# CANCELLATION OF DIGITAL NARROWBAND INTERFERENCE FOR MULTI-CARRIER SYSTEMS

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**Abstract** Spectrum-overlay scenarios for wideband multi-carrier (MC) systems bring new technical challenges that must be considered during the system design. In such a scenario, the performance of the multi-carrier system is affected by the presence of possibly strong in-band interference. To improve the performance of the MC communication link in an interference corrupted environment (without increasing transmit bandwidth), the interference must be estimated and removed. In this paper, we propose a new low complexity algorithm to estimate and suppress digitally modulated interference suppression is evaluated in an analytical way. Further, simulations have been carried out to verify the validity of approximations in the analysis.

Keywords: Interference cancellation, narrowband interference

## 1. Introduction

The scarcity of available bandwidth typically necessitates spectrum sharing between legacy and new multi-carrier (MC) systems. The broadband very high frequency (B-VHF) project [1], which aims to develop a new integrated broadband VHF system for aeronautical voice and data link communications based on multi-carrier technology, is a good example of an overlay system. In this project, the MC system is intended to share parts of the VHF spectrum which are currently used by narrowband (NB) systems. Due to the spectral leakage of the discrete Fourier Transform (DFT) demodulation, many MC subcarriers near the interference frequency will suffer from serious interference, limiting the effectiveness of the multi-carrier system. In order to improve the performance of the MC communication link, some means of interference removal should be used. The techniques to cope with NBI in wireless MC systems can be divided into two categories. The first category is based on NBI suppression. Receiver windowing is a well known NBI suppression technique [2] using samples from the cyclic prefix to construct a window that reduces the NBI component of the received signal without affecting the data component. The result is that the NBI is convolved in frequency domain with a window that has smaller side-lobes than the sinc function, limiting the leakage to other subchannels. However, this technique requires a cyclic prefix length that is sufficiently long, reducing the efficiency of the MC system. Other techniques for NBI suppression are based on spreading the data over the whole MC bandwidth by either using orthogonal carrier interferometry spreading codes as in[3] or using orthogonal Hadamard sequences as in [4]. However, the complexity of both techniques is rather high.

The second category of NBI suppression techniques is based on interference cancellation: the interference is estimated and then subtracted from the received signal. In [5], the linear minimum mean square estimator (LMMSE) is adopted to estimate the NBI in the frequency domain: however this technique requires prior information about the power spectral density (PSD) of the NBI signals. In [6], unmodulated subcarriers close to the NBI central frequency are used to demodulate the NBI signal. Then, the NBI signal is reconstructed by passing the demodulated NBI data through the NBI transmit pulse. The complexity of this technique is rather high. It is worth to mention that former NBI suppression and cancellation techniques are done in the frequency domain, i.e. after FFT. This approach will lead to several practical issues like the design of the D/A converter (especially when the NBI signal is stronger the MC signal) and synchronization of the MC system [7–9]. This motivates us to propose a new low complexity algorithm to estimate NBI signals in time domain, before synchronization. The rest of the paper is organized as follows. In section 2, a MC system model and a narrow-band interference model are described. The proposed narrowband interference cancellation technique is illustrated in section 3. Simulation and analytical results are shown in section 4. Finally, the conclusions are given in section 5.

## 2. System Description

Fig. 1 shows the simplified Orthogonal Frequency Division Multiplexing (OFDM) system including the digital NBI. The sequence of input data symbols is segmented into blocks of length  $N_u$ . Define the input data as  $a_l(k)$ ,  $k = 0, ..., N_u - 1$ , where the subscript l is used to indicate

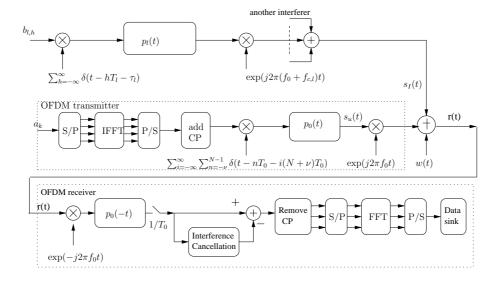


Figure 1. Schematic model of the OFDM system with NBI signals

the *l*th block of data, and *k* refers to the *k*th subchannel. The OFDM transmitter takes an *N* point IFFT of the *l*th block, and copies the last  $\nu$  samples of the result as a cyclic prefix to form  $x_l(n)$ :

$$x_l(n) = \sqrt{\frac{1}{N+\nu}} \sum_{k \in I_u} a_i(k) e^{\frac{j2\pi kn}{N}} - \nu \le n \le N-1$$
(1)

where  $I_u$  is a set of  $N_u$  carrier indices. The data symbols are assumed i.i.d.<sup>1</sup> random values with zero mean and variance  $E[|a_l(k)|^2] = E_s$ . The time domain baseband OFDM signal  $s_u(t)$  consists of the concatenation of all time domain blocks  $x_l(n)$ :

$$s_u(t) = \sum_{i=-\infty}^{\infty} \sum_{n=-\nu}^{N-1} x_l(n) \, p_0(t - nT_0 - i(N+\nu)T_0) \tag{2}$$

where  $p_0(t)$  is the unit-energy transmit pulse of the OFDM system and  $1/T_0$  is the sample rate. The baseband signal (2) is up-converted to the radio frequency  $f_0$ . At the receiver, the signal is first down-converted. Next, the OFDM receiver discards the first  $\nu$  samples of the received block, and takes an *N*-point FFT of the result. The OFDM signal is disturbed by additive white Gaussian noise with uncorrelated real and imaginary parts, each having variance  $\sigma_n^2$ . The signal to noise ratio at the output of the matched filter is defined as  $SNR = \sigma_s^2/\sigma_n^2$  where  $\sigma_s^2$  is the

variance of the time domain OFDM signal per real dimension. Further, the signal is disturbed by narrowband interference residing within the same frequency band as the wideband OFDM signal. In this paper, we assume the NBI signal consists of  $N_I$  digitally modulated signals. Following the interference model from [7, 8], the interfering signal  $s_I(t)$ may be written as

$$s_I(t) = \sum_{l=1}^{N_I} s_l(t) e^{j2\pi(f_0 + f_{c,l})t}$$
(3)

where  $s_l(t)$  is a baseband narrowband signal and  $f_{c,l}$  is the carrier frequency deviation for the *l*th interference from the MC carrier frequency  $f_0$ . The baseband interference  $s_l(t)$  is modeled as a digitally modulated signal

$$s_{l}(t) = \sum_{h=-\infty}^{\infty} b_{h,l} p_{l}(t - hT_{l} - \tau_{l})$$

$$\simeq \sum_{h=0}^{q} b_{h,l} p_{l}(t - hT_{l} - \tau_{l})$$

$$(4)$$

where  $p_l(t)$  is the time domain impulse response of the transmit filter of the *l*th interferer,  $b_{h,l}$  is the *h*th interfering data symbol,  $\tau_l$  is its delay, and  $1/T_l$  its sample rate. Let  $B_l$  be the bandwidth of  $p_l(t)$  and  $B_0$ the bandwidth of the OFDM signal. In an MC symbol duration  $T_{FFT}$ , there are *q* symbols of  $s_l(t)$ , where *q* is an integer equal to or less  $\frac{B_l}{B_0}$ . Because  $p_l(t)$  degrades rapidly in time, symbols  $\{b_{h,l} \forall h < 0 \text{ or } h > q\}$ have a negligible effect on the signal  $s_l(t)$  ( $0 \le t \le T_{FFT}$ ). Therefore, the approximation in (4) is valid. The total NBI signal at the output of the matched filter of the MC receiver yields

$$r_I(t) \simeq \sum_{l=1}^{N_I} \sum_{h=0}^{q} b_{h,l} \, e^{j2\pi f_{c,l}hT_l} \, g_l(t-hT_l) \tag{5}$$

where  $g_l(t)$  is the convolution of  $p_0(-t)$  and  $p_l(t-\tau_l) \exp(j2\pi f_{c,l}t)$ . The normalized location of the interferer within the MC spectrum may be defined as  $f'_{c,l} = f_{c,l}/B_0$ . It is assumed that the interfering symbols are uncorrelated with each other, i.e.  $E[b_{h,l}b^*_{h',l'}] = E'_l \delta_{ll'} \delta_{hh'}$ , where  $E'_l$  is the energy per symbol of the *l*th interferer. Further, the interfering data symbols  $b_{h,l}$  are statistically independent of the OFDM data symbols  $a_i(n)$ . The signal to interference ratio (SIR) at the input of the receiver is defined as [8] Cancellation of Digital Narrowband Interference for Multi-Carrier Systems 5

$$SIR = \frac{2\sigma_s^2/T_0}{\sum_{l=1}^{N_I} \frac{E_l'}{T_l}}.$$
(6)

#### 3. Narrowband Interference Cancellation

In the proposed algorithm, each NBI signal  $s_l(t)$  is estimated separately and subtracted from the received signal. Let us assume that we use the available samples at the MC receiver to estimate the interference. The sample  $r(mT_0)$  at the output of the matched filter of the OFDM receiver consists of a useful signal  $r_u(mT_0)$ , an interfering  $r_I(mT_0)$  component, and noise  $w(mT_0)$ :

$$r(mT_0) = r_u(mT_0) + r_I(mT_0) + w(mT_0).$$
(7)

Further, in our analysis we assume that the frequency  $f_{c,l}$  is perfectly known. In practice, a simple estimate of  $f_{c,l}$  can be obtained by using the squared magnitude of the FFT outputs, as in a periodogram, searching for the subcarriers with the strongest interference; the estimate of  $f_{c,l}$ can then be found by interpolation [5]. Although a perfect knowledge of the frequency  $f_{c,l}$  is assumed, it turns out that the proposed algorithm is insensitive to small estimation errors in  $f_{c,l}$ . The samples (7) are multiplied with  $\exp(-j2\pi f_{c,l}mT_0)$  to down-convert the *l*th NBI to baseband. Next, the samples are averaged over a (2K + 1) size sliding window.

$$\hat{s}_l(nT_0) = \frac{1}{2K+1} \sum_{k=-K}^{K} r((n+k)T_0) \cdot e^{-j2\pi(n+k)f_{c,l}T_0}$$
(8)

This averaging acts as a low-pass filter with bandwidth  $1/(2K+1)T_0$ , which reduces the effects of the noise, the OFDM signal, and the contributions of other NBI signals on the estimation of the wanted NBI signal. Increasing K will reduce the bandwidth of the equivalent low-pass filter, and results in a reduction of the effects of noise, OFDM, and other disturbing NBI signals. However, if K is selected too large, the sliding window will not be able to track the small variations of the wanted NBI signal, i.e. the bandwidth of the equivalent low-pass filter must be larger than the bandwidth of the NBI signals. Let  $\sigma_l^2(nT_0)$  be the mean squared error of *l*th interferer at instant  $nT_0$ ,  $\sigma_l^2(nT_0) = E[|s_l(nT_o) - \hat{s}_l(nT_0)|^2]$ . After tedious computations, it follows that  $\sigma_l^2(nT_0)$  can be written as

$$\sigma_l^2(nT_0) = \sigma_{SI_l}^2(nT_0) + \sigma_{MI_l}^2(nT_0) + \sigma_{AWGN_l}^2(nT_0)$$
(9)

where the self noise  $\sigma_{SI_l}^2(nT_0)$  is the variance resulting from the unablity to track the small variations of the wanted NBI signal,  $\sigma_{MI_l}^2(nT_0)$  is the variance resulting from other NBI signals, and  $\sigma^2_{AWGN_l}(nT_0)$  is the variance resulting from AWGN and the OFDM signal<sup>2</sup>. These variances can be written as

$$\sigma_{SI_{l}}^{2}(nT_{0}) = E_{l} \sum_{h=-\infty}^{\infty} |\xi_{l,h}(n,0)|^{2} + \left(\frac{1}{2K+1}\right)^{2} \sum_{k,k'=-K}^{K} E_{l} \sum_{h=-\infty}^{\infty} \xi_{l,h}(n,k) \xi_{l,h}^{*}(n,k')$$
(10)

$$-Re\left\{\frac{2}{2K+1}\sum_{k=-K}^{K}E_{l}\sum_{h=-\infty}^{\infty}\xi_{l,h}(n,0)\xi_{l,h}^{*}(n,k)\ e^{J2\pi f_{c,l}(n+k)T_{o}}\right\}$$

where  $\xi_{l,h}(n,k) = g_l ((n+k)T_0 - hT_l).$ 

$$\sigma_{MI_l}^2(nT_0) = \left(\frac{1}{2K+1}\right)^2 \sum_{k,k'=-K}^K \sum_{l'=1,l'\neq l}^{N_I} E_{l'} \sum_{h=-\infty}^\infty \xi_{l',h}(n,k) \xi_{l',h}^*(n,k')$$
(11)

$$\sigma_{AWGN_l}^2(nT_0) = \frac{2(\sigma_s^2 + \sigma_n^2)}{2K + 1}.$$
(12)

Note that  $\sigma_l^2(nT_0)$  is a periodic function with period  $T_l$ , i.e.  $\sigma_l^2(nT_0) = \sigma_l^2\left(nT_0 + \frac{T_l}{T_0}T_0\right)$ . Therefore, the average variance  $\sigma_l^2$  can be written

$$\sigma_l^2 = \frac{1}{[[T_l/T_0]]} \sum_{n=0}^{[[T_l/T_0]]-1} \sigma_l^2(nT_0)$$
(13)

where [[x]] rounds x to the nearest integer. Assuming that the NBI signals are independent, the total variance  $\sigma_I^2$  of the estimator equals  $\sum_{l=1}^{N_I} \sigma_l^2$ .

## 4. Numerical Results

For the numerical and simulation results, we assume that the number of sub-carriers N = 256 and the number of active sub-carriers is  $N_u = 256$  i.e. all carriers are modulated. The guard interval equals  $\nu = 20$ and the bandwidth of OFDM signal is  $B_0 = 1024$  kHz. The bandwidth of NBI signal equals  $B_l = 25$  kHz and  $\tau_l = 0$ . We use 8-PSK and QPSK modulation for the data symbols of the OFDM and the interferer signals respectively. Transmit filters are square-root raised-cosine filters with roll off factors  $\alpha_0 = 0.25$  and  $\alpha_l = 0.5$  for OFDM and interfering signals, respectively.

Fig. 2 shows the simulated and analytical variance of the estimation error  $(\sigma_I^2)$  as a function of the window length (2K + 1) assuming that the OFDM system transmits data on all carriers. As can be observed, at high signal to interference SIR values,  $\sigma_I^2$  decreases with window size. At low SIR,  $\sigma_I^2$  shows a minimum at intermediate window size. This can be explained with the aid of Fig. 3. In the estimator, there are two types of noise. The first type comes from the OFDM signal and AWGN noise (N1) while the second type is the noise (N2) that results from the unability of the estimator to track the variations of the NBI signal. At high SIR values, the former dominates. Since increasing the window size reduces the effect of this type of noise, we notice that  $\sigma_I^2$  decreases with the window size. At low SIR, i.e. high interference power, the first type of noise diminishes but the second type increases with increasing window size. Therefore,  $\sigma_I^2$  increases again with the window size.

Fig. 4 shows the variance of the estimation error  $(\sigma_I^2)$  as a function of the signal to interference ratio (SIR) at a window size equal to 33 assuming that the OFDM system does not transmit data on M carriers around the location of the NBI. Since at low SIR, the second type of noise dominates, the OFDM signal has only a small effect on the estimator. Therefore,  $\sigma_I^2$  is approximately independent of M as can be observed in the figure. For large SIR ( $\geq 0$  dB), the effect of the second type of noise diminishes and the first type of noise dominates. Increasing M will reduce the effect of this type of noise, as the the spectral leakage from the OFDM signal to the NBI signal reduces. Therefore, increasing M leads to a reduction of  $\sigma_I^2$ .

Fig. 5 shows the power spectrum of the original interference signal and residual interference signal (after cancellation) in two cases. In the first case, we have assumed that the received signal only consists of NBI signal, i.e. the OFDM signal and noise are not present. This result gives us an indication of the maximum possible interference reduction. In the second case, we also consider the presence of the OFDM signal and noise. We observe a strong reduction in power for frequencies close to the frequency of NBI. The reduction of larger NBI frequency components will be smaller.

Fig. 6 shows the bit error rate (BER) performance of the OFDM system with and without NBI cancellation for different values for M. As can be observed, NBI cancellation achieves a great improvement in

BER performance for small SIR values i.e. when the interference signal is strong as compared to the OFDM signal. At high SIR values, the proposed cancellation algorithm has worse performance than the case of no NBI cancellation. This is explained as at high SIR, the NBI signal is very small as compared to the OFDM signal, and therefore, the NBI is very difficult to extract from the received signal. Further, at high SIR the presence of the estimator will cause a noise enhancement to the OFDM system with variance equal to  $(\sigma_s^2 + \sigma_n^2) / (2K + 1)$  per real dimension (see (12)). However, at high SIR, the effect of the NBI on the OFDM system (without cancellation) becomes negligible and the BER reaches an asymptote. This asymptote depends on the SNR, as AWGN is the dominating disturbance on the OFDM system. Note however that the difference between in BER no cancellation for the proposed algorithm becomes small when the gap M increases. Therefore, we can conclude that the proposed algorithm works well when M is chosen sufficiently large.

Fig 7 shows the total variance  $\sigma_I^2$  of the estimator as function of the number  $N_I$  of NBI signals in two cases. In case 'A', we consider that the SIR is fixed per interferer, so the total SIR decreases inversely proportional to  $N_I$ . In case 'B', we consider a fixed total SIR, i.e. SIRper interferer decreases linearly as  $N_I$  increases. As can be observed, the variance of the estimator increases with  $N_I$ . This is because the noise caused by other NBI signal increases with  $N_I$ . The corresponding BER performance of the MC system is shown in Fig. 8. We note that the BER is essentially independent of  $N_I$  at M = 16: the spectrum leakage from the NBI signals on the MC signal becomes very small when Mincreases.

#### 5. Conclusions

In this paper, a new NBI cancellation scheme for MC systems has been proposed. The estimator is based on averaging the received baseband samples over a sliding window. It can be used to suppress the spectral leakage that occurs when many digitally modulated NBI signals reside in the same frequency band of the MC signal. Further, we have derived the mean squared error (MSE) of the estimator in an analytical way. Simulation results show that the theoretical expressions for the MSE agree well with the simulation. Moreover, bit error rate performance shows that the proposed estimator performs well especially if the MC system avoids the use of a number of subcarriers around the NBI frequencies.

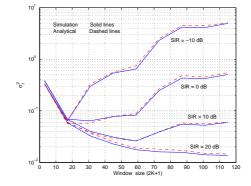


Figure 2. Averaging variance,  $\sigma_I^2$  at SNR = 8 dB,  $N_I = 1$ .

## Notes

1. i.i.d. = independently and identically distributed

2. From the estimator viewpoint, the OFDM signal can be modeled (according to central limit theorem) as zero-mean Gaussian distributed.

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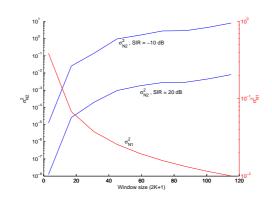


Figure 3. Averaging variance,  $\sigma_{N1}^2$ ,  $\sigma_{N2}^2$  at SNR = 8 dB,  $N_I = 1$ .

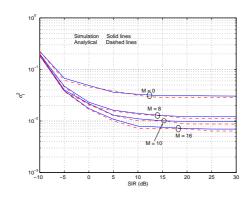


Figure 4. Averaging variance,  $\sigma_I^2$  at SNR = 8 dB, and  $N_I = 1$ 

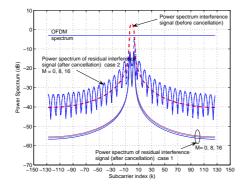


Figure 5. Power spectrum at SNR = 17 dB, SIR = 10 dB, and  $N_I = 1$ 

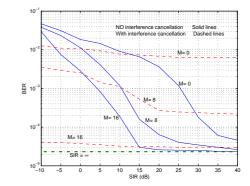


Figure 6. BER at SNR = 17dB and  $N_I = 1$ 

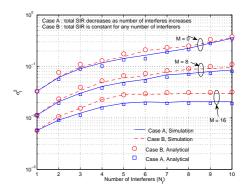


Figure 7. Averaging variance,  $\sigma_I^2$  at SIR = 10 dB and SNR=17dB

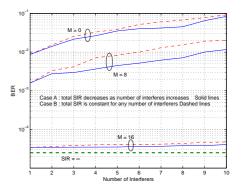


Figure 8. BER at SNR=17 dB and SIR = 10 dB