Iterative Residual Frequency Offset Correction for OFDM Systems

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ABSTRACT

The estimation and tracking of the fractional carrier frequency offset (CFO) is a crucial issue in the implementation of orthogonal frequency division multiplexing (OFDM) systems. In this contribution, we present a novel code-aided fractional CFO estimation algorithm based on the Expectation-Maximization (EM) algorithm. The proposed algorithm exchanges soft information from the channel decoder, in the form of a posteriori probabilities of the coded symbols, between the demapper, the decoder, and the CFO estimator in an iterative way. The proposed estimation scheme can work with any detector as long as the detector is able to compute the a posteriori probabilities (APPs) of the data symbols.

Index-Terms- Orthogonal Frequency Division Multiplexing (OFDM), Carrier Frequency Offset (CFO), Coding, Expectation-Maximization (EM) algorithm.

I. Introduction

The Orthogonal Frequency Division Multiplexing (OFDM) transmission system has received great interest in wireless communication due to its high spectral efficiency and robustness to multipath channels [1], [2]. It has been adopted for several types of high data rate wireless communication systems, including digital video/audio broadcasting [3] and wireless local area networking (WLAN) [4]. In such systems, the high rate data stream is split in many low-rate parallel streams, each modulating an orthogonal sub-carrier. Since the individual sub-carrier signal spectra are affected by frequency flat fading rather than frequency selective fading, equalization is drastically simplified.

Unfortunately, OFDM is known to be vulnerable to carrier frequency offset (CFO) [5]. One main source of CFO is the mismatch between the carrier frequencies at the transmitter and the receiver. Another possible source is the Doppler shift caused by relative motion between the transmitter and the receiver. The CFO is usually divided into an integer part, which is a multiple of the sub-carrier spacing, and a fractional part, which is less than the sub-carrier spacing. The former causes a circular shift of the transmitted symbols resulting in a high BER [6], but does not cause inter-carrier interference (ICI); i.e., the orthogonality of the subcarriers is maintained. The latter leads to a reduction and rotation of the signal amplitude and to a loss of sub-carrier orthogonality. This loss introduces inter-carrier interference (ICI) which results in a degradation of the global system performance. A frequency offset as small as a few percent of the sub-carrier spacing is sufficient to impair the performance of an OFDM receiver, as pointed out in [5]. In order to operate correctly, an OFDM receiver needs accurate compensation of the carrier frequency offset in the input signal.

In this paper, we concentrate on the compensation of a fractional CFO with respect to the carrier spacing. The previously proposed methods for fractional carrier frequency synchronization of OFDM systems can be classified into two main categories, namely data aided methods [7], [8] and blind methods [9]–[12]. In the data aided methods, the CFO estimate is acquired through sending training symbols. Since training symbols carry no information, these methods induce a loss of bandwidth efficiency. On the other hand, blind methods are able to avoid the need of training symbols by exploiting the inherent structure of received data. Typical structures include virtual sub-carrier [10], [13], cyclic data structure due to cyclic prefix [9], [11] and cyclostationarity [12].

The proper action of the frequency synchronization can be strongly disturbed by operating at very low signal to noise ratios, which occurs in turbo or LDPC coded systems. This motivates the design of a new fractional frequency offset estimation algorithm in which the phase of the decoded data is tracked; the algorithm is initialized by a conventional data aided or blind frequency estimator. Starting from the Maximum Likehood (ML) principle, we derive an estimator that is based on the Expectation-Maximization (EM) algorithm [14]. The proposed algorithm iterates between data detection and estimation, improving the estimate of CFO. In the first iteration, we decode the received data without having any information about the residual CFO, or using a few pilots to obtain a rough estimate of the residual CFO. Next, based on a posteriori probabilities provided by the decoder, the a posteriori expectations of the transmitted symbols can be computed; they are used to estimate the CFO in an iterative way. In [15], we exploited the same idea to estimate the common phase error resulting from carrier phase noise (PN). Instead of modeling ICI resulting from PN as a Gaussian noise as in [15], ICI resulting from CFO can be mitigated by using the estimated CFO and the decoder output.

The rest of the paper is organized as follows. In section II, we consider the system model, where the proposed transmitter and receiver are described. The CFO estimation algorithm is derived in section III. Simulation results are provided in section IV. Finally, we end with conclusions in section V.

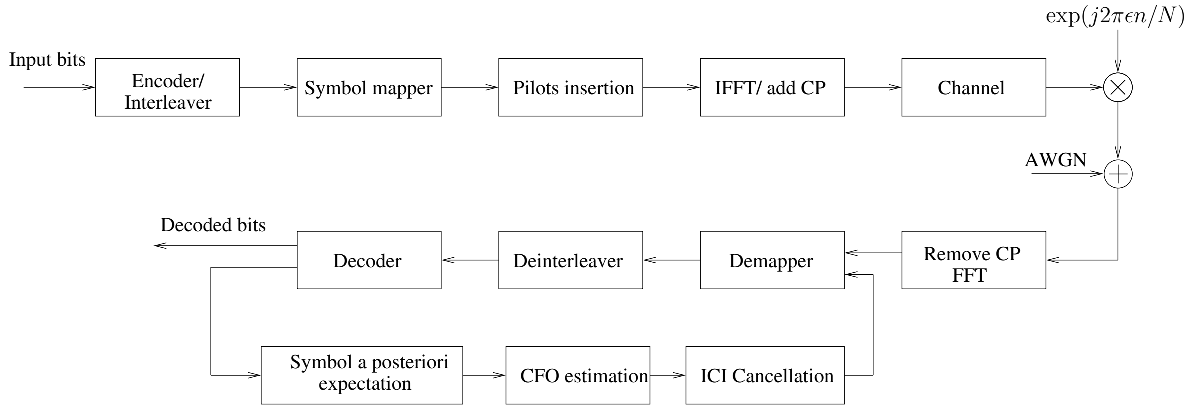

Fig. 1. The conceptual block diagram of the proposed system.

II. SYSTEM MODEL

Fig. 1 depicts the discrete time baseband equivalent block diagram for the proposed OFDM system. At the transmitter, a binary information sequence $\mathbf{b} = \{b(0), b(1), \cdots, b(N_b - 1)\}$ is passed through a channel encoder with coding rate R_c. The resulting binary codeword $\mathbf{c} = \{c(0), c(1), \cdots, c(N_c - 1)\}$, $N_c = N_b/R_c$, is then interleaved and mapped onto N_d symbols $\mathbf{d} = \{d(0), d(1), \cdots, d(N_d - 1)\}$ where $d(i)$ belongs to a 2^q -point constellation, $N_d = N_c/q$. We further assume that a total of N_p pilot symbols $\mathbf{p} = \{p(0), p(1), \cdots, p(N_p)\}$ are inserted at known sub-carriers in each OFDM symbol. The set of sub-carrier indices on which pilots are transmitted is denoted as Γ. The block of $N = N_d + N_p$ symbols is converted to the time domain using an N-point inverse Fourier transform (IFFT). The OFDM symbol is extended using a cyclic prefix of ν samples. We can write the n-th time-domain sample ($n = -\nu, \ldots, N-1$) of the OFDM block as

$$s(n) = \sqrt{\frac{1}{N+\nu}} \sum_{k=0}^{N-1} z(k)\, e^{\frac{j2\pi kn}{N}} \tag{1}$$

where

$$z(k) = \begin{cases} d(k) & k \notin \Gamma \\ p(k) & k \in \Gamma \end{cases} . \tag{2}$$

The data symbols are independently and identically distributed random values with zero mean and variance $E\left[|d(k)|^2\right] = E_s$. The transmitted OFDM block propagates to the receiver through an L- tap channel with overall channel impulse response (CIR) $\mathbf{h} = [h(0), \ldots, h(L-1)]$. This CIR incorporates the transmit filter, physical propagation channel and receiver filter.

Assuming perfect time synchronization, the nth time domain received sample $r(n)$ can be written as:

$$r(n) = [s(n) \star h(n)]\, e^{j(2\pi\epsilon n/N)} + w(n) \tag{3}$$

where the symbol $\star$ stands for convolution, ϵ represents the carrier frequency offset normalized to the carrier spacing, $w(n)$ is the white Gaussian noise with variance σ_n^2, and $n = -\nu, \ldots N + L - 2$. After removing the cyclic prefix and applying the Fast Fourier Transform (FFT), the demodulated symbol $Y(k')$ at carrier k' can be written as [5]

$$\begin{aligned} Y(k') &= z(k')H(k')\vartheta(\epsilon) \\ &+ \sum_{\substack{k=0,\\ k\neq k'}}^{N-1} z(k)H(k)\eta(k-k',\epsilon) + \xi(k') \end{aligned} \tag{4}$$

where $\xi(k')$ is the Gaussian noise contribution, $\eta(m,\epsilon)$ is given by

$$\eta(m,\epsilon) = \frac{1}{\sqrt{N(N+\nu)}} \frac{\sin(\pi(m+\epsilon))}{\sin(\pi((m+\epsilon)/N))} e^{j\pi(m+\epsilon)(N-1)/N} \tag{5}$$

and $\vartheta(\epsilon) = \eta(0,\epsilon)$. Equation (4) shows that the CFO has two detrimental effects; one is the reduction and rotation of each FFT output and the second is the introduction of ICI from other carriers, as the CFO destroys the orthogonality between subcarriers.

III. CPE ESTIMATION ALGORITHM

In this section, we give a brief outline of the EM algorithm, and apply it to the observation vector $\mathbf{Y} = [Y(0), \cdots, Y(N-1)]$ from (4) to estimate the CFO ϵ.

A. EM Algorithm-Principle

The EM algorithm is an iterative technique to approach the ML estimate of a parameter (set) ρ from an observation $\mathbf{r}$ [14]. It is based on the concept of the so-called missing data $\mathbf{a}$, such that, if the missing[1] data were known, the estimation of ρ would be easy. We denote the iteration index by ι. Starting from a first estimate $\hat{\rho}(\iota = 1)$, we iteratively apply the following two steps:

- E-step:

$$Q(\rho\,|\widehat{\rho}(\iota)) = \int \log(\mathrm{p}(\mathbf{r}\,|\mathbf{a},\rho))\, \mathrm{p}(\mathbf{a}\,|\mathbf{r},\widehat{\rho}(\iota))\, d\mathbf{a} \tag{6}$$

[1] In this paper, missing data refers to transmitted data.

- M-step:

$$\widehat{\rho}\,(\iota+1) = \arg\max_{\iota} \left\{ Q\left(\rho\,|\widehat{\rho}\,(\iota)\right) \right\}. \tag{7}$$

The EM algorithm terminates when the estimate has converged or a certain stopping criterion has been met. For continuous parameters, the final estimate converges to the ML estimate as long as the initial estimate is sufficiently accurate [16], [17].

B. Soft Decision-Directed CFO Estimation

Let us consider the received vector $\mathbf{Y} = [Y(0), \cdots, Y(N-1)]$ and the transmitted data vector $\mathbf{z} = [z(0), \cdots z(N-1)]$. We represent the estimated ICI term in (4) at iteration (ι) by the vector $\mathbf{I}^{(\iota)} = \left[I^{(\iota)}(0), \cdots, I^{(\iota)}(N-1)\right]$ where $I^{(\iota)}(k')$ is given by

$$I^{(\iota)}(k') = \sum_{\substack{k=0,\\ k \neq k'}}^{N-1} \widehat{z}^{(\iota)}(k')H(k')\eta\left(k-k', \widehat{\epsilon}^{(\iota)}\right) \tag{8}$$

where $\widehat{q}^{(\iota)}$is the estimated value of q at iteration ι. Let us define the vector $\mathbf{R}^{(\iota)} = \mathbf{Y} - \mathbf{I}^{(\iota)}$ and assume that $\mathbf{R}^{(\iota)}$ is jointly Gaussian distributed. Accordingly, we can write:

$$\log \mathrm{p}\left(\mathbf{R}^{(\iota)}, \mathbf{z}\,|\vartheta(\epsilon)\right) \quad \propto \quad \log \mathrm{p}\left(\mathbf{R}^{(\iota)}\,|\mathbf{z},\, \vartheta(\epsilon)\right) \tag{9}$$

and

$$\log \mathrm{P}\left(\mathbf{R}^{(\iota)}\,|\mathbf{z},\, \vartheta(\epsilon)\right) \propto \sum_{k=0}^{N-1} \left| R^{(\iota)}(k) - z(k)H(k)\vartheta(\epsilon)\right|^2. \tag{10}$$

Using (10) in (6) yields

$$\begin{aligned}
&Q\left(\vartheta(\epsilon)\left|\widehat{\vartheta(\epsilon)}\,(\iota)\right.\right) = \\
&\textstyle\sum_{k\notin\Gamma}\Big(\left|R^{(\iota)}(k)\right|^2 + \left|\vartheta(\epsilon)H(k)\right|^2\psi(k) \\
&-2\mathrm{Re}\left\{R^{(\iota)}(k)\vartheta^*(\epsilon)H^*(k)\mu^*(k)\right\}\Big) \\
&+\textstyle\sum_{k\in\Gamma}\Big(\left|R^{(\iota)}(k)\right|^2 + \left|\vartheta(\epsilon)H(k)\right|^2|p(k)|^2 \\
&-2\mathrm{Re}\left\{R^{(\iota)}(k)\vartheta^*(\epsilon)H^*(k)p^*(k)\right\}\Big)\ .
\end{aligned} \tag{11}$$

The a posteriori expectations $\mu(k)$ and $\psi(k)$ are obtained by exploiting the a posteriori probabilities (APPs) computed by the decoder:

$$\mu(k) = \sum_{\omega\in\Omega} \omega \cdot \mathrm{P}\left(z(k) = \omega\left|\mathbf{R}^{(\iota)}, \widehat{\vartheta(\epsilon)}\,(\iota)\right.\right) \tag{12}$$

$$\psi(k) = \sum_{\omega\in\Omega} |\omega|^2 \cdot \mathrm{P}\left(z(k) = \omega\left|\mathbf{R}^{(\iota)}, \widehat{\vartheta(\epsilon)}\,(\iota)\right.\right). \tag{13}$$

Using (11), it follows that the updated estimate for the CPE is given as

$$\widehat{\vartheta(\epsilon)}\,(\iota+1) = \arg\max_{\vartheta(\epsilon)} \frac{\partial Q\left(\vartheta(\epsilon)\left|\widehat{\vartheta(\epsilon)}\,(\iota)\right.\right)}{\partial\vartheta(\epsilon)} \tag{14}$$

After some mathematical manipulations, the closed form for the updated estimate of $\vartheta(\epsilon)$ is given by:

$$\widehat{\vartheta(\epsilon)}\,(\iota+1) = \frac{\sum_{k=0}^{N-1} R^{(\iota)}(k)H^*(k)g^*(k)}{\sum_{k=0}^{N-1}|H(k)|^2 q(k)} \tag{15}$$

where

$$g(k) = \begin{cases} p(k) & \text{if } \ k\in\Gamma \\ \mu(k) & \text{if } \ k\notin\Gamma \end{cases} \tag{16}$$

and

$$q(k) = \begin{cases} |p(k)|^2 & \text{if } \ k\in\Gamma \\ \psi(k) & \text{if } \ k\notin\Gamma \end{cases}\ . \tag{17}$$

In the following, (15) is denoted as the pilot assisted estimator, where the initial value of $\widehat{\vartheta(\epsilon)}$ (the first iteration) is given as

$$\widehat{\vartheta(\epsilon)}\,(1) = \frac{\sum_{k\in\Gamma} R^{(\iota)}(k)H^*(k)p^*(k)}{\sum_{k\in\Gamma}|H(k)|^2|p(k)|^2}. \tag{18}$$

In the absence of pilot symbols, i.e. if Γ is empty, we assume that the initial value of $\widehat{\vartheta(\epsilon)}$ equals one: in the first iteration, we decode the received data without having any information about the CFO. This is referred to as the blind estimator.

At iteration ι, the estimated CFO $\hat{\epsilon}(\iota)$ can be obtained using (5):

$$\hat{\epsilon}(\iota) = \frac{N}{\pi(N-1)}\arg\left(\widehat{\vartheta}(\epsilon)(\iota)\right). \tag{19}$$

Based on $\hat{\epsilon}(\iota)$ and the decoder output, the ICI term in (4) can be calculated and subtracted from the received signal to improve the estimation accuracy in the next iteration. However, in the first iteration, we assume $\mathbf{I}^{(1)}$ is zero as we do not have any information about the CFO value nor the transmitted information.

IV. Simulation Results

To validate the proposed algorithm, we have carried out Monte Carlo simulations. We consider an OFDM system, using a convolutional code with constraint length 5, rate 1/2 and polynomial generators $(23)_8$ and $(35)_8$. The BCJR algorithm is used for decoding. A block length of $N_b = 128$ information and pilot bits was chosen, leading to $N_c = 256$ coded bits. The coded bits are Gray-mapped on a 8-PSK constellation, resulting in $N_d + N_p = 64$ symbols. We take $N_p = 4$ for the pilot estimator while $N_p = 0$ for the blind estimator. The channel consists of $L = 4$ statistically independent taps, each being a zero-mean complex Gaussian random variable with an exponential power delay profile [18]:

$$E\left[|h\,(l)|^2\right] = E_h \exp\left(-l/5\right),\, l = 0, \ldots, L-1 \tag{20}$$

where E_h is chosen such that the average energy per sub-carrier is normalized to unity. To avoid ISI, a cyclic prefix of length $\nu = L-1$ is employed. We assume perfect channel knowledge to isolate the effect of the CFO error.

Figures 2 and 3 show the mean squared estimation error (MSE) of the CFO, $(\mathrm{MSE})_{\mathrm{CFO}} = E\left[(\epsilon-\hat{\epsilon})^2\right]$, and the bit error error rate (BER) performance of the proposed schemes as a function of E_b/N_o for a normalized CFO $\epsilon = 0.1$,

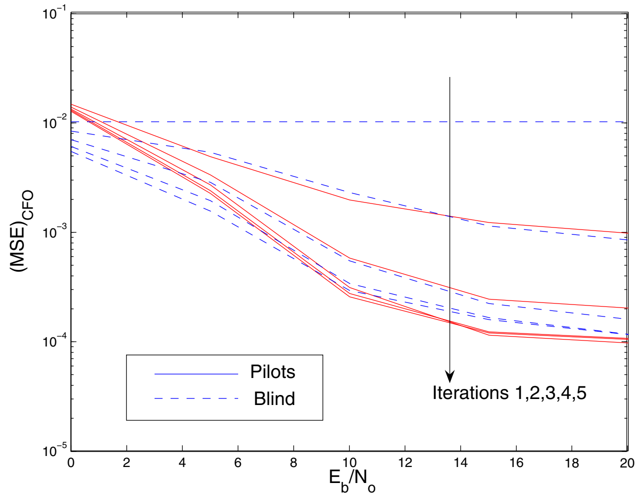

Fig. 2. Mean squared error (MSE) with normalized CFO, $\epsilon = 0.1$.

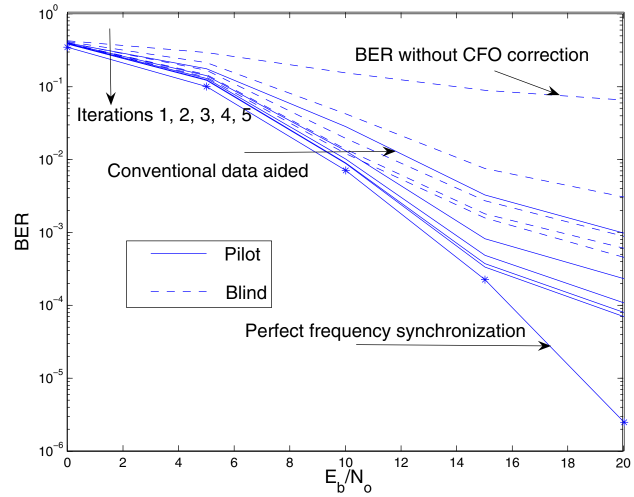

Fig. 3. BER with normalized CFO, $\epsilon = 0.1$.

respectively. Since we assume that at the first iteration the estimated CFO equals zero for the blind estimator, this curve illustrates the effect of the CFO on the OFDM system. Further, the performance of the pilot estimator at the first iteration represents the performance of the conventional data aided estimator. For all iterations, the pilot estimator outperforms the blind estimator. For both estimators, a strong improvement of the performance is achieved after only three EM iterations.

Figures 4 and 5 show the MSE and the BER performance of the proposed schemes as a function of the normalized CFO ϵ at E_b/N_o= 15 dB, respectively. We note that for small values of ϵ, the blind estimator is better than the pilot estimator because the initial estimate for the blind estimator (i.e. $\widehat{\epsilon} = 0$) is typically closer to the true CFO ϵ than the initial estimate for the pilot estimator, which is based on a few pilots. However, when ϵ increases, the pilot estimator outperforms the blind estimator: the initial estimate for the blind estimator is no longer close the true CFO. For larger ϵ, it follows from the figures that the MSE and BER increase: the ICI component is the dominating effect and disturbs the estimator. It can be observed from the figures that the increase in MSE and BER for the blind estimator starts at lower CFO than the pilot estimator: the pilot estimator is able to track larger CFO than the blind estimator.

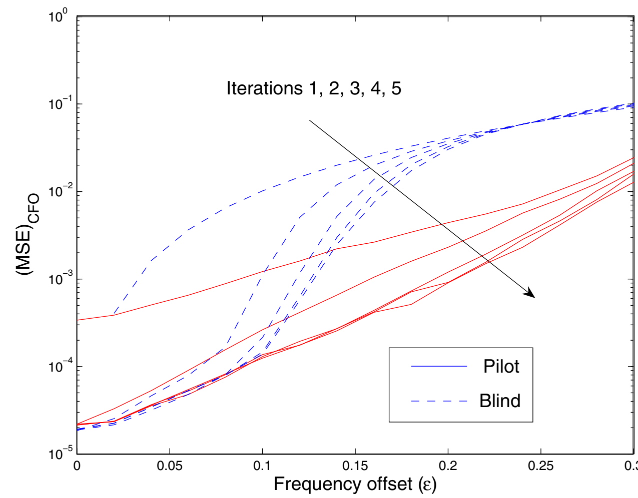

Fig. 4. Mean squared error (MSE), $E_b/N_0 = 15$ dB.

V. Conclusions

In this paper, we have proposed a novel soft code-aided fractional carrier frequency offset (CFO) estimation algorithm for OFDM systems. Based on the EM algorithm, the receiver iterates between data detection and estimation, with the exchange of soft information. The results indicate that the proposed algorithm achieves a significant improvement in the MSE and the BER performance after 3 iterations as compared to the case of no fractional CFO correction.

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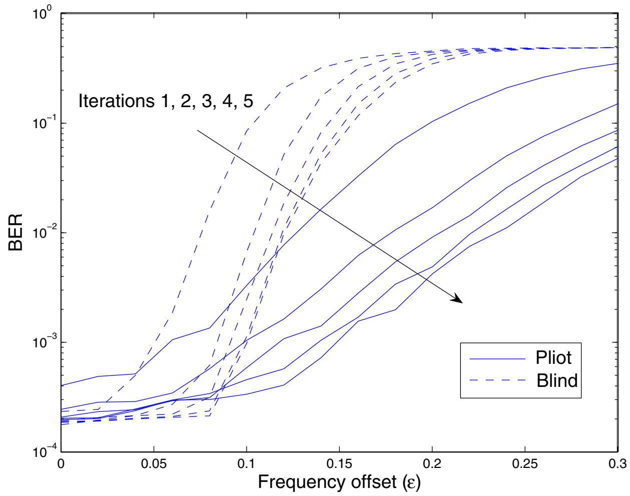

Fig. 5. BER, $E_b/N_0 = 15$ dB.

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