Soft Information Aided Phase Noise Correction for OFDM Systems

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Abstract- Orthogonal Frequency Division Multiplexing (OFDM) systems suffer from severe performance degradation in the presence of phase noise. In particular, phase noise leads to a common phase error (CPE) as well as intercarrier interference (ICI) in the frequency domain. In this contribution, we present a novel code aided CPE estimation algorithm. The Expectation-Maximization (EM) algorithm is used to approach the Maximum-Likelihood (ML) estimate of the CPE. The estimator accepts soft information from the decoder in the form of a posteriori probabilities of the coded symbols, which can be interpreted as performing joint data detection and decoding. The performance of the proposed algorithm is verified through computer simulations. Impressive performance gains are obtained as compared to conventional data-aided CPE correction.

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is recognized to be one of the best transmission techniques for wideband wireless communication systems. However, OFDM is known to be vulnerable to synchronization errors due to the narrow spacing between subcarriers [1]. One of these synchronization errors is phase noise (PN) i.e. the random fluctuation in the difference between the phase of the carrier and the phase of the local oscillator (LO). The influence of PN on the performance of an OFDM system has been extensively analyzed in the literature [2], [3], [4]. Generally, the influence can be split into a multiplicative part, which is common to all subcarriers and therefore often referred to as common phase error (CPE), and an additive part, which is often referred to as intercarrier interference (ICI). It turns out that the CPE dominates for slow PN (PN varies slowly in comparison to the OFDM symbol duration), while ICI is dominant for fast PN [5].

Most of the approaches estimate CPE by using pilot symbols [6], [7], [8], [9] at the cost of decreased throughput. Further, with the advent of powerful error-correcting codes (including turbo and LDPC codes), these conventional data-aided estimation algorithms can not always be applied successfully. Since such codes operate at low E_b/N_o values, many pilot symbols may be necessary to acquire reliable estimates, resulting in a significant loss in terms of power and bandwidth efficiency. It is thus of interest to develop schemes to estimate the PN by taking into account the presence of the error correcting codes. In this paper, we propose a code aided CPE estimation algorithm. Starting from the Maximum Likehood (ML) principle, we derive the estimator based on the Expectation-Maximization (EM) algorithm [10]. The proposed algorithm iterates between data detection and estimation, improving the estimate of CPE. In the first iteration, we decode the received data without having any information about the CPE, or using a few pilots to obtain a rough estimate of the CPE. Next, based on a posteriori probabilities provided by the decoder, the posteriori expectations of the transmitted symbols can be computed; they are used to estimate the CPE in an iterative way. It is noteworthy that the designed estimation scheme can work with any detector as long as the detector is able to compute the a posteriori probabilities (APPs) of the data symbols.

The paper is organized as follows: the system model is described in section II. The CPE estimation algorithm is derived in section III. Simulation results are provided in section IV. Finally, we end with conclusions in section V.

II. System Model

The block diagram of the proposed system is shown in Fig. 1. At the transmitter, the X_b information bits are sent to the encoder, resulting in X_c coded bits. After interleaving, these X_c coded bits are mapped on X_d symbols $\{d(k)\}_{k=0}^{X_d-1}$ belonging to a 2^q -point constellation, $X_d = X_c/q$. We further assume that a total of N_p pilot symbols $\{p(k)\}_{k=0}^{N_p-1}$ are inserted at known locations in each OFDM symbol. The set of subcarrier indices on which pilots are transmitted will be denoted as Υ. The block of

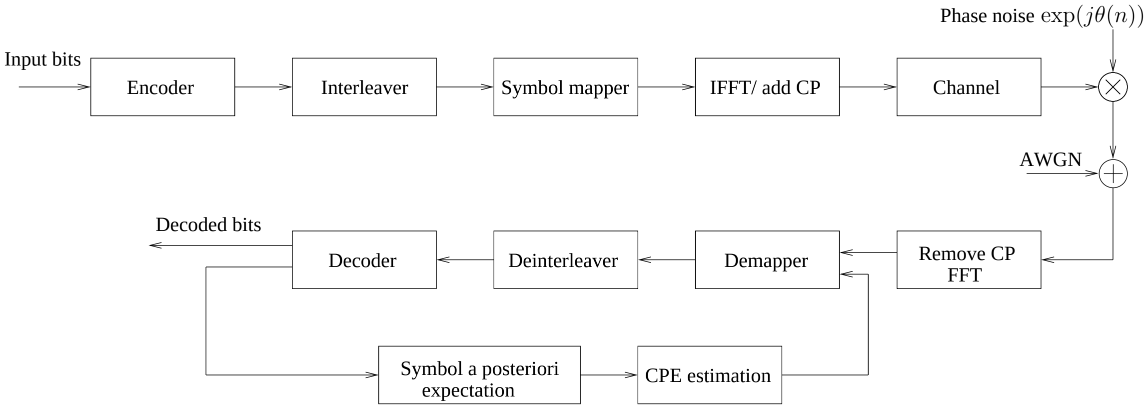

Fig. 1. The conceptual block diagram of the proposed system

length $X_d + N_p$ symbols is converted to the time domain using an N-point inverse Fourier transform (IFFT). A cyclic prefix of length N_g samples is inserted to cope with channel multipath fading and to enable simple channel equalization at the receiver. We can write the n-th time-domain sample ($n = -N_g, \ldots, N-1$) of the OFDM block as

$$s(n) \quad = \quad \sqrt{\frac{1}{N}} \sum_{k=0}^{N-1} a(k)\, e^{\frac{j2\pi kn}{N}} \tag{1}$$

where

$$a(k) = \begin{cases} d(k) & k \notin \Upsilon \\ p(k) & k \in \Upsilon \end{cases} \tag{2}$$

The data symbols are independently and identically distributed random values with zero mean and variance $E\left[|d(k)|^2\right] = E_s$. The transmitted OFDM block propagates to the receiver through an L- tap channel with overall channel impulse response (CIR) $\mathbf{h} = [h(0), \ldots, h(L-1)]$. This CIR incorporates the transmit filter, physical propagation channel and receive filter. At the receiver, the signal is multiplied by the PN disturbance $\phi(n) = \exp(j\theta(n))$. The received sample $r(n)$ can be expressed as

$$r(n) = [s(n) \star h(n)]\, e^{j(\theta(n))} + w(n) \tag{3}$$

where the symbol $\star$ stands for convolution, $w(n)$ represents white Gaussian noise with variance σ_n^2, and $n = -N_g, \ldots N + L - 2$. After removing the cyclic prefix and applying the Fast Fourier Transform (FFT), the demodulated symbol $R(k')$ at carrier k' can be written as

$$\begin{aligned} R(k') \;&=\; a(k')H(k')I(0) \\ &+ \sum_{\substack{k=0,\\ k \neq k'}}^{N-1} a(k)H(k)I(k-k') + \xi(k') \end{aligned} \tag{4}$$

where $\xi(k')$ is the Gaussian noise contribution, and $I(i)$ is given by

$$I(i) = \frac{1}{N} \sum_{n=0}^{N-1} e^{[j2\pi in/N + \theta(n)]} \tag{5}$$

Equation (4) shows that the PN has two different effects. First, each information symbol $a(k')$ is multiplied by $I(0)$ (referred to as the common phase error (CPE)) which depends on the phase noise, but it is independent of the particular subcarrier index. Secondly, the PN causes intercarrier interference (ICI) depending on the data from all subcarriers.

III. CPE Estimation Algorithm

In this section, we give a brief outline of the EM algorithm, and apply it to the observation vector $\mathbf{R} = [R(0), \cdots, R(N-1)]$ from (4).

A. EM algorithm-principle

The EM algorithm is an iterative approach to acquire the ML estimates when evaluation of the likelihood is difficult due to some unknown data [10]. The original ML problem involves the estimation of a parameter (set) λ from an observation $\mathbf{r}$, by maximizing the likelihood function $\mathrm{p}(\mathbf{r}\,|\lambda)$. In the presence of unknown data (e.g. unknown transmitted symbols), finding the ML solution can be very difficult. The main idea behind the EM algorithm is to define the so-called missing (or unobserved) data

$\mathbf{a}$, such that, if the missing data were known, estimating λ would be easy, i.e. maximizing $\mathrm{p}\left(\mathbf{r}\left|\mathbf{a},\lambda\right.\right)$ is feasible. However, since we do not know the missing data, an iterative approach starting from an initial estimate λ (say, $\widehat{\lambda}\left(0\right)$) is used. Consider $\mathbf{r}$ as the "incomplete" observation and $\mathbf{z} \doteq [\mathbf{r}, \mathbf{a}]$ as the "complete" observation. At iteration ζ, the EM algorithm consists of two steps :

1) E-step: given the current estimate $\widehat{\lambda}\left(\zeta\right)$ and "incomplete" observation $\mathbf{r}$, we first take the expectation of the log-likelihood of the complete data $\mathbf{z} \doteq [\mathbf{r}, \mathbf{a}]$ with respect to the unknown data $\mathbf{a}$:
$$Q\left(\lambda\left|\widehat{\lambda}\left(\zeta\right)\right.\right) = E_{\mathbf{a}}\left[\log \mathrm{p}\left(\mathbf{z}\left|\lambda\right.\right)\left|\mathbf{r}, \widehat{\lambda}\left(\zeta\right)\right.\right]. \quad (6)$$
2) M-step: we maximize $Q\left(\lambda\left|\widehat{\lambda}\left(\zeta\right)\right.\right)$ with respect to λ to find a new estimate:
$$\widehat{\lambda}\left(\zeta+1\right) = \arg\max_{\lambda}\left\{Q\left(\lambda\left|\widehat{\lambda}\left(\zeta\right)\right.\right)\right\}. \quad (7)$$

The EM algorithm terminates when the estimate has converged or a certain stopping criterion has been met. For continuous parameters, the final estimate converges to the ML estimate as long as the initial estimate is sufficiently accurate.

B. Soft Information CPE Estimation Algorithm

Let us consider the received vector $\mathbf{R} = [R(0), \cdots, R(N-1)]$, the pilot vector $\mathbf{P} = [p(0), \cdots, p(N_p-1)]$, and the transmitted data vector $\mathbf{a} = [a(0), \cdots a(N-1)]$. Based on the observation model (4), we can write:

$$\log \mathrm{p}\left(\mathbf{R}, \mathbf{a}\left|I\left(0\right)\right.\right) \quad \propto \quad \log \mathrm{p}\left(\mathbf{R}\left|\mathbf{a},\, I\left(0\right)\right.\right) \quad (8)$$

Assuming the ICI plus noise in (4) can be modeled as a Gaussian random variable, we can write

$$\log \mathrm{P}\left(\mathbf{R}\left|\mathbf{a},\, I\left(0\right)\right.\right) \propto \sum_{i=0}^{N-1} \left|R(i) - a(i)H(i)I(0)\right|^2 \quad (9)$$

Using (8) and (9) in (6) yields

$$\begin{aligned} &Q\left(I\left(0\right)\left|\widehat{I\left(0\right)}\left(\zeta\right)\right.\right) = \\ &\textstyle\sum_{i\notin\Upsilon}\left(\left|R(i)\right|^2 + \left|I\left(0\right)H(i)\right|^2\psi(i)\right. \\ &\left.-2\mathrm{Re}\left\{R(i)I^*(0)H^*(i)\mu^*(i)\right\}\right) \\ &+\textstyle\sum_{i\in\Upsilon}\left(\left|R(i)\right|^2 + \left|I\left(0\right)H(i)\right|^2\left|p(i)\right|^2\right. \\ &\left.-2\mathrm{Re}\left\{R(i)I^*(0)H^*(i)p^*(i)\right\}\right) \end{aligned} \quad (10)$$

where $\mu(i) = E\left[a(i)\left|\mathbf{R}, \widehat{I\left(0\right)}\left(\zeta\right)\right.\right]$ and $\psi(i) = E\left[\left|a(i)\right|^2\left|\mathbf{R}, \widehat{I\left(0\right)}\left(\zeta\right)\right.\right]$. The a posteriori expectations $\mu(i)$ and $\psi(i)$ are obtained by using the a posteriori probabilities (APPs) computed by the decoder:

$$\mu(i) = \sum_{\omega\in\Omega} \omega \cdot \mathrm{p}\left(a(i) = \omega\left|\mathbf{R}, \widehat{I\left(0\right)}\left(\zeta\right)\right.\right). \quad (11)$$

$$\psi(i) = \sum_{\omega\in\Omega} \left|\omega\right|^2 \cdot \mathrm{p}\left(a(i) = \omega\left|\mathbf{R}, \widehat{I\left(0\right)}\left(\zeta\right)\right.\right) \quad (12)$$

Using (10), it follows that the updated estimate for the CPE is given as

$$\widehat{I\left(0\right)}\left(\zeta+1\right) = \arg\max_{I(0)} \frac{\partial Q\left(I\left(0\right)\left|\widehat{I\left(0\right)}\left(\zeta\right)\right.\right)}{\partial I(0)} \quad (13)$$

After some mathematical manipulations, the closed form for the updated estimate is given by:

$$\widehat{I\left(0\right)}\left(\zeta+1\right) = \frac{\sum_{i=0}^{N-1} R(i)H^*(i)\chi(i)}{\sum_{i=0}^{N-1} \left|H(i)\right|^2 \xi(i)} \quad (14)$$

where $\chi(i) = p^*(i),\ \xi(i) = \left|p(i)\right|^2\ \forall\, i \in \Upsilon$, $\chi(i) = \mu^*(i),\ \xi(i) = \psi(i)\ \forall\, i \notin \Upsilon$. In the sequel, (14) is denoted as the pilot assisted estimator (PAE), where the initial value of $\widehat{I\left(0\right)}$ (the first iteration) is given as

$$\widehat{I\left(0\right)}\left(1\right) = \frac{\sum_{i\in\Upsilon} R(i)H^*(i)p^*(i)}{\sum_{i\in\Upsilon} \left|H(i)\right|^2 \left|p(i)\right|^2} \quad (15)$$

In the absence of pilot symbols, i.e. if Υ is empty, we assume that the initial value of $\widehat{I\left(0\right)}$ equals one, i.e. in the first iteration, we decode the received data without having any information about the CPE. This is referred to as the blind estimator (BE).

To obtain a lower bound on the mean squared error of the estimate, we assume all the data symbols are known, i.e. the number of pilots N_p equals N. Then,

$$\widehat{I\left(0\right)}_{lower} = \frac{\sum_{i=0}^{N-1} R(i)H^*(i)p^*(i)}{\sum_{i=0}^{N-1} \left|H(i)\right|^2 \left|p(i)\right|^2} \quad (16)$$

IV. Simulation Results

To validate the proposed algorithm, we have carried out Monte Carlo simulations over a frequency selective channel. We consider an OFDM system, using a convolutional code with constraint length 5, rate 1/2 and polynomial generators $(23)_8$ and $(35)_8$. The BCJR algorithm is used for decoding. A block length of $X_b = 128$ information and pilot bits was chosen, leading to $X_c = 256$ coded bits. The coded bits are Gray-mapped on a 16-QAM constellation, resulting in $X_d + N_p = 64$ symbols. We take $N_p = 4$ for the pilot estimator while $N_p = 0$ for the blind estimator. The channel consists of $L = 4$ statistically independent taps, each being a zero-mean complex Gaussian random variable with an exponential power delay profile [11]:

$$E\left[|h(l)|^2\right] = E_h \exp(-l/5), \, l = 0, \ldots, L-1 \tag{17}$$

where E_h is chosen such that the average energy per subcarrier is normalized to unity. To avoid ISI, a cyclic prefix of length $N_g = L - 1$ is employed.

Figure 2 shows the mean squared estimation error of the CPE for both estimators, $\text{MSE} = E\left[\left|I(0) - \widehat{I(0)}\right|^2\right]$ with Wiener phase noise rate $\beta T = 0.008$ as a function E_b/N_0, for a frequency selective channel. As can be observed, since we assume that the estimated value of $I(0)$ for the blind estimator is 1 at the first iteration, the MSE is independent of E_b/N_0. On the other hand, at the first iteration, the MSE for the pilot estimator decreases with E_b/N_0. At low PN rate $I(0)$ is almost one, therefore our approximation for the blind estimator ($\widehat{I(0)}(1) = 1$) is good enough. However, for the pilot estimator, a few pilot symbols is not enough to obtain a reliable estimate at low or even at intermediate E_b/N_0. This explains why the performance of the blind estimator is better than the pilot estimator at the first iteration over a wide range of E_b/N_0. The pilot estimator slightly outperforms the blind estimator after the first iteration. A strong improvement of the MSE for both estimators is achieved after only two iterations. Further, we notice that no improvement in the MSE is visible after 3 EM iterations for both estimators for any E_b/N_0. Moreover, the MSE for both estimators converges to the lower bound obtained with (16) for a wide range of E_b/N_0.

Figure 3 shows the corresponding bit error rate (BER) performance for both estimators as function of E_b/N_0 for a frequency selective channel. We observe a large improvement of the BER performance with the proposed algorithms as compared to the case of no CPE correction. In the figure, we also show the BER when the CPE could perfectly be corrected and the case with no PN. The difference between these curves is caused by the presence of the ICI. The proposed pilot and blind schemes have a BER that reaches the BER of perfect CPE correction for a large range of E_b/N_0. At high E_b/N_0, the influence of the ICI however becomes dominant, such that the decoder can not deliver reliable APP's. This results in a slight increase of the BER as compared to perfect CPE correction especially for the blind estimator.

Figures 4 and 5 show the MSE and BER performance of the proposed schemes as function of the phase noise rate βT for a frequency selective channel at E_b/N_o= 25 dB, respectively. In Fig. 5, the BER is added for the cases where no phase noise is present and perfect CPE estimation. We note that at the first iteration, the performance of the pilot estimator is worse than that of the blind estimator for small values of the PN rate βT. As we explained earlier, $I(0)$ is almost one for small PN rates. Therefore, the approximation of $I(0)$ by 1 gives a better performance as compared to the estimation of $I(0)$ with a few pilots. However, both estimators give approximately the same performance after the first iteration. Moreover, both estimators reach lower bound for a wide range of the PN rate. Further, it can be observed that for the large values of the PN rate, $\beta T > 0.05$, the performance of the estimators strongly degrades with an increasing number of iterations. At high rates, the ICI component becomes dominant over the useful component [3], disturbing the proper action of the decoder. However in many practical applications βT is smaller than 10^{-2}, in which case the proposed schemes work properly.

V. CONCLUSIONS

In this paper, we have proposed a novel soft code-aided estimation algorithm for common phase error correction in an OFDM system. Based on the EM algorithm, the receiver iterates between data detection and estimation, with the exchange of the soft information. The results indicate that the proposed algorithm achieves a significant improvement in MSE and BER performance after 2 iterations as compared to the case of no CPE correction.

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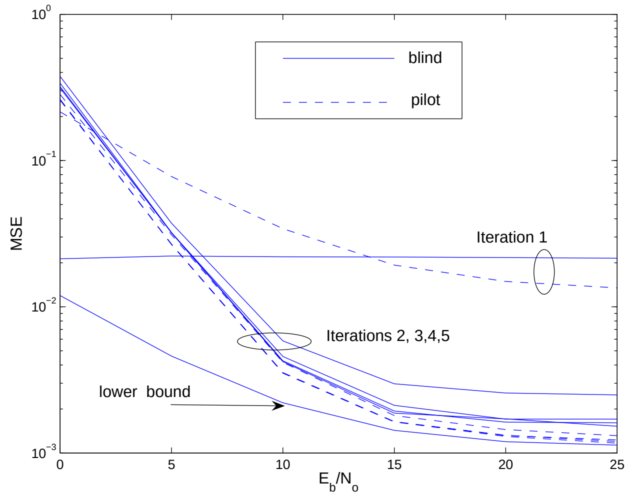

Fig. 2. Mean squared error (MSE) as function of E_b/N_0 over frequency selective channel with phase noise rate $BT = 8(10)^{-3}$

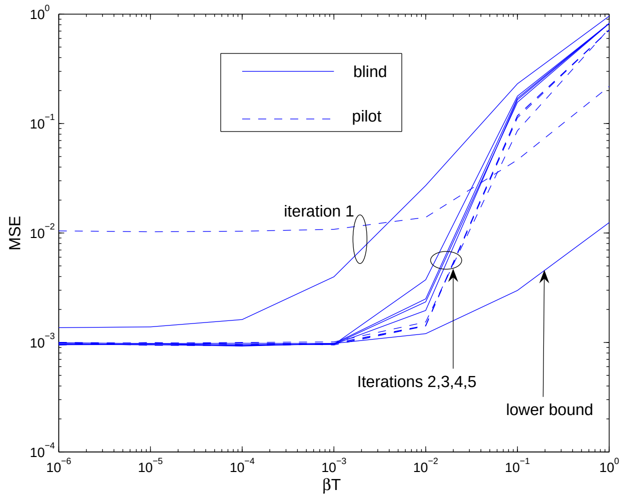

Fig. 4. Mean squared error (MSE) as function of phase noise rate βT over frequency selective channel at $E_b/N_0 = 25$ dB

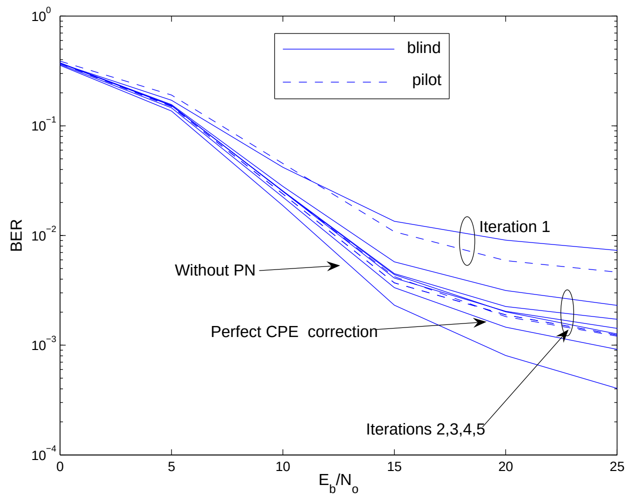

Fig. 3. BER as function of E_b/N_0 over frequency selective channel with phase noise rate $BT = 8(10)^{-3}$

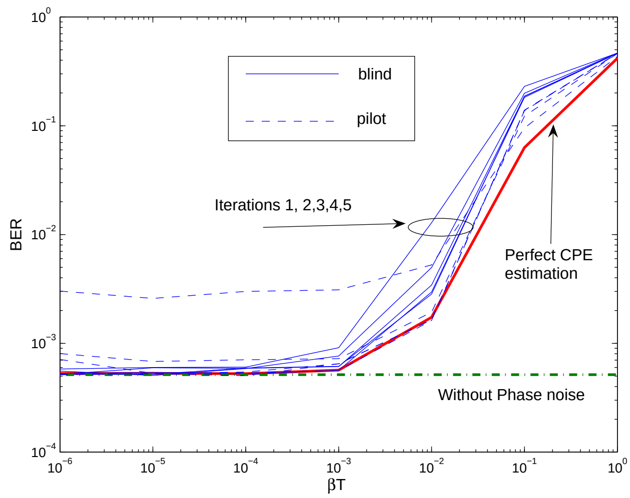

Fig. 5. BER as function of phase noise rate βT over frequency selective channel at $E_b/N_0 = 25$ dB

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