Interference Cancellation of AM Narrowband Interference Signals

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Abstract— In this paper, we consider an overlay system where narrowband AM signals interfere with a broadband multicarrier system [1]. To reduce the effect of the AM narrowband interference on the multicarrier system, we propose a low complexity algorithm to estimate AM narrowband interference. We derive analytical expressions for the performance of the estimator and verify the results with simulations. The proposed estimator is able to produce accurate estimates of the frequencies, and track the time-varying amplitudes of the AM signals. The estimator can reduce the power of the AM signal to a level that is approximately 20 dB lower than the multicarrier power, independent of the AM signal power.

I. INTRODUCTION

In future communications, a continuously increasing amount of information must be transmitted at higher and higher data rates. To transmit this information, broadband transmission schemes are developed, such as multicarrier (MC) transmission [2], wideband CDMA (WCDMA) [3] and Ultra Wide Band (UWB) [4]. However, as frequency resources are scarce, these broadband transmission schemes may have to coexist with narrowband legacy systems. These narrowband signals disturb the broadband signals, resulting in a reduction of the performance [5]-[6].

The system under consideration in this paper is a broadband multicarrier system for aeronautical communication in the 118-137 MHz band [1],[7]. In this frequency band, the multicarrier system has to coexist with legacy narrowband VHF signals, consisting of mainly analog amplitude modulated (AM) signals for voice communication with 8.033 kHz or 25 kHz bandwidth. These narrowband signals strongly affect the performance of the multicarrier system [8]. To avoid the reduction of the performance of the multicarrier system, the AM narrowband signals have to be estimated and canceled.

In this paper, we propose a technique to estimate and eliminate AM signals. The performance of the AM narrowband estimator is derived in an analytical way, and the results are verified through computer simulations.

II. AM SIGNAL ESTIMATOR

At the receiver of the multicarrier system, the received signal r(t) consists of the sum of the multicarrier signal $r_{MC}(t)$, the AM interference signals $r_{AM}(t)$ and noise w(t). Assuming the bandwidth of the AM narrowband signal is small as compared of the coherence bandwidth of the channel,

the AM interference signal can be written as

$$r_{AM}(t) = \sum_{\ell=1}^{L} A_{\ell}(1 + mx_{\ell}(t))e^{j(2\pi f_{c,\ell}t + \theta_{\ell})}$$
(1)

where L is the number of AM interferers, m the modulation index, $x_{\ell}(t)$ the voice signal of interferer ℓ , A_{ℓ} its amplitude, $f_{c,\ell}$ its central frequency and θ_{ℓ} its phase. A typical value of the modulation index for voice communication is m = 0.85.

In the multicarrier system, the data is transmitted in parallel over N_{MC} subcarriers at a rate 1/T. The time-domain signal of the multicarrier system consists of the sum of contributions of the N_{MC} carriers. If the number of carriers in the multicarrier system is sufficiently large ($N_{MC} \ge 64$) and the transmitted data symbols have zero mean, the time-domain samples can be modeled according to the central limit theorem as zero-mean Gaussian distributed. Further, the noise signal w(t) is assumed to be AWGN. Hence, the received signal can be rewritten as

$$r(t) = r_{AM}(t) + \tilde{w}(t) \tag{2}$$

where $\tilde{w}(t)$ is the equivalent noise, taking into account the effects of the multicarrier signal and the AWGN. The equivalent noise $\tilde{w}(t)$ is zero-mean Gaussian distributed. Its variance is assumed to be σ^2 , and consists mainly of the MC power.

At the multicarrier receiver, samples at a rate 1/T are available. Let us assume we want to estimate the AM signal using the available samples. To do this, we rewrite the AM signal samples as

$$r_{AM}(kT) = \sum_{\ell=1}^{L} \tilde{A}_{\ell}(kT) e^{j2\pi f_{c,\ell}kT}$$
(3)

where the ℓ th equivalent amplitude equals

$$\tilde{A}_{\ell}(kT) = A_{\ell}(1 + mx_{\ell}(kT))e^{j\theta_{\ell}}.$$
(4)

As the bandwidth of the AM signal is small as compared to the bandwidth 1/T of the multicarrier system, the equivalent amplitudes $\tilde{A}_{\ell}(kT)$ are slowly varying functions of the time index k. From (3) it follows that the parameters to be estimated are the number L of interferers, the frequencies $\{f_{c,\ell}\}$ and the time-varying amplitudes $\{\tilde{A}_{\ell}(kT)\}, \ell = 1, \ldots, L$.

To determine the number L of interferers and the frequencies $\{f_{c,\ell}\}$, we select a block of N samples $\{r(kT)|k = 0, \ldots, N-1\}$ and apply a discrete Fourier transform (DFT) –

which can easily be implemented as a fast Fourier transform (FFT) – on the N selected samples. The output of the DFT is given by

$$y(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r(kT) e^{-j2\pi \frac{kn}{N}}$$

= $y_{AM}(n) + W(n)$ (5)

where

$$y_{AM}(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r_{AM}(kT) e^{-j2\pi \frac{kn}{N}}$$
(6)

and W(n) is the noise component with zero mean and variance σ^2 .

Let us assume that the equivalent amplitudes $\{\tilde{A}_{\ell}(kT)\}\$ are approximately constant for k = 0, ..., N - 1, then the DFT output corresponding to the AM signal approximately yields

$$y_{AM}(n) \approx \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \sum_{\ell=1}^{L} \tilde{A}_{\ell}(0) e^{j2\pi f_{c,\ell}kT} e^{-j2\pi \frac{kn}{N}} \\ = \sum_{\ell=1}^{L} \tilde{A}_{\ell}(0) D_{N} \left(\frac{n}{N} - f_{c,\ell}T\right)$$
(7)

where

$$D_N(x) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{-j2\pi kx} = e^{-j\pi(N-1)x} \frac{\sin(\pi Nx)}{\sin(\pi x)}.$$
 (8)

The function $|D_N(x)|$ is shown in figure 1. As can be observed, the periodic function $|D_N(x)|$ shows a peak with maximum value \sqrt{N} for integer values of x, while for |x| >1/N, the amplitude sharply drops. The width of the peak decreases for increasing N, and at the same time, the value of $\frac{1}{\sqrt{N}}|D_N(x)|$ for non-integer values of x decreases. Hence, assuming the spectra of the AM signals do not overlap¹, the peaks corresponding to the different contributions in (7) will not interfere: for $n \approx f_{c,\ell}NT$, the sum in (7) reduces to one term. Therefore, the DFT output corresponding to the AM signal will show peaks at the indices n_ℓ for which $|f_{c,\ell} - \frac{n_\ell}{NT}| < \frac{1}{NT}$, $\ell = 1, \ldots, L$.

To determine the number L of interferers, we observe the amplitude of the output of the DFT, and count the maxima. However, in the presence of noise, some peaks will not be visible (when the power of the AM interferer is small) and the noise may introduce unwanted peaks. It is not necessary to estimate AM signals of which the peak is not visible because of the noise (which mainly consists of the multicarrier signal), as the goal of the estimator is to estimate and eliminate the AM signals disturbing the multicarrier signal, and weak AM signals will only slightly disturb the multicarrier signal. Estimation and elimination of an unwanted peak, on the other hand, will cause distortion of the multicarrier signal and must be avoided. Therefore, only peaks larger than a threshold will be selected for estimation and elimination. The threshold is selected such

¹This assumption is valid as the AM signals may not interfere with each other.



Fig. 1. The function $\frac{1}{\sqrt{N}}|D_N(x)|$, N = 128.

that the probability of an unwanted peak is smaller than a predetermined value that depends on the multicarrier signal $power^2$.

Let us assume the DFT results in the detection of \hat{L} peaks larger than the threshold. The position of the maximum, i.e. n_{ℓ} yields a coarse estimate for the frequency of the ℓ th AM signal, $\ell = 1, ..., \hat{L}$. A fine estimate for the frequency $f_{c,\ell}$ can be found by observing (7) and figure 2, where the influence of the position of $f_{c,\ell}$ on the output of the DFT is shown. Taking into account that for values of n close to n_{ℓ} , (7) reduces to $\tilde{A}_{\ell}(0)D_N\left(\frac{n}{N} - f_{c,\ell}T\right)$ (the contributions of the other terms in the sum can be neglected), we can determine the shift $\epsilon_{\ell} = \frac{n_{\ell}}{N} - f_{c,\ell}T$ by observing $y_{AM}(n_{\ell} \pm 1)$. The effect of $\tilde{A}_{\ell}(0)$ can be eliminated by computing the ratios $\frac{y_{AM}(n_{\ell})}{y_{AM}(n_{\ell} \pm 1)}$. Using (8), these ratios can be approximated by

$$\frac{y_{AM}(n_{\ell})}{y_{AM}(n_{\ell}+1)} = e^{-j\frac{\pi}{N}} \frac{\sin(\pi(\epsilon_{\ell} + \frac{1}{N}))}{\sin(\pi\epsilon_{\ell})}$$

$$\approx e^{-j\frac{\pi}{N}} \left(1 + \frac{1}{N\epsilon_{\ell}}\right) \qquad (9)$$

$$\frac{y_{AM}(n_{\ell})}{y_{AM}(n_{\ell}-1)} = e^{+j\frac{\pi}{N}} \frac{\sin(\pi(\epsilon_{\ell} - \frac{1}{N}))}{\sin(\pi\epsilon_{\ell})}$$

$$\approx e^{+j\frac{\pi}{N}} \left(1 - \frac{1}{N\epsilon_{\ell}}\right) \qquad (10)$$

Taking into account (9) and (10), we define $\Delta_{\pm 1} = \frac{y(n_{\ell})}{y(n_{\ell}\pm 1)}$. The noise will affect the estimation of the shift ϵ_{ℓ} . Noting that the effect of the noise on the estimate will be smaller when the amplitude of $y_{AM}(n_{\ell}\pm 1)$ is larger, we use only the largest of the two DFT outputs $|y(n_{\ell}\pm 1)|$ for estimating ϵ_{ℓ} . The shift

²Let us assume that the unwanted peak is caused by the noise only, and the AM contribution is zero. The real and imaginary parts of the noise are independently Gaussian distributed with zero mean and variance $\sigma^2/2$. Then the probability that the amplitude of the noise contribution is larger than the threshold α equals $Pr(\text{amplitude noise} > \alpha) = \exp(-\frac{\alpha^2}{\sigma^2})$.



Fig. 2. Influence of $f_{c,\ell}$ on the output of the DFT.

 ϵ_{ℓ} can then be approximated by

$$\hat{\epsilon}_{\ell} \approx \frac{1}{N(\Re(\Delta_{+1}e^{+j\frac{\pi}{N}}) - 1)} \quad \text{if } |\Delta_{+1}| < |\Delta_{-1}| \hat{\epsilon}_{\ell} \approx \frac{-1}{N(\Re(\Delta_{-1}e^{-j\frac{\pi}{N}}) - 1)} \quad \text{if } |\Delta_{+1}| > |\Delta_{-1}| (11)$$

where $\Re(\cdot)$ denotes the real part. However, in the case that $n_{\ell} \approx f_{c,\ell}NT$, both $y_{AM}(n_{\ell} \pm 1)$ are close to zero (see figure 2) such that $y(n_{\ell} \pm 1)$ is dominated by noise. As a result, the estimated $\hat{\epsilon}_{\ell}$ will be unreliable and can be larger than 1/(2N). Note however that in this case, the coarse estimate will be accurate. Therefore, when $|\hat{\epsilon}_{\ell}| > 1/2N$, no correction is applied, and the frequency estimate equals $\hat{f}_{c,\ell} = n_{\ell}/NT$. Note that increasing N will cause the coarse frequency estimate to be more accurate, as the difference $|f_{c,\ell} - \frac{n_{\ell}}{NT}| < \frac{1}{NT}$, i.e. the difference decreases with increasing N. However, by increasing N, the assumption that $\tilde{A}_{\ell}(kT)$ is approximately constant over the N samples will not hold, such that the fine estimates from (11) will become less reliable.

The last step is the estimation of the time-varying amplitudes of the AM signals. We propose the following estimator:

$$\hat{\tilde{A}}_{\ell}(nT) = \frac{1}{2K+1} \sum_{k=-K}^{K} r((n+k)T) e^{-j2\pi(n+k)\hat{f}_{c,\ell}T}.$$
 (12)

To obtain the amplitude of the ℓ th AM signal, we downconvert the ℓ th AM signal to baseband by multiplying the received signal samples with $\exp(-j2\pi k \hat{f}_{c,\ell}T)$. Next, the samples are averaged over a (2K+1) size sliding window. This averaging acts as a low-pass filter with bandwidth $\frac{1}{(2K+1)T}$, which reduces the effects of the noise and the contributions of the other AM signals with frequencies $\hat{f}_{c,\ell'} - \hat{f}_{c,\ell}$, so containing mainly high frequency components with respect to the window size. Increasing K will reduce the bandwidth of the equivalent low-pass filter, and hence results in a further reduction of the effects of the noise and disturbing AM signals. However, if K is selected too large, the sliding window will not be able to track the small variations of the wanted AM signal, i.e. the

TABLE I The AM interference estimator.

Determine number of interferers \hat{L} + coarse estimate of frequencies • take DFT of N time domain samples
• count number of maxima at DFT output $>$ threshold
• coarse estimate of frequencies = positions n_{ℓ} of maxima
Fine estimate of frequencies
• compute $\Delta_{\pm 1} = \frac{y(n_\ell)}{y(n_\ell \pm 1)}$
• compute $\hat{\epsilon}_{\ell}$ using (11)
• if $ \hat{\epsilon}_{\ell} < 1/2N$: $\hat{f}_{c,\ell} = \frac{n_{\ell}}{NT} - \frac{\hat{\epsilon}_{\ell}}{T}$ else $\hat{f}_{c,\ell} = \frac{n_{\ell}}{NT}$
Amplitude estimates
• compute $\hat{\tilde{A}}_{\ell}(nT)$ using (12)

bandwidth of the equivalent low-pass filter must be larger than the bandwidth of the AM signals.

III. PERFORMANCE EVALUATION

In this section, we derive the mean squared error (MSE) for the amplitude, assuming perfect frequency estimation. In the numerical results, it will be shown that the MSE for the frequency estimation is very low, such that this assumption is valid. Assuming A_{ℓ} is real valued, and the real-valued voice signal $x_{\ell}(t)$, $\ell = 1, \ldots, L$ has zero mean, autocorrelation function $R_{\ell}(t) = E[x_{\ell}(\tau)x_{\ell}(t+\tau)]$ and spectrum $S_{\ell}(f) =$ $FT(R_{\ell}(t))$ (FT(·) denotes the Fourier transform), the MSE of the amplitude can be written as

$$\sigma_{\tilde{A}_{\ell}}^{2} = A_{\ell}^{2}m^{2} \int_{-\infty}^{+\infty} S_{\ell}(f)(F_{K}(fT) - 1)^{2} df + \sum_{\ell'=1;\ell\neq\ell}^{L} A_{\ell'}^{2} \Big((F_{K}(\Delta f_{\ell,\ell}T)^{2} + m^{2} \int_{-\infty}^{+\infty} S_{\ell'}(fT) |F_{K}((f - \Delta f_{\ell',\ell})T)|) df \Big) + \frac{\sigma^{2}}{2K + 1}$$
(13)

where $\Delta f_{\ell',\ell} = f_{c,\ell'} - f_{c,\ell}$ and

$$F_K(x) = \frac{1}{2K+1} \sum_{k=-K}^{K} e^{j2\pi kx}.$$
 (14)

When the amplitude $\tilde{A}_{\ell}(nT)$ can be considered as constant over the interval [(n - K)T, (n + K)T], i.e. for sufficiently small K, the MSE (13) reduces to

$$\sigma_{\tilde{A}_{\ell}}^{2} = \frac{\sigma^{2}}{2K+1} + \sum_{\ell'=1;\ell'\neq\ell}^{L} P_{\ell'}(F_{K}(\Delta f_{\ell',\ell}T))^{2}$$
(15)

where $P_{\ell} = A_{\ell}^2 (1 + m^2 R_{\ell}(0))$ is the power of the ℓ th AM signal.

IV. NUMERICAL RESULTS

For the numerical results and the simulations, the sample rate of the MC system $1/T = 1/(0.9375\mu s)$. The FFT length equals N = 512. The voice signals $x_{\ell}(t)$ are generated with



Fig. 3. MSE and attenuation, one AM interferer (L=1), $A_{\ell} = 1$, $Nf_{c,\ell}T = 10.76$, m = 0.85, $\sigma^2 = -10$ dB.

the B-VHF voice signal generator [7]. The sum of the MC signals and noise is generated with an AWGN generator.

In figure 3, simulation results are shown for the MSE of the frequency estimate and the amplitude estimate for the case of one AM signal (L = 1) with amplitude $A_{\ell} = 1$ and the noise power level equals $\sigma^2 = -10$ dB. As can be observed, the MSE of the frequency is very low, which implies that the proposed frequency estimator is very accurate. The MSE of the frequency estimator is as expected independent of the window length K. Further, the MSE of the amplitude, obtained with the expressions (13) and (15) are shown. The simulation results agree well with the theoretical result (13) for all values of K. However, to compute (13), the spectrum of the AM signal must be known. If only the power of the AM signal is available, the expression (15) can be used, which shows a very good accordance with the simulation results for sufficiently small window size K, where the approximation that the timevarying amplitude $\hat{A}_{\ell}(nT)$ is constant over the window is valid. Further, the attenuation, which is defined as the difference between the power of the AM signal after and before elimination, is shown. This attenuation well corresponds to the MSE of the amplitude, indicating that the estimation error of the frequencies can be neglected.

Figure 4 shows the MSE of the frequency and amplitude estimates together with the attenuation for the case of two AM signals (L = 2). Similar conclusions as in figure 3 can be drawn. Note the effect of the second AM signal on the estimation error: the MSE of the amplitude shows an oscillation. This is caused by the second term in (13) and (15): the function $F_K(\Delta f_{\ell',\ell}T)$ is a periodic function with frequency $\Delta f_{\ell',\ell}$.

In figure 5, simulation results for the attenuation of the AM signal obtained with the proposed estimator are shown for different values of the MC signal plus noise level σ^2 for L = 1. Further, theoretical results are shown for the cases of AM signal only (no noise present), corresponding to the first term in (13), and noise only (no AM signal present), corresponding to the last term in (13). As can be observed, the attenuation is dominated by the noise for small values of K.



Fig. 4. MSE and attenuation, two AM interferers (*L*=2), $A_{\ell} = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $Nf_{c,\ell}T = \begin{bmatrix} 10.76 & 92.36 \end{bmatrix}$, m = 0.85, $\sigma^2 = -20$ dB.



Fig. 5. Attenuation, one AM interferer (L=1), $A_{\ell} = 1$, $Nf_{c,\ell}T = 10.76$, m = 0.85.

This is explained as for small values of K, the time variation of the amplitude can be neglected and by increasing K, and the effect of the noise is better averaged out. For large K, the time variation of the amplitude becomes important. This is caused by the fact that the estimator cannot track the small variations of the amplitude as the window size is too large: for large K, the AM signal is approximated by the estimator as a sinusoid with constant amplitude. The optimum value of K depends on the bandwidth of the AM signal and the noise level. For decreasing noise level, the optimum value of Kdecreases, as the effect of the noise averaging becomes less important than the time-variation of the AM signal. Note that the optimum value of the attenuation is approximately 20 dB larger than the noise level σ^2 : the estimator is able to reduce the AM signal power to roughly 20 dB below the power of the MC signal.

V. CONCLUSIONS

In this paper, we have proposed a AM signal estimator to reduce the effect of the AM interference on an overlay multicarrier system. Further, we have derived the mean squared error of the amplitude estimate in an analytical way. Simulation results showed that the theoretical expressions for the performance agree well with the simulations. Further, the attenuation of the AM signal that can be obtained with the proposed estimator is independent of the frequency estimation error, but is mainly determined by the amplitude estimation error. An attenuation of the AM signal of approximately 20 dB below the multicarrier signal power can be obtained.

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