where $N > N_u$, and a cyclic prefix (CP) of length ν is inserted. The kth time domain sample of the ith OFDM block can be written as

$$s_u^i(k) = \sqrt{\frac{1}{N+\nu}} \sum_{n \in I_u} a_{n,i} \, e^{\frac{j2\pi kn}{N}} \quad -\nu \leq k \leq N-1 \quad (1)$$

where I_u is a set of N_u carrier indices and $a_{n,i}$ is the nth data symbol of the ith OFDM block; the data symbols are i.i.d[1] random values with zero mean and variance $E\left[|a_{n,i}|^2\right] = E_s$. The time domain signal of the baseband OFDM signal $s_u(t)$ consists of the concatenation of all time domain blocks $s_u^i(k)$:

$$s_u(t) = \sum_{i=-\infty}^{\infty} \sum_{k=-\nu}^{N-1} s_u^i(k)\, p_0(t - kT_0 - i(N+\nu)T_0) \quad (2)$$

where $p_0(t)$ is the unit-energy transmit pulse of the OFDM system and $1/T_0$ is the sample rate. The baseband signal (2) is up-converted to the radio frequency f_0. At the receiver, the signal is first down-converted to $-(f_0 - \Delta f)$, where Δf represents the frequency difference between transmitter and receiver oscillator, then fed to the matched filter and finally sampled at rate $1/T_0$. Note that, when the number N_u of modulated carriers is large, the sample $s_u^i(k)$ consists of a large number of contributions. Hence, taking into account the central limit theorem, the real and imaginary parts of $s_u^i(k)$ can be modeled as Gaussian random variables with zero mean and variance $\sigma_s^2 = \frac{E_s \cdot N_u/2}{N+\nu}$. The OFDM signal is disturbed by additive white Gaussian noise with uncorrelated real and imaginary parts, each having variance σ_n^2. The signal to noise

[1] i.i.d = independently and identically distributed

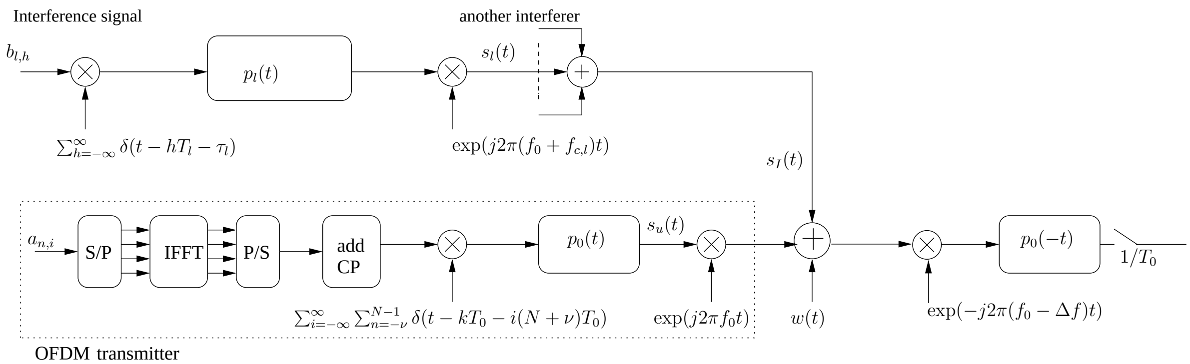

Fig. 1. Block diagram of OFDM system including interfering signals

ratio (SNR) at the output of the matched filter is defined as $\frac{\sigma_s^2}{\sigma_n^2}$.

Further, the signal is disturbed by narrowband interference residing within the same frequency band as the wideband OFDM signal as shown in Fig. 2. The interfering signal $s_I(t)$ may be modeled as the sum of N_I narrowband interfering signals

$$s_I(t) = \sum_{l=1}^{N_I} s_l(t) \tag{3}$$

where $s_l(t)$ is the lth NBI component:

$$s_l(t) = \sum_{h=-\infty}^{\infty} b_{l,h} p_l(t - hT_l - \tau_l) \cdot e^{j2\pi(f_0+f_{c,l})t} \tag{4}$$

where $b_{l,h}$ is the hth interfering data symbol of the lth interferer, $p_l(t)$ is the unit-energy transmit pulse of the lth interferer, τ_l is its delay, and $1/T_l$ its sample rate. The lth interfering signal is modulated to radio frequency $f_0 + f_{c,l}$, where $f_{c,l}$ is the carrier frequency deviation from f_0 for the lth interferer. The total NBI signal may be seen at the output of the matched filter of the OFDM receiver as

$$r_I(t) = \sum_{l=1}^{N_I} \sum_{h=-\infty}^{\infty} b_{l,h}\, e^{j2\pi f_{c,l} hT_l}\, g_l(t - hT_l) \tag{5}$$

where $g_l(t)$ is the convolution of $p_0(-t)$ and $p_l(t-\tau_l)\exp\left(j2\pi\left(f_{c,l}+\Delta f\right)t\right)$. The normalized location of the interferer within the OFDM spectrum may be defined as $f'_{c,l} = \left(\frac{f_{c,l}+\Delta f}{B_0}\right)$. It is assumed that the interfering symbols are uncorrelated with each other, i.e. $E[b_{l,h}b^*_{l',h'}] = E'_l\delta_{ll'}\delta_{hh'}$, where E'_l is the energy per symbol of the lth interferer. Further, the interfering data symbols $b_{l,h}$ are statistically independent of the OFDM data symbols $a_{n,i}$. The signal to interference ratio (SIR) at the input of the receiver is defined as [6]

$$SIR = \frac{2\sigma_s^2/T_0}{\sum_{l=1}^{N_I}\frac{E'_l}{T_l}} \tag{6}$$

III. ML Integer CFO Estimator

As the frequency offset Δf is generally larger than the sub-carrier spacing, it is useful to split it into an integer part m and fractional part ϵ, where $\epsilon \in [-.5,\ .5[$, with respect to the carrier spacing $\delta f = \frac{1}{NT_0}$, i.e. $\Delta f = \frac{m}{NT_0} + \frac{\epsilon}{NT_o}$. As in [7], we made the following assumptions: 1) The parameter ϵ, has already been estimated and is perfectly corrected. 2) A total of N_p pilots symbols are inserted at known locations in each OFDM block. They satisfy the relation $a_{n,0}a^*_{n,1} = d_n \ \forall\, n \in P$, where d_n is a pseudonoise sequence known at the receiver and P is the set of the pilot-symbol locations (P is empty when $N_p = 0$). 3) All symbols (known and unknown) belong to a PSK constellation, have zero mean and the following second order statistics:

$$E\left[a_{n1,j}a_{n2,f}\right] = 0 \quad -N_u \le n1,\, n2 \le N_u \quad j,f \in \{0,1\} \tag{7}$$

$$E\left[a_{n1,j}a^*_{n2,f}\right] = \begin{cases} 1 & n1 = n2 \quad\quad j = f \in \{0,1\} \\ d_k & n1,\, n2 \in P \quad j=0 \quad f=1 \\ d^*_k & n1,\, n2 \in P \quad j=1 \quad f=0 \\ 0 & \quad\quad elsewhere \end{cases} \tag{8}$$

We assume that two consecutive OFDM blocks (with indices $i = 0$ and $i = 1$) are observed. The time domain samples outside the CP are given by

$$x_i(k) = e^{j[2\pi(m+\epsilon)(k+i(N+\nu))/N]} s^i_u(k) + w^i(k) + r^i_I(k) \tag{9}$$

where $0 \le k \le N-1,\ \ i = 0,1,\ s^i_u(k)$ is given in (1) , $w^i(k)$ is the AWGN component and $r^i_I(k)$ is the kth interference sample in ith block given in (5).

In this paper, it is assumed that the effect of the fractional frequency offset $\frac{\epsilon}{NT_0}$ is compensated. This is done by rotating the samples $x_i(k)$ in (9) at the angular speed $\frac{-2\pi\epsilon}{N}$ per sample, resulting in samples $z_i(k) = x_i(k)\, e^{-j2\pi\epsilon(k+i(N+\nu))/N}\ \forall\ i = 0,1$. The rotated samples $z_i(k)$ are fed to an N-point FFT. The nth FFT output of the ith received OFDM block is given by $Z_i(n)$. To estimate m, we use the ML estimator from [7]

$$\widehat{m} = arg\ max_{|\tilde{m}|\le M}\ \left\{\Lambda\left(\tilde{m}\right)\right\} \tag{10}$$

where $\widehat{m}$ is the estimated value of m, $\tilde{m}$ is the trial value of m, M represents the largest expected value of $|m|$ and $\Lambda\left(\tilde{m}\right)$ is given as

$$\Lambda\left(\tilde{m}\right) = \begin{array}{l} \sum_{n=-N_u}^{N_u}\left[\left|Z_0(n+\tilde{m})\right|^2 + \left|Z_1(n+\tilde{m})\right|^2\right] \\ +2Re\left\{\sum_{n\in P} d_n Z^*_0(n+\tilde{m}) Z_1(n+\tilde{m}) e^{-j\vartheta(\tilde{m})}\right\} \end{array} \tag{11}$$

where $\vartheta(\tilde{m}) = 2\pi\tilde{m}(N+\nu)/N$. In the absence of pilot symbols, P is empty and (11) reduces to

$$\Lambda(\tilde{m}) = \sum_{n=-N_u}^{N_u} \left[|Z_0(n+\tilde{m})|^2 + |Z_1(n+\tilde{m})|^2 \right] \quad (12)$$

In the following, the estimator corresponding to (12) is referred to as the blind estimator (BE), whereas the estimator corresponding to (11) is called the pilot estimator (PE).

To evaluate the effect of NBI on these estimators, we derive the upper bound on the probability of failure $P_f = Pr\{\widehat{m} \neq m\}$ of the estimators. Let $A(\widetilde{m}, m)$ be the event that $\Lambda(\widetilde{m}) > \Lambda(m)$ where $\widetilde{m} \neq m$. Therefore P_f can be expressed as

$$P_f = Pr\left\{ \bigcup_{\widetilde{m}=-M, \widetilde{m}\neq m}^{M} A(\widetilde{m}, m) \right\} \quad (13)$$

This can be upper bounded using the union bound approximation [8],

$$P_f \leq \sum_{\widetilde{m}=-M, \widetilde{m}\neq m}^{M} Pr\{\Lambda(\widetilde{m}) > \Lambda(m)\} \quad (14)$$

Note that when the number of modulated subcarriers N_u is large, $\Lambda(\tilde{m})$ consists of a large number of contributions. Hence taking into account the central limit theory, $\Lambda(\tilde{m})$ can be modeled as a Gaussian random variable. Let us assume $\Lambda(\widetilde{m}) \sim N\left(\mu_{\tilde{m}}, \sigma^2_{\tilde{m}}\right)$[2], $|\widetilde{m}| \leq M$ and the covariance between $\Lambda(\widetilde{m})$ and $\Lambda(m)$, $\widetilde{m} \neq m$ is given by $\sigma^2_{\tilde{m}m}$. We define $H(\widetilde{m}, m) = \Lambda(\widetilde{m}) - \Lambda(m)$; $H(\widetilde{m}, m)$ is a Gaussian random variable with mean $\mu_H(\widetilde{m}, m) = \mu_{\tilde{m}} + \mu_m$ and variance $\sigma^2_H(\widetilde{m}, m) = \sigma^2_{\tilde{m}} + \sigma^2_m - 2\sigma^2_{\tilde{m}m}$. Hence

$$Pr\{\Lambda(\widetilde{m}) > \Lambda(m)\} = Q\left(\frac{-\mu_H(\widetilde{m}, m)}{\sigma_H(\widetilde{m}, m)} \right) \quad (15)$$

Therefore, the upper bound on the probability of failure is given as

$$P_f \leq \sum_{\widetilde{m}=-M, \widetilde{m}\neq m}^{M} Q\left(\frac{-\mu_H(\widetilde{m}, m)}{\sigma_H(\widetilde{m}, m)} \right) \quad (16)$$

where $\mu_H(\widetilde{m}, m)$ and $\sigma_H(\widetilde{m}, m)$ are derived in the Appendix. In the next section, we will check the validity of the assumptions leading to (16) by means of simulations.

IV. Numerical Results

The numerical results in this paper are obtained with the following OFDM and interference parameters: Transmit filters are square-root raised-cosine filters with roll off factors $\alpha_0 = 0.25$ and $\alpha_l = 0.5$ for OFDM and interfering signals, respectively. The total number of subcarriers is N = 1024. The total number of active subcarriers is N_u = 1000. The carriers close to the edge of the OFDM spectrum are not used (virtual carriers). The guard interval is set to about 10 % of the useful part, $\nu = 102$. The bandwidth of the OFDM spectrum, $B_0 = \frac{1}{T_0} = 1024$ kHz. We use QPSK modulation for the data symbols of the OFDM and the interferer signals. The pilot symbols are uniformly distributed over the used carriers. The time delay of the interferers $\tau_l = 0$.

In Fig. 3, the probability of failure P_f , based on the obtained upper bound expression (16), is shown as function of the signal to interference ratio (SIR) for blind $(N_p = 0)$ and pilot estimators $(N_p \neq 0)$. We have assumed that there is one interference signal $(N_I = 1)$ with normalized interference bandwidth, $NBW = \frac{B_1}{B_0} = 0.0244$ (where B_1 is the interference bandwidth) and normalized interference frequency $f'_{c,l} = 0.5$. Further, simulation results on the probability of failure are shown. As expected, the pilot estimator outperforms the blind estimator. In the figure, the probability of failure is added for the case where no NBI is present $(SIR = \infty)$. At high SIR, the probability of failure clearly converges to the curve corresponding to no NBI: at high SIR, the effect of the NBI diminishes and AWGN dominates. Further, it can clearly be observed that increasing the number of pilot symbols leads to better performance. Although the upper bound agrees with the simulation results, it can be observed that the upper bound slightly overestimates the simulated probability of failure, especially for the blind estimator $(N_p = 0)$. As can be observed from (11), $\Delta(\tilde{m})$ for the pilot estimator contains more contributions than the blind estimator. Therefore, it follows that the Gaussian approximation of $\Delta(\tilde{m})$ for the pilot estimator is better than for the blind estimator. Hence, the upper bound and the simulation results better agree in the case of pilot estimator. It can easily be shown that although $\Lambda(\widetilde{m})$ can be well approximated by a Gaussian variable, the small deviations cause an overestimation of the probability of failure in (16).

Fig. 4 compares the probability of failure P_f obtained with (16) and through simulation, as function of the normalized interference bandwidth$\left(NBW = \frac{B_1}{B_0}\right)$ for different values of the SIR. We have assumed that $SNR = 8$ dB, $N_I = 1$, $f'_{c,l} = 0.5$. Note that increasing NBW does not have a large influence on P_f, especially at high values of the SIR. This is explained as at high SIR, the effect of interference signal diminishes.

Fig.5 illustrates the upper bound and simulation results for the probability of failure P_f as function of the normalized interference carrier frequency deviation $f'_{c,1}$ assuming $N_I = 1$ and $NBW = .0244$. As can be observed, when the NBI signal is located outside the OFDM spectrum, i.e, $\left|f'_{c,l}\right| > 0.625$, the position of the interferer has no effect on the estimators. Further, the probability of failure of the blind and the pilot estimator is very sensitive to the location of the interferer within the OFDM spectrum. For the blind estimator, the result indicates that the interference does not have a large effect on P_f as long as the interferer is located the within OFDM spectrum but far away from the region of virtual carriers. This can easily be explained as follows. The blind estimator calculates the sum of the power of the received OFDM subcarriers located in the region $[-N_u,\ N_u]$ (12) for all $\tilde{m}$, and estimates $\hat{m}$ that maximizes $\Lambda(\tilde{m})$ (10). If the interferer is located in the OFDM spectrum but sufficiently far from the virtual carriers, i.e $\left|f'_{c,l}\right| \ll \frac{N_u B_0}{2N}$, the interference

[2] $\Lambda(\tilde{m}) \sim N\left(\mu_{\tilde{m}}, \sigma^2_{\tilde{m}}\right)$ means that $\Lambda(\tilde{m})$ is Gaussian distributed with average $\mu_{\tilde{m}}$ and variance $\sigma^2_{\tilde{m}}$.

contribution to $\Lambda(\tilde{m})$ is nearly the same for all values of $\tilde{m}$. Therefore, NBI will not affect the maximization of $\Lambda(\tilde{m})$. The worst case scenario occurs when the interferer is located in the virtual carrier region. In that case, the optimization will strongly depend on the position of the interferer as the contribution of the NBI on $\Lambda(\tilde{m})$ will strongly vary as function of $\tilde{m}$. As the probability that an incorrect value of m is selected in the optimization process is very high in this case, it results in a dramatically increase of P_f. On the other hand, the pilot estimator is very sensitive to the location of NBI within the OFDM spectrum. This can easily be explained as follows. If the NBI signal is located sufficiently far from the virtual carriers, i.e $\left|f'_{c,l}\right| \ll \frac{N_u B_0}{2N}$, the first term in (11) has nearly no effect on the maximization of $\Lambda(\tilde{m})$ similarly as for the blind estimator. Therefore, the interference contribution to the first term in (11) can be neglected. However, if the interferer is located near a pilot, the interference contribution to the second term in (11) can not be neglected. As in the simulations, we have assumed that the pilots are uniformly distributed over OFDM spectrum, the probability of failure seems a periodic function of the interference frequency.

Fig. 6 shows the upper bound and simulation results for the probability of failure P_f as function of the number of interfering signals, N_I, in two cases. In case 'A', we consider a fixed total interference power, hence SIR , i.e. interference power per interferer decreases linearly as N_I increases. While, in case 'B', we consider that the interference power, hence SIR is fixed per interferer, so the total interference power increases proportional to N_I. The location of the interferers is assumed to be uniformly distributed in the region where the OFDM spectrum differs from zero. It is clear that the probability of failure in case 'B' is larger than in case 'A': the total interference power in the former case is larger than in the latter case. Note that the blind estimator does essentially not depend on the number of interferers for both cases 'A' and 'B'. As the location of the interferer has nearly no effect on the performance of the blind estimator, the number of interferers has nearly no effect on the performance of the blind estimator. For the pilot estimator, the probability of failure slightly changes for an increasing number of interferers. From Fig. 5, we can conclude that the number of interferers has only a small effect on the performance of the estimator.

V. Conclusion

This paper evaluates the performance of the blind and pilot maximum likehood frequency ambiguity resolution algorithms in the presence of narrowband interference. An upper bound on the performance of the algorithms is derived, and simulations have been carried out to check the validity of the analytical results. Generally, the bandwidth of the interference and number of interferers do not have a large influence on the performance of the estimators. However, it turns out that the position of the interferers has a large influence of the performance. If the interferer is located in the region of the virtual carriers or, in the case of pilot estimator, close to a pilot symbol, the performance of the estimators is dramatically affected by NBI.

Appendix

The nth symbol of the ith received block may be represented as

$$Z(n) = Ba_{z,i} + W_i(n) \tag{17}$$

where $B = \sqrt{\frac{N\,E_s}{N+\nu}}$, $z = modulo(n-m,\, N)$ and $W_i(n)$ is the total noise resulting from AWGN noise and NBI signals. Using (17) in (11), and after tedious computations, it follows that

$$\begin{aligned}\mu_{\tilde{m}} = \;& 2B^2 N_p \delta_{\tilde{m}m} + 2B^2\left(2N_u + 1 - |m-\tilde{m}|\right)\\ &+ \textstyle\sum_{n=-N_u}^{N_u} \chi(n,\tilde{m})\end{aligned} \tag{18}$$

where $\tilde{m} \in [-M,\ M]$ and $\chi(n,\tilde{m}) = E\left[\left|W_0\left(n+\tilde{m}\right)\right|^2\right] + E\left[\left|W_1\left(n+\tilde{m}\right)\right|^2\right]$; $E\left[\left|W_i\left(n+\tilde{m}\right)\right|^2\right] = 2\sigma_n^2 + \eta_i(n,\tilde{m})$, where $i = 0,\ 1,$ and

$$\begin{aligned}\eta_i(n,\tilde{m}) = \;& \tfrac{1}{N}\textstyle\sum_{k,k'=0}^{N-1}\sum_{l=1}^{N_I} E_l \sum_{h=-\infty}^{\infty}\\ & A_l\left(k',h,i\right) A_l^*(k,h,i)\, e^{j2\pi(n+\tilde{m})(k-k')/N}\end{aligned} \tag{19}$$

where $A_l(z,h,i) = g_l\left(zT_0 + i\left(N+\nu\right)T_0 - hT_l\right)$. The variance $\sigma^2_{\tilde{m}}$ is given by

$$\begin{aligned}\sigma^2_{\tilde{m}} = \;& \left(1-\delta_{\tilde{m}m}\right)2B^4 N_p + \left(1+2\delta_{\tilde{m}m}\right)2B^2 \textstyle\sum_{n\in N_P}\chi(n,\tilde{m})\\ &+ \textstyle\sum_{n\in N_P}\psi(n,\tilde{m}) + \sum_{n,n=-N_u}^{N_u}\sum_{i=0}^{1}\Phi_i(n,n')\\ &+ \Omega - \left(\textstyle\sum_{n=-N_u}^{N_u}\chi(n,\tilde{m})\right)^2\end{aligned} \tag{20}$$

where

$$\Omega = \; 2B^2 \sum_{n=max(-N_u,-Nu-m+\tilde{m})}^{min(N_u,Nu-m+\tilde{m})} \chi(n,\tilde{m}) \tag{21}$$

and $\Phi_i(n,n') = E\left[\left|W_i\left(n+\tilde{m}\right)\right|^2\left|W_i\left(n'+\tilde{m}\right)\right|^2\right]$:

$$\begin{aligned}\Phi_i(n,n') = \;& 4\sigma_n^2\left(\sigma_n^2 + \eta_i(n,\tilde{m})\right)\delta_{nn'} + 4\sigma_n^4\\ &+2\sigma_n^2\left(\eta_i(n,\tilde{m}) + \eta_i(n',\tilde{m})\right)\\ &+4\sigma_n^2\eta_i(n,\tilde{m})\delta_{nn'} + \eta_i(n,\tilde{m})\eta_i(n',\tilde{m})\end{aligned} \tag{22}$$

and $\psi(n,\tilde{m}) = E\left[\left|W_0\left(n+\tilde{m}\right)\right|^2\left|W_1\left(n'+\tilde{m}\right)\right|^2\right]$:

$$\begin{aligned}\psi(n,\tilde{m}) = \;& 4\sigma_n^4 + 2\sigma_n^2\left(\eta_0(n,\tilde{m}) + \eta_1(n,\tilde{m})\right)\\ &+\eta_0(n,\tilde{m}).\eta_1(n,\tilde{m})\end{aligned} \tag{23}$$

The covariance $\sigma^2_{\tilde{m}m}$where $\tilde{m} \neq m$ is given by

$$\begin{aligned}\sigma^2_{\tilde{m}m} = \;& 2B^2\textstyle\sum_{n'=-N_u}^{Nu}\sum_{n\in P,n=n'+-m+\tilde{m}}\chi(n,\tilde{m})\\ &+2B^2\textstyle\sum_{n,n'=-N_u,n=n'+m\tilde{-}m}^{Nu}\chi(n,\tilde{m})\\ &+\textstyle\sum_{n,n'=-N_u}^{Nu}\Phi_0(n,n') + \Phi_1(n,n')\\ &-\textstyle\sum_{n,n'=-N_u}^{Nu}\left(2\sigma_n^2 + \eta_0(n,m)\left(2\sigma_n^2 + \eta_0(n',\tilde{m})\right)\right.\\ &-\textstyle\sum_{n,n'=-N_u}^{Nu}\left(2\sigma_n^2 + \eta_1(n,m)\left(2\sigma_n^2 + \eta_1(n',\tilde{m})\right)\right.\end{aligned} \tag{24}$$

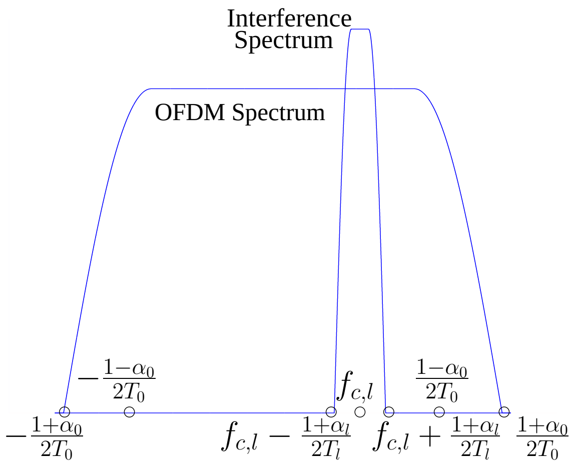

Fig. 2. Baseband OFDM and one interfering signal spectrum

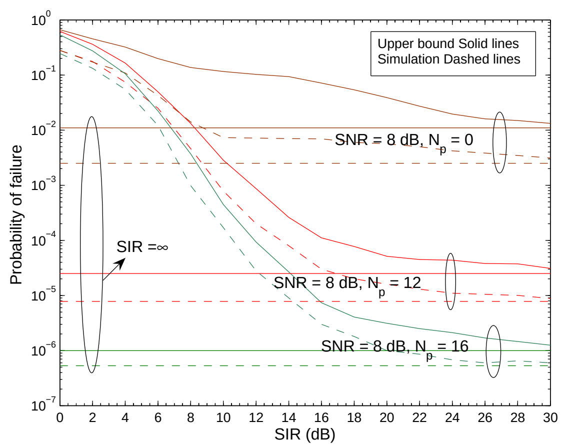

Fig. 3. Probability of failure versus signal to interference ratio (SIR) , dB, NBW= .0244, $f^{'}_{c,1} = .5$, and N_I=1

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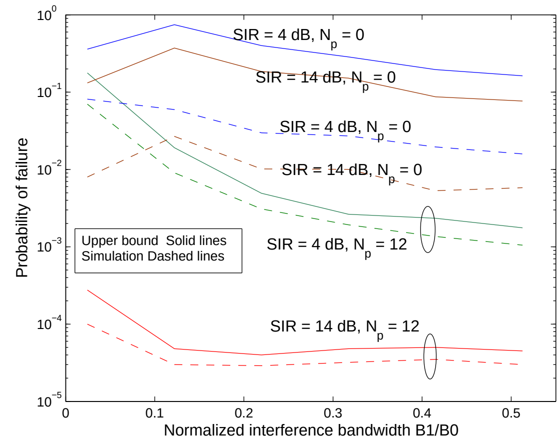

Fig. 4. Probability of failure versus normalized interference bandwidth, NBW , $f^{'}_{c,1} = .5$ and $N_I = 1$ and $SNR = 8$ dB

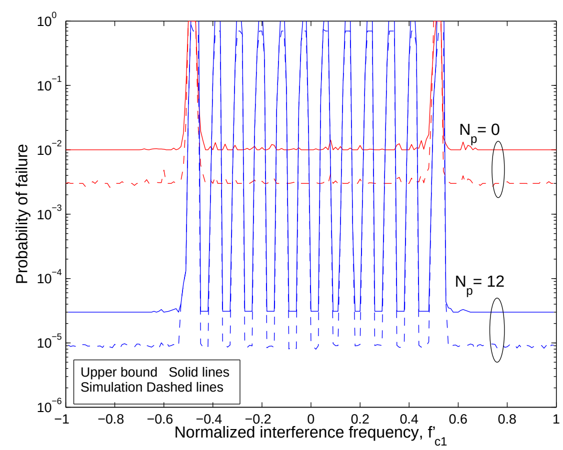

Fig. 5. Probability of failure versus normalized interference carrier frequency, $f^{'}_{c,1}$, $SNR = 8$ dB, NBW= .0244, and N_I=1

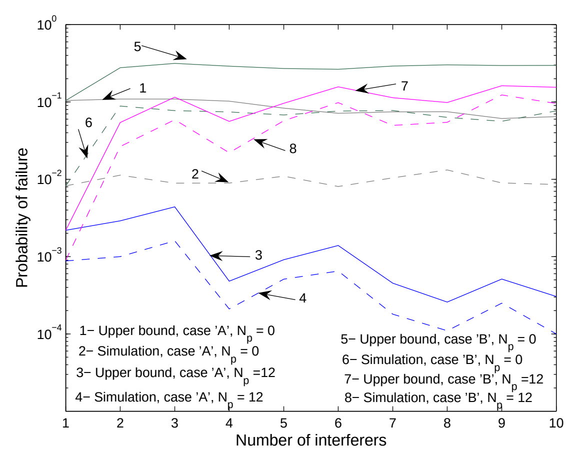

Fig. 6. Probability of failure versus number of interference signals, N_I, $NBW = .0244$ and $SNR = 10$ dB