

# Iterative DA/DD Channel Estimation for KSP-OFDM

Dieter Van Welden, Heidi Steendam, Marc Moeneclaey  
DIGCOM research group, TELIN Dept., Ghent University  
Sint-Pietersnieuwstraat 41, 9000 GENT, BELGIUM

E-mail: {Dieter.Vanwelden,Heidi.Steendam,Marc.Moeneclaey}@telin.ugent.be

**Abstract**—In this paper, we propose an iterative joint DA/DD channel estimation algorithm for known symbol padding (KSP) OFDM. The pilot symbols used to estimate the channel are not only located in the guard interval, but also on some of the OFDM carriers. Initially, the channel is estimated using the pilot symbols only. Next, a decision is made with respect to the transmitted data symbols. We consider both hard and soft decision of the data symbols. The decisions on the data symbols are then used to update the channel estimate in a joint DA/DD estimation algorithm. The algorithm iterates between data detection and channel estimation until convergence is reached. At high SNR, the MSE performance of the iterative estimator converges to the MSE performance of the case where all data symbols are prior known at the receiver, i.e. the all pilot DA estimator. It turns out that the MSE performance of hard decision of the data symbols reaches the MSE of the all pilot DA estimator at lower SNR than that of soft decision of the data symbols; this is caused by some approximations needed to simplify the estimation algorithm.

## I. INTRODUCTION

In multicarrier (MC) transmission [1], the effect of channel dispersion is mitigated by inserting a guard interval between successively transmitted MC symbols. Several types of guard intervals are discussed in the literature. The most popular type of guard interval is the cyclic prefix (CP) [2]-[3]. In CP-OFDM, the guard interval consists of a cyclic extension of the transmitted MC block: the last samples of each MC block are copied and added as a prefix to the MC block. Another guard interval type is zero-padding (ZP) [2]-[3]. In ZP-OFDM, no signal is transmitted during the guard interval. In these two guard interval techniques data-aided channel estimation is typically obtained by replacing some of the data carriers by pilot carriers and estimating the channel in the frequency domain [2],[4]. A drawback of these two guard interval techniques, however, is the ambiguity problem in timing synchronization. In low complexity timing synchronization algorithms like Schmid&Cox [5], the correct borders of a MC block can only be determined with an ambiguity equal to the guard interval length.

In the guard interval technique that is considered in this paper, i.e. known symbol padding (KSP) [6]-[7], the guard interval consists of known samples. By properly selecting the known samples, the ambiguity problem in timing synchronization as occurs in CP-OFDM and ZP-OFDM can be avoided [8]. The known samples from the guard interval can also be used for channel estimation. However, the guard interval length is usually selected to be only slightly larger

than the channel impulse response length, which means that the number of pilots in the guard interval is typically too small to accurately estimate the channel. To solve this problem, additional pilots can be inserted on some of the carriers, by replacing data symbols by pilot symbols [9]-[10]. However, channel estimation in KSP-OFDM is harder than in CP-OFDM and ZP-OFDM, as optimal maximum likelihood (ML) channel estimation is very complex [10]. Therefore, suboptimal channel estimation techniques are developed.

In this paper, we propose an iterative joint DA/DD (data aided/decision directed) channel estimation algorithm for KSP-OFDM. In the first step, the channel is estimated by means of a DA algorithm. Using this channel estimate, a decision is made about the transmitted data symbols. In this paper, we will consider both hard and soft decision. The decisions about the data symbols are then used to update the channel estimate by means of a joint DA/DD channel estimator. The algorithm iterates between the data decisions and the channel estimation until convergence is reached. In the literature, different DA channel estimation techniques for KSP-OFDM can be found. In [10]-[11], two ML-based estimators are proposed. However, at high SNR, the mean squared error (MSE) performance of these estimators shows an error floor as the presence of the unknown data symbols disturbs the channel estimation. In [12], a frequency domain DA channel estimation technique is used. Although this estimator has a slightly worse MSE performance at low SNR than the estimators from [10]-[11], it does not suffer from an error floor at high SNR. Therefore, in this paper, we will use the frequency domain estimator from [12] as the DA estimator in the first step of our algorithm.

## II. SYSTEM DESCRIPTION

The KSP-OFDM system under consideration consists of  $N$  carriers, and a guard interval of  $\nu$  samples is included in each OFDM block. During each block  $M - \nu$  pilot symbols and  $N + \nu - M$  data symbols are transmitted on the carriers. We denote the  $M - \nu$  pilots transmitted on the carriers during block  $i$  as  $\mathbf{b}_c^{(i)} = (b_c^{(i)}(0), \dots, b_c^{(i)}(M - \nu - 1))^T$  and the  $N + \nu - M$  data symbols  $\mathbf{a}_d^{(i)} = \{a_d^{(i)}(0), \dots, a_d^{(i)}(N + \nu - M - 1)\}$ . The sets  $I_p$  and  $I_d$  are the sets of carriers modulated by pilots and data, respectively, where  $I_p \cup I_d = \{0, \dots, N - 1\}$ . The pilot and data symbols are mapped on a vector  $\mathbf{a}^{(i)}$  of length  $N$ . The data and pilot symbols are then modulated on the different

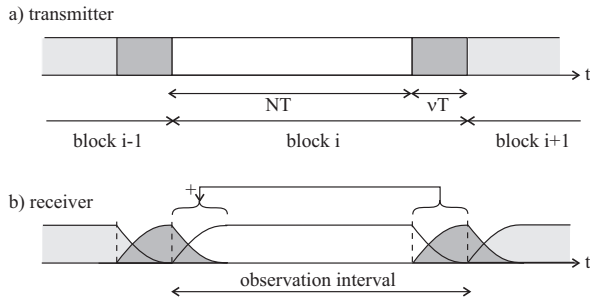


Fig. 1. Time-domain signal of KSP-OFDM a) transmitted signal b) received signal and observation interval.

carriers of the multicarrier system, by applying the vector  $\mathbf{a}^{(i)}$  to an  $N$ -point inverse fast Fourier transform (IFFT). We insert a guard interval of  $\nu$  known samples after each OFDM symbol to avoid interference between successive OFDM blocks, as shown in figure 1; the dark gray area in this figure corresponds to the guard interval. The time-domain samples during block  $i$  are then given by

$$\mathbf{s}^{(i)} = \sqrt{\frac{N}{N+\nu}} \begin{pmatrix} \mathbf{F}^+ \mathbf{a}^{(i)} \\ \mathbf{b}_g^{(i)} \end{pmatrix}. \quad (1)$$

The  $N \times N$  matrix  $\mathbf{F}$  in (1) corresponds to the FFT operation, with  $\mathbf{F}_{k,\ell} = \frac{1}{\sqrt{N}} e^{-j2\pi \frac{k\ell}{N}}$ ,  $\mathbf{F}^+$  is the Hermitian transpose of  $\mathbf{F}$ , and  $\mathbf{b}_g^{(i)} = (b_g^{(i)}(0), \dots, b_g^{(i)}(\nu-1))^T$  are the  $\nu$  known samples of the guard interval. We assume that the data symbols are independent identically distributed (i.i.d.) and have energy per symbol  $E[|a_d^{(i)}(n)|^2] = E_s$ . Further, we assume that  $E[|b_c^{(i)}(n)|^2] = E[|b_g^{(i)}(m)|^2] = E_s$ . The normalization factor  $\sqrt{N/(N+\nu)}$  in (1) implies that  $E[|s^{(i)}(m)|^2] = E_s$ .

The KSP-OFDM signal is transmitted over a dispersive channel with channel impulse response  $\mathbf{h} = (h(0), \dots, h(L-1))^T$ . We assume the guard interval length  $\nu$  is chosen longer than the length  $L$  of the channel impulse response, i.e.  $\nu \geq L-1$ , in order to avoid intersymbol interference between successively transmitted OFDM symbols. Further, the signal is disturbed by additive white Gaussian noise  $\mathbf{w}$ ; the components  $w(k)$  of  $\mathbf{w}$  are statistically independent and have zero mean and variance  $N_0$ . Without loss of generality, we restrict our attention to the detection of the OFDM block with index  $i = 0$ , and we drop the block index for notational simplicity. At the receiver, we consider the  $N + \nu$  time-domain samples corresponding to the observation interval shown in figure 1b:

$$\mathbf{r} = \mathbf{H}_{ch}\mathbf{s} + \mathbf{w} \quad (2)$$

where the  $(N + \nu) \times (N + \nu)$  channel matrix is given by  $(\mathbf{H}_{ch})_{k,k'} = h(k - k')$ .

### III. ITERATIVE CHANNEL ESTIMATION

The channel is estimated using the  $N + \nu$  time-domain samples from the observation interval shown in figure 1b. The observation vector (2) can be rewritten as

$$\mathbf{r} = \mathbf{B}\mathbf{h} + \mathbf{A}\mathbf{h} + \mathbf{w}. \quad (3)$$

In (3), the  $(N + \nu) \times L$  matrix  $\mathbf{A}$  contains the contributions from the data symbols  $\mathbf{a}_d$  transmitted during the observed OFDM block:

$$(\mathbf{A})_{k,\ell} = \sqrt{\frac{N}{N+\nu}} s_d(k-\ell) \quad (4)$$

where  $\mathbf{s}_d = \mathbf{F}_d \mathbf{a}_d$  and  $\mathbf{F}_d$  is a  $N \times (N + \nu - M)$  matrix that consists of the subset of columns of  $\mathbf{F}^+$  corresponding to the set  $I_d$  of data carriers, i.e.  $\mathbf{s}_d$  equals the  $N$ -point IFFT of the data carriers only. The matrix  $\mathbf{B}$  has dimension  $(N + \nu) \times L$  and contains the contributions from the pilot symbols from both the guard interval and the pilot carriers:  $\mathbf{B} = \mathbf{B}_g + \mathbf{B}_c$ . The contribution from the pilot carriers is contained in the matrix  $\mathbf{B}_c$  and is given by

$$(\mathbf{B}_c)_{k,\ell} = \sqrt{\frac{N}{N+\nu}} s_p(k-\ell). \quad (5)$$

The vector  $\mathbf{s}_p$  corresponds to the  $N$ -point IFFT of the pilot carriers only, i.e.  $\mathbf{s}_p = \mathbf{F}_p \mathbf{b}_c$ , where  $\mathbf{F}_p$  is a  $N \times (M - \nu)$  matrix consisting of a subset of columns of the IFFT matrix  $\mathbf{F}^+$  corresponding to the set  $I_p$  of pilot carriers. Note that  $s_p(k) = 0$  for  $k < 0$  or  $k \geq N$ . The matrix  $\mathbf{B}_g$  contains the contributions from the guard interval pilots and is given by

$$(\mathbf{B}_g)_{k,\ell} = \sqrt{\frac{N}{N+\nu}} b_g(|k-\ell+\nu|_{N+\nu}) \quad (6)$$

where  $|x|_K$  is the modulo- $K$  operation of  $x$  yielding a result in the interval  $[0, K]$ , and  $b_g(k) = 0$  for  $k \geq \nu$ .

#### A. Step 1: Data-Aided Channel Estimation

To initialize the iterative algorithm, the channel is first estimated using the pilot symbols only. The data-aided estimation algorithm that is used in this paper, is the frequency-domain estimator from [12]. In this algorithm, first the  $\nu$  samples from the guard interval are added to the first  $\nu$  samples of the data part of the OFDM block as shown in figure 1b, and then the first  $N$  samples of the OFDM block are applied to the  $N$ -point FFT, i.e. we convert the received samples to the frequency domain. Because of the orthogonality of the carriers, data carriers and pilot carriers will not interfere. Hence, if we use as observations the  $M - \nu$  pilot carriers only, ML estimation of the channel is simple.

The  $M - \nu$  observations corresponding to the pilot carrier positions can be written as

$$\mathbf{r}' = \mathbf{B}'\mathbf{h} + \mathbf{w}'. \quad (7)$$

The autocorrelation matrix  $\mathbf{R}'$  of the zero-mean Gaussian distributed noise component  $\mathbf{w}'$  equals

$$(\mathbf{R}')_{k,k'} = N_0 \left( \delta_{k,k'} + \frac{1}{N} \sum_{\ell=0}^{\nu-1} e^{-j2\pi \frac{(n_k - n_{k'})\ell}{N}} \right). \quad (8)$$

with  $n_k, n_{k'} \in I_p$ . The  $(M - \nu) \times L$  matrix  $\mathbf{B}'$  consists of the contributions from the pilot carriers and the guard interval

pilots:  $\mathbf{B}' = \mathbf{B}'_p + \mathbf{B}'_g$ . The matrix  $\mathbf{B}'_p$  corresponds to the contributions from the pilot carriers, i.e.

$$(\mathbf{B}'_p)_{k,\ell} = \sqrt{\frac{N}{N+\nu}} b_c(k) e^{-j2\pi \frac{n_k \ell}{N}} \quad (9)$$

with  $n_k \in I_p, \ell = 0, \dots, L-1$  and  $\mathbf{B}'_g$  to the contributions from the pilots in the guard interval, i.e.

$$\mathbf{B}'_g = \sqrt{\frac{N}{N+\nu}} \mathbf{F}_{\nu,p} \mathbf{B}_{g,\nu} \quad (10)$$

where  $(\mathbf{F}_{\nu,p})_{k,\ell} = \frac{1}{\sqrt{N}} e^{-j2\pi \frac{n_k \ell}{N}}$ ,  $n_k \in I_p, \ell = 0, \dots, \nu-1$  and  $(\mathbf{B}_{g,\nu})_{k,\ell} = b_g(|k-\ell|_\nu)$ . In [12], the ML channel estimate of  $\mathbf{h}$  based on the observation of the  $M-\nu$  FFT outputs that correspond to the pilot carrier positions is derived:

$$\hat{\mathbf{h}}_{DA} = (\mathbf{B}'^+ \mathbf{R}'^{-1} \mathbf{B}')^{-1} \mathbf{B}'^+ \mathbf{R}'^{-1} \mathbf{r}' \quad (11)$$

and its MSE yields

$$MSE_{DA} = \text{trace} \left( (\mathbf{B}'^+ \mathbf{R}'^{-1} \mathbf{B}')^{-1} \right). \quad (12)$$

Note that the estimate (11) is a minimum variance unbiased (MVU) estimate, i.e. the MSE (12) of this estimate coincides with the Cramer Rao bound (CRB) [13] assuming only the  $M-\nu$  pilot carrier positions are used for estimation.

### B. Step 2: Detection of the Data Symbols

To estimate the data symbols, we need to compute the posterior distribution  $p(\mathbf{a}_d | \mathbf{r}, \mathbf{h}, \mathbf{b})$  where  $\mathbf{b} = (\mathbf{b}_c, \mathbf{b}_g)$ . It can easily be verified that

$$p(\mathbf{a}_d | \mathbf{r}, \mathbf{h}, \mathbf{b}) \propto p(\mathbf{r} | \mathbf{a}_d, \mathbf{h}, \mathbf{b}) p(\mathbf{a}_d). \quad (13)$$

Taking into account (3), the observation  $\mathbf{r}$  given  $\mathbf{a}_d, \mathbf{h}$  and  $\mathbf{b}$  is Gaussian distributed with mean  $\mathbf{B}\mathbf{h} + \mathbf{A}\mathbf{h}$  and autocorrelation matrix  $N_0 \mathbf{I}_{N+\nu}$ :

$$p(\mathbf{r} | \mathbf{a}_d, \mathbf{h}, \mathbf{b}) \propto p(\mathbf{a}_d) e^{-\frac{1}{N_0} (\mathbf{r} - \mathbf{B}\mathbf{h} - \mathbf{A}\mathbf{h})^+ (\mathbf{r} - \mathbf{B}\mathbf{h} - \mathbf{A}\mathbf{h})}. \quad (14)$$

To obtain the distribution  $p(\mathbf{a}_d | \mathbf{r}, \mathbf{h}, \mathbf{b})$ , we first rewrite the contribution from the data symbols in (3):  $\mathbf{A}\mathbf{h} = \mathbf{H}_d \mathbf{a}_d$ , where  $\mathbf{H}_d = \mathbf{H} \mathbf{F}_d$  and the  $(N+\nu) \times N$  matrix  $\mathbf{H}$  has components  $\mathbf{H}_{k,k'} = \sqrt{N/(N+\nu)} h(k-k')$ . Substituting (14) in (13), and assuming all data sequences are equiprobable, the distribution  $p(\mathbf{a}_d | \mathbf{r}, \mathbf{h}, \mathbf{b})$  can be rewritten as

$$p(\mathbf{a}_d | \mathbf{r}, \mathbf{h}, \mathbf{b}) \propto e^{-(\mathbf{a}_d - \mathbf{m}_a)^+ \mathbf{Q}_a^{-1} (\mathbf{a}_d - \mathbf{m}_a)} \quad (15)$$

where

$$\mathbf{m}_a = (\mathbf{H}_d^+ \mathbf{H}_d)^{-1} \mathbf{H}_d^+ (\mathbf{r} - \mathbf{B}\mathbf{h}) \quad (16)$$

and

$$\mathbf{Q}_a = \frac{1}{N_0} \mathbf{H}_d^+ \mathbf{H}_d. \quad (17)$$

In the previous equations, it is assumed that the channel vector  $\mathbf{h}$  is known at the receiver. As the channel vector is not prior known at the receiver, we use the DA channel estimate (11) instead, i.e. we replace in all expressions  $\mathbf{h}$  by  $\hat{\mathbf{h}}_{DA}$ .

Hard decisions about the transmitted data symbols are obtained by optimizing the posterior distribution  $p(\mathbf{a}_d | \mathbf{r}, \hat{\mathbf{h}}_{DA}, \mathbf{b})$  over all possible sequences:

$$\hat{\mathbf{a}}_d = \arg \max_{\mathbf{a}_d} p(\mathbf{a}_d | \mathbf{r}, \hat{\mathbf{h}}_{DA}, \mathbf{b}). \quad (18)$$

Soft decisions about the data symbols are obtained by averaging the data symbols over all possible data sequences:

$$E[\mathbf{a}_d] = \sum_{\mathbf{a}_d} \mathbf{a}_d p(\mathbf{a}_d | \mathbf{r}, \hat{\mathbf{h}}_{DA}, \mathbf{b}). \quad (19)$$

As the number  $N+\nu-M$  of data symbols is large, the complexity of (18) and (19) is very high. Note however that for large  $N$ , the matrix  $\mathbf{Q}_a$  can be approximated by a diagonal matrix. This implies that the components of  $\mathbf{a}_d$  given  $\mathbf{r}, \hat{\mathbf{h}}_{DA}$  and  $\mathbf{b}$  can be considered as essentially statistically independent. In that case, the decisions about the data symbols can be done componentwise, i.e. for hard decision

$$\hat{a}_d(i) = \arg \max_{a_d(i)} p(a_d(i) | \mathbf{r}, \hat{\mathbf{h}}_{DA}, \mathbf{b}) \quad (20)$$

and for soft decision

$$E[a_d(i)] = \sum_{a_d(i)} a_d(i) p(a_d(i) | \mathbf{r}, \hat{\mathbf{h}}_{DA}, \mathbf{b}) \quad (21)$$

where

$$p(a_d(i) | \mathbf{r}, \mathbf{h}, \mathbf{b}) \propto e^{-\frac{1}{(\mathbf{Q}_a)_{i,i}} |a_d(i) - m_a(i)|^2}. \quad (22)$$

Note from (22) and (20) that the hard decision is the constellation point that is closest to  $m_a(i)$ .

### C. Step 3: Joint DA/DD Channel Estimation

Let us consider the all pilots case, i.e. all data symbols are prior known at the receiver. Hence, the matrix  $\mathbf{A}$  from (3) is known at the receiver. The ML estimate of  $\mathbf{h}$ , based on the observation  $\mathbf{r}$  (3) can easily found to be

$$\hat{\mathbf{h}}_{\text{all pilots}} = (\mathbf{C}^+ \mathbf{C})^{-1} \mathbf{C}^+ \mathbf{r} \quad (23)$$

and its MSE yields

$$MSE_{\text{all pilots}} = \text{trace} \left( (\mathbf{C}^+ \mathbf{C})^{-1} \right) \quad (24)$$

where  $\mathbf{C} = \mathbf{A} + \mathbf{B}$ . The estimate (23) is a MVU estimate, i.e. the MSE (24) corresponds to the CRB on the estimation of  $\mathbf{h}$  based on the observation of  $\mathbf{r}$  (3) in the all pilots case.

In the considered system, the data symbols are not prior known at the receiver. Instead, we use the hard or soft estimates on the data symbols. To use the estimates of the data symbols in a decision directed way, we replace all data symbols that are included in (23) by their hard (20) or soft (21) estimates to update the channel estimate. In the iterative joint DA/DD algorithm, steps 2 and 3 are repeated until convergence is reached.

#### IV. NUMERICAL RESULTS

In this section, the performance of the iterative joint DA/DD estimator is investigated. Without loss of generality, we consider the comb-type pilot arrangement [14]-[15] for the pilots transmitted on the carriers. We assume the channel has  $L = 8$  channel taps that are linearly decreasing:  $h(k) = h(0)(L - k)$ ,  $k = 0, \dots, L - 1$ , and  $h(0)$  is selected such that the channel impulse response is normalized:  $\sum_{\ell=0}^{L-1} |h(\ell)|^2 = 1$ . The pilot symbols are randomly generated and BPSK modulated. Simulations show that the MSE performance only slightly depends on the pilot carrier positions. Therefore, the MSE performance in this section is averaged out over a large number of randomly generated pilot carrier positions. Unless mentioned otherwise, all MSE results for the iterative joint DA/DD estimator are given after 10 iterations, i.e. after convergence of the algorithm.

In figure 2, the MSE of the iterative joint DA/DD estimator for hard and soft data decisions is shown as function of the  $E_s/N_0$  for different number of iterations. In addition, the MSE of the DA (12) and the all pilot estimator (24) are shown. It can be observed that the MSE of the DA estimator essentially coincides with the curve  $\frac{L}{M-\nu}SNR^{-1}$  and the MSE of the all pilots estimator with  $\frac{L}{N+\nu}SNR^{-1}$  where  $SNR = \frac{E_s}{N_0} \frac{N}{N+\nu}$ , indicating that the MSE of the DA and all pilots estimator are inversely proportional to the number of known symbols that are used for the estimation. At high  $E_s/N_0$ , the MSE of the iterative joint DA/DD estimator converges to the MSE of the all pilot estimator, whereas at low  $E_s/N_0$ , the decisions on the data symbols are not reliable and disturb the channel estimation; at low  $E_s/N_0$ , the unreliable data decisions may even increase the MSE as compared to the DA only case. For both hard and soft data decisions, the MSE converges after 2-4 iterations. It can be observed that the MSE for hard decisions converges at lower  $E_s/N_0$  to the all pilots curve than the MSE for soft decisions. This effect is caused by the approximations made in the algorithm, i.e. the componentwise data decisions and the fact that the algorithm proposed in section III-C is a suboptimal algorithm.

The effect of the number of pilots on the iterative joint DA/DD estimator is shown in figure 3. Evidently, the MSE corresponding to the all pilot case does not depend on  $M$ : according to figure 2, this MSE behaves like  $\frac{L}{N+\nu}SNR^{-1}$ , which is not a function of  $M$ . As expected, the MSE of the DA algorithm, which according to figure 2 behaves like  $\frac{L}{M-\nu}SNR^{-1}$ , approaches the MSE of the all pilots case when  $M$  gets close to  $N + \nu$  and  $\nu \ll N$ . Similarly, the MSE resulting from the iterative DA/DD algorithm also converges to the MSE of the all pilot case when  $M$  approaches  $N + \nu$ : when the number of data symbols is much smaller than the number of pilot symbols, the quality of the data symbol decisions has a negligible effect on the MSE. Finally, we observe that the MSE of the iterative joint DA/DD estimator is essentially independent of  $M$  for a large range of  $M$ : for sufficiently large SNR, the reliability of the data decisions is high and the data symbols can be considered as known, i.e. the proposed

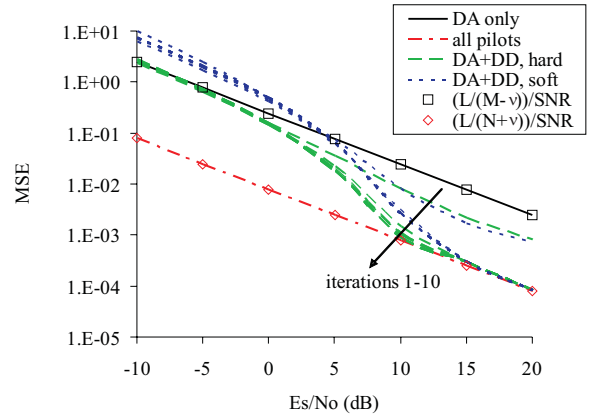


Fig. 2. MSE performance,  $\nu = 7$ ,  $N = 1024$ ,  $M = 40$ .

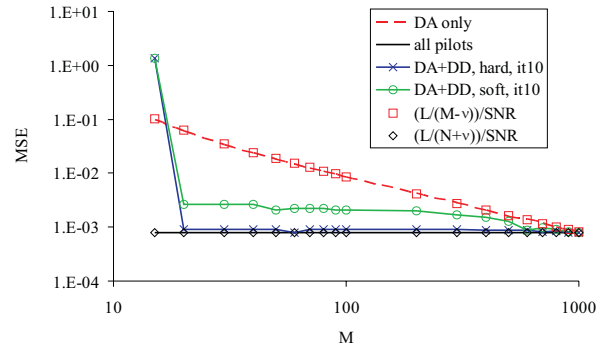


Fig. 3. Influence of  $M$  on the MSE,  $\nu = 7$ ,  $N = 1024$ ,  $E_s/N_0 = 10$  dB.

estimator converges to the all pilot estimator.

In figure 4, the influence of  $N$  on the MSE of the iterative joint DA/DD estimator is shown. At low  $E_s/N_0$ , the MSE of the proposed estimator is essentially independent of the FFT size  $N$ , whereas the MSE decreases with increasing  $N$  at high  $E_s/N_0$ . The dependency on  $N$  at high  $E_s/N_0$  is also shown in figure 5. It can be observed that for sufficiently large  $N$ , the MSE is inversely proportional to  $N$ . This can be explained by noting that at high  $E_s/N_0$ , the MSE of the proposed estimator converges to the all pilots estimator and the behavior of the MSE of this latter case: the MSE of the all pilots case behaves like  $\frac{L}{N+\nu}SNR^{-1}$  which can be approximated by  $\frac{L}{N}SNR^{-1}$  for  $N \gg \nu$ .

In figure 6, the effect of the guard interval length  $\nu$  on the performance of the iterative joint DA/DD estimator is shown. From the figure, it follows that the MSE of the proposed estimator is essentially independent of  $\nu$ . Indeed, at sufficiently high SNR, the MSE behaves like  $\frac{L}{N+\nu}SNR^{-1}$ , which, taking into account that  $\nu \ll N$ , is essentially independent of  $\nu$ .

#### V. CONCLUSIONS AND REMARKS

In this paper, we have proposed an iterative joint DA/DD algorithm for channel estimation in KSP-OFDM. The algorithm

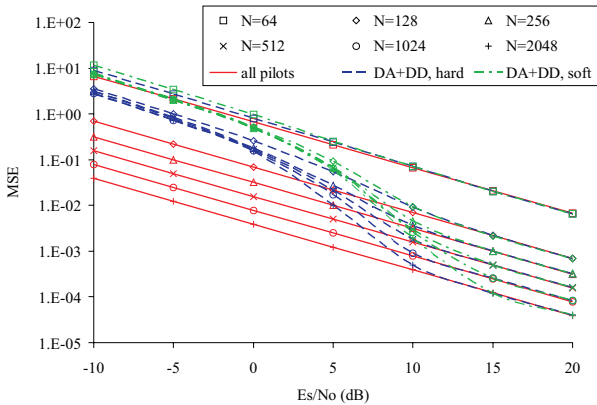


Fig. 4. Influence of  $N$  on the normalized MSE,  $\nu = 7$ ,  $M = 40$ .

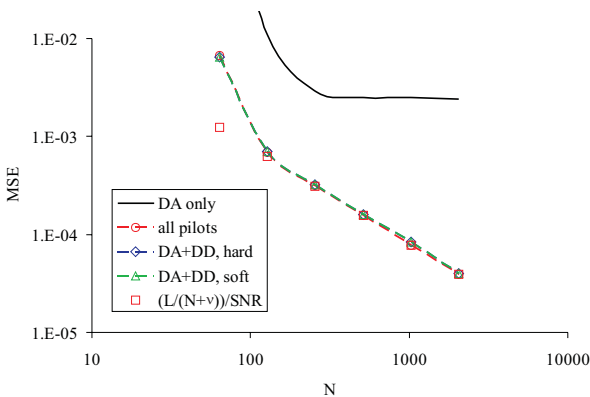


Fig. 5. Influence of  $N$  on the normalized MSE,  $\nu = 7$ ,  $M = 40$ ,  $E_s/N_0 = 20$  dB.

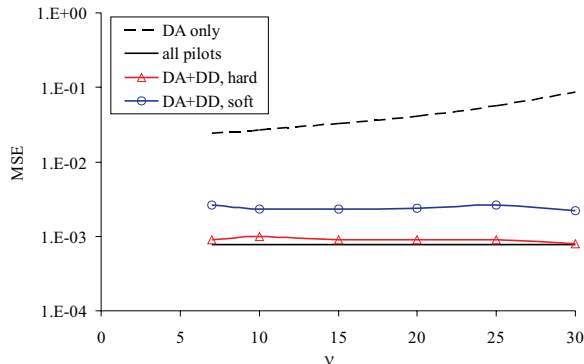


Fig. 6. Influence of  $\nu$  on the MSE,  $M = 40$ ,  $N = 1024$ ,  $E_s/N_0 = 10$  dB.

is initialized by means of the DA channel estimate from [12]. Using this DA channel estimate, we make a decision about the transmitted data symbols. In this paper, we consider both hard and soft decisions on the data symbols. The decisions on the data symbols are then used to update the channel estimate in a joint DA/DD way. The algorithm iterates between data decision and DA/DD channel estimation until convergence is reached. When the decisions on the data symbols are reliable, i.e. at high SNR, the MSE performance of the proposed estimator converges to the case of the all pilots estimator.

At high SNR, the MSE of the proposed estimator behaves like  $\frac{L}{N+\nu}SNR^{-1}$ , which for  $N \gg \nu$  is approximated by  $\frac{L}{N}SNR^{-1}$ . Hence, the MSE is essentially inversely proportional to the SNR and to the number  $N$  of carriers, and independent of the guard interval length  $\nu$ .

Note that in this paper, the data symbols were uncoded. However, the algorithm can easily be extended to coded transmission: in that case, the posterior probability  $p(\mathbf{a}_d|\mathbf{r}, \mathbf{h}, \mathbf{b})$  of the data symbols is then delivered by the (turbo) decoder.

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