Iterative EM Based Channel Estimation for KSP-OFDM

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***Abstract*—This paper proposes a new iterative channel estimation algorithm for known symbol padding (KSP) Orthogonal Frequency Division Multiplexing (OFDM) based on the Expectation Maximization (EM) algorithm . The guard interval is filled with pilot symbols and the remaining part of the pilot symbols is put on some of the OFDM carriers. To start up the EM algorithm an initial channel estimate is obtained by using only the pilot symbols. Then the EM algorithm is applied until convergence is reached. The performance of this estimator is compared with the performance of the iterative joint data-aided (DA) / decision-directed (DD) estimator proposed in [1]. The EM algorithm converges to the performance of the all pilot estimator for lower SNR than the iterative joint DA/DD estimator, but the gain in performance results in a higher computational complexity.**

I. Introduction

OFDM is a promising technique to cope with frequency selective channels [2]. The different OFDM blocks are separated by a guard interval to avoid intersymbol interference caused by the channel. There exist different types guard intervals in literature. Among the most popular guard interval techniques we find the cyclic prefix (CP) and zero padding (ZP) [3]. In CP-OFDM the last samples of each OFDM block are put in front of the OFDM block, while in ZP-OFDM the guard interval consists of zeros, i.e. no signal is transmitted during the guard interval.

In this contribution we consider another type of guard interval called known symbol padding (KSP) [4], where the guard interval consists of pilot symbols. This guard interval technique can be useful to solve the ambiguity in timing synchronization which occurs with other guard interval techniques [5]. Usually the length of the guard interval is not much larger than the duration of the channel impulse response, so the number of pilot symbols in the guard interval is not sufficient to perform channel estimation. Extra pilot symbols are inserted on some carriers of the OFDM system in order to increase the accuracy of the channel estimate. In the literature several estimators have been proposed. In [6], [7] suboptimal ML-based estimation methods are proposed. For both methods, the mean squared error (MSE) exhibits an error floor at high signal to noise ratios (SNR). This error floor is caused by the presence of the unknown data symbols. In [8], a data aided (DA) channel estimator is proposed that operates in the frequency domain. Only the pilot carriers are used to estimate the channel. Because of the orthogonality of the carriers, the pilot carriers can easily be separated from the data carriers. At low SNR the performance is slightly worse than the estimators from [6], [9], but at high SNR there exists no error floor. In [1], an iterative joint DA/DD estimator is proposed based on the frequency domain estimator from [8]. At high SNR the MSE performance of this estimator converges to the case of the all pilots estimator.

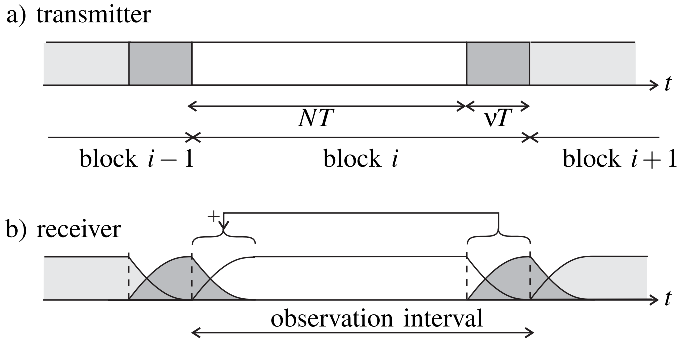

Figure 1. Time-domain signal of KSP-OFDM a) transmitted signal b) received signal and observation interval

Another way to deal with the unknown data symbols is by applying the Expectation Maximization (EM) algorithm [10], which is an iterative algorithm that converges to the maximum likelihood (ML) estimate. In [11], EM based channel estimation algorithms operating in the frequency domain, are proposed for CP-OFDM. In this paper, we propose an iterative channel estimator for KSP-OFDM based on the EM algorithm, operating in the time domain . In the first step, an initial estimate of the channel is obtained by means of the data-aided estimator proposed in [8]. The performance of the proposed algorithm is compared with the iterative estimator from [1] and with the all pilots estimator.

II. System Model

Let us consider a KSP-OFDM system with N carriers and a guard interval of length ν. The guard interval is filled with ν known pilot symbols denoted as $\mathbf{b}_g = (b_g(0), \ldots, b_g(\nu-1))^T$. The i-th block of transmitted symbols is denoted $\mathbf{a}_i = (a_i(0), \ldots, a_i(N-1))^T$ and consists of $M-\nu$ pilot symbols and $N-M+\nu$ data symbols, denoted as $\mathbf{b}_c^{(i)} = \left(b_c^{(i)}(0), \ldots, b_c^{(i)}(M-\nu-1)\right)^T$ and $\mathbf{a}_d^{(i)} =$

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$\left(a_d^{(i)}(0),\ \ldots,\ a_d^{(i)}(N-M+\nu-1)\right)^T$ respectively. We define M as the total number of pilot symbols transmitted in the guard interval and on the carriers. The symbols $\mathbf{a}_i$ are modulated on the different carriers using the N-point inverse FFT and the guard interval is inserted. The transmitted time domain samples $\mathbf{s}_i$ are given by

$$\mathbf{s}_i = \sqrt{\frac{N}{N+\nu}} \begin{pmatrix} \mathbf{F}^H \mathbf{a}_i \\ \mathbf{b}_g \end{pmatrix} \tag{1}$$

where $\mathbf{F}$ is the $N \times N$ FFT matrix with $\mathbf{F}_{k,l} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{kl}{N}};\ k,l = 0,\ \ldots,\ N-1$. The time-domain signal is shown in figure 1.

The sequence $\mathbf{s}_i$ is transmitted over a block fading frequency selective channel with an impulse response consisting of L taps $\mathbf{h} = (h(0),\ \ldots,\ h(L-1))^T$. The length of the guard interval ν is chosen so that the duration of the guard interval exceeds the duration of the channel impulse response in order to avoid interblock interference. The transmitted sequence is disturbed by additive white Gaussian noise (AWGN) $\mathbf{w}$. The noise components $w(k)$ are zero-mean and have variance N_0. Without loss of generality we focus on the detection of the OFDM block with index $i = 0$ and we drop the block index to simplify the notation. The $N+\nu$ received time-domain samples corresponding to the considered transmitted OFDM block can be written as

$$\mathbf{r} = \mathbf{H}_{ch}\mathbf{s} + \mathbf{w} \tag{2}$$

where $\mathbf{H}_{ch}$ is the $(N+\nu) \times (N+\nu)$ channel matrix with $(\mathbf{H}_{ch})_{k,k'}$ given by

$$(\mathbf{H}_{ch})_{k,k'} = h\left(\left|k-k'\right|_{N+\nu}\right) \tag{3}$$

where $|x|_K$ is the modulo-K operation of x yielding a result in the interval $[0,\ K[$. For data detection, the contribution from the ν pilot samples is subtracted from the received signal and the last ν samples from the observation interval are added to the first ν samples of the OFDM symbol. The resulting block of N samples is applied to the FFT. However as the pilot samples are affected by the unknown channel and their contribution to the received signal is unknown, the channel impulse response needs to be estimated.

III. Channel Estimation

For channel estimation, we rewrite the observation model (2) as

$$\mathbf{r} = \mathbf{B}\mathbf{h} + \mathbf{A}\mathbf{h} + \mathbf{w} \tag{4}$$

where $\mathbf{B}$ is a $(N+\nu) \times L$ matrix which depends on the pilot symbols. The matrix $\mathbf{B}$ can be written as a sum of a matrix $\mathbf{B}_g$ which depends on the pilot symbols from the guard interval and a matrix $\mathbf{B}_c$ which depends on the pilots symbols from the pilot carriers: $\mathbf{B} = \mathbf{B}_g + \mathbf{B}_c$. The matrix $\mathbf{B}_g$ is defined as

$$(\mathbf{B}_g)_{k,l} = \sqrt{\frac{N}{N+\nu}} b_g\left(|k-l+\nu|_{N+\nu}\right) \tag{5}$$

where $b_g(k) = 0$ for $k \geq \nu$. The contribution from the pilot carriers $\mathbf{B}_c$ is given by

$$(\mathbf{B}_c)_{k,l} = \sqrt{\frac{N}{N+\nu}} s_p(k-l) \tag{6}$$

where $\mathbf{s}_p$ is the N-point IFFT of the pilot carriers only: $\mathbf{s}_p = \mathbf{F}_p \mathbf{b}_c$, $\mathbf{F}_p$ consists of the $M-\nu$ columns of $\mathbf{F}^H$ which correspond to the pilot carriers. Note that $s_p(k) = 0$ for $k < 0$ or $k \geq N$. In (4), the contributions from the unknown data symbols $\mathbf{a}_d$ are collected in the $(N+\nu) \times L$ matrix $\mathbf{A}$:

$$(\mathbf{A})_{k,l} = \sqrt{\frac{N}{N+\nu}} s_d(k-l). \tag{7}$$

where $\mathbf{s}_d = \mathbf{F}_d \mathbf{a}_d$ and $\mathbf{F}_d$ consists of the $N+\nu-M$ columns of $\mathbf{F}^H$ which correspond to the data carriers. We introduce the matrix $\mathbf{C}$ to write (4) in a more compact form

$$\mathbf{r} = \mathbf{C}\mathbf{h} + \mathbf{w}$$

where $\mathbf{C} = \mathbf{B} + \mathbf{A}$.

A. EM Estimation

The EM algorithm is an iterative method to obtain an ML estimate of a parameter vector θ based on an observation $\mathbf{r}$ [10], where $\mathbf{r}$ depends on unobserved data $\mathbf{y}$. Each iteration consists of an expectation (E) step, and a maximization (M) step. In the E-step the log likelihood $\log p(\mathbf{r}|\mathbf{y},\theta)$ is averaged over the unobserved data, given the observation $\mathbf{r}$ and the last estimate of θ

$$Q\left(\theta\left|\hat{\theta}_{k-1}\right.\right) = \int \log p(\mathbf{r}|\mathbf{y},\theta)\, p\left(\mathbf{y}\left|\mathbf{r},\hat{\theta}_{k-1}\right.\right) d\mathbf{y} \tag{8}$$

where k is the iteration index and $\hat{\theta}_{k-1}$denotes the estimate of θ obtained in the previous iteration. The M-step comprises the maximization of (8) with respect to θ:

$$\hat{\theta}_{k+1} = \arg\max_{\theta} Q\left(\theta\left|\hat{\theta}_k\right.\right). \tag{9}$$

The EM algorithm starts from an initial estimate denoted $\hat{\theta}_0$.

B. Step 1: Data-Aided Channel Estimation

An initial estimate is needed to start the iterative EM algorithm. In this paper, we consider the data-aided estimation method described in [8]. This estimator adds first the ν samples of the guard interval to the first ν samples of the OFDM block. The resulting N samples are transformed to the frequency domain by applying an N-point FFT.

$$\mathbf{r}' = \begin{pmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_\nu \end{pmatrix} \begin{pmatrix} \mathbf{I}_N & \begin{matrix} \mathbf{I}_\nu \\ \mathbf{0} \end{matrix} \\ \mathbf{0} & \mathbf{I}_\nu \end{pmatrix} \mathbf{r}$$

where $\mathbf{I}_K$ is the $K \times K$ identity matrix. The data carriers can be separated from the pilot carriers because the carriers are orthogonal. ML estimation of the channel can be performed by using the observations from the $M-\nu$ pilot carriers only. The $M-\nu$ observations from the pilot carriers are collected in the vector $\mathbf{r}_2$ which can be written as

$$\mathbf{r}_2 = \mathbf{B}_2\mathbf{h} + \mathbf{w}_2$$

where $\mathbf{w}_2$ is Gaussian noise with zero mean and an autocorrelationmatrix $\mathbf{R}_2$ defined in [8] and $\mathbf{B}_2$ contains the rows from $\mathbf{FB}$ corresponding to the pilot carriers. The ML estimate of $\mathbf{h}$ based on the observation $\mathbf{r}_2$ is given by [8]

$$\hat{\mathbf{h}}_{ML} = \left(\mathbf{B}_2^H \mathbf{R}_2^{-1} \mathbf{B}_2\right)^{-1} \mathbf{B}_2^H \mathbf{R}_2^{-1} \mathbf{r}_2. \quad (10)$$

C. Step 2: Decision-Directed Channel Estimation

The obtained channel estimate (10) can be used to start up the EM algorithm. In our case, the E-step (8) can be rewritten as

$$\mathrm{E}_{\mathbf{a_d}}\left[\log p\left(\mathbf{r}\left|\mathbf{a}_d, \mathbf{b}_c, \mathbf{b}_g, \mathbf{h}\right.\right)\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right] = -\frac{1}{N_0}\left(\mathbf{r}^H\mathbf{r} - \mathbf{r}^H\tilde{\mathbf{C}}\mathbf{h} - \mathbf{h}^H\tilde{\mathbf{C}}^H\mathbf{r} + \mathbf{h}^H\tilde{\mathbf{R}}_{\mathbf{C}}\mathbf{h}\right) \quad (11)$$

where

$$\tilde{\mathbf{C}} = \mathrm{E}_{\mathbf{a_d}}\left[\mathbf{C}\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right] \quad (12)$$

$$\tilde{\mathbf{R}}_{\mathbf{C}} = \mathrm{E}_{\mathbf{a_d}}\left[\mathbf{C}^H\mathbf{C}\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right]. \quad (13)$$

See the appendix for the computation of $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{R}}_{\mathbf{C}}$. In (11) $\mathrm{E}_{\mathbf{a}_d}[.]$ is the average over the data symbols.

The new estimate $\hat{\mathbf{h}}_k$ obtained in the M-step can be written as a closed form expression

$$\hat{\mathbf{h}}_k = \left(\tilde{\mathbf{R}}_{\mathbf{C}}\right)^{-1}\tilde{\mathbf{C}}\mathbf{r}.$$

The algorithm terminates once the estimate has reached convergence.

IV. Simulation Results

In this section some simulation results are shown to illustrate the performance of the proposed channel estimator. For the pilot symbols transmitted on the carriers, the comb-type pilot arrangement [12], [13] can be assumed without loss of generality. We consider a channel with $L = 8$ taps and an impulse response given by $h(l) = h(0)(L - l)$, for $l = 0, \ldots, L - 1$, with $\sum_{l=0}^{L-1} |h(l)|^2 = 1$. A random BPSK sequence is used as pilot sequence. The shown MSE results are the results obtained after convergence of the algorithm unless mentioned otherwise.

Figure 2 shows the MSE of the EM algorithm as a function of E_s/N_0 for different numbers of iterations. Also shown are the MSE of the DA estimator and the all pilot estimator. As mentioned in [1] the MSE of the DA estimator coincides with $\frac{L}{M-\nu}SNR^{-1}$ and the MSE of the all pilot estimator with $\frac{L}{N+\nu}SNR^{-1}$ where $SNR = \frac{E_s}{N_0}\frac{N}{N+\nu}$. The MSE of the EM algorithm converges to the MSE of the all pilot estimator for high E_s/N_0 while for low E_s/N_0 this is not the case, but still there is an improvement in performance if we compare with the MSE of the DA estimator. The MSE results for the iterative joint DA/DD estimator from [1] are also shown. For low E_s/N_0 it can be observed that the EM algorithm results in a lower MSE. The EM algorithm reaches the performance of the all pilot estimator at lower E_s/N_0 than the iterative joint DA/DD estimator from [1].

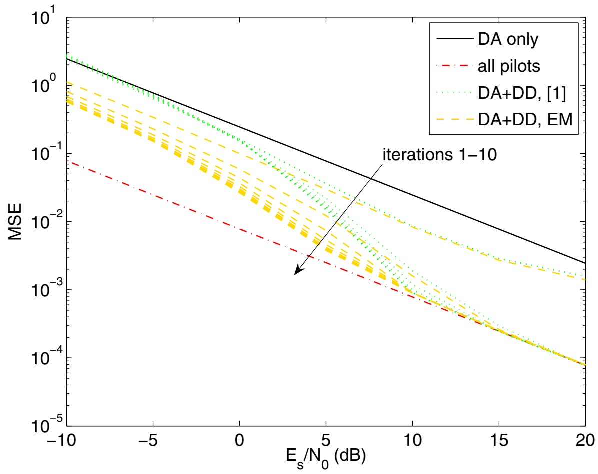

Figure 2. MSE results, $\nu = 7$, $N = 1024$, $M = 40$

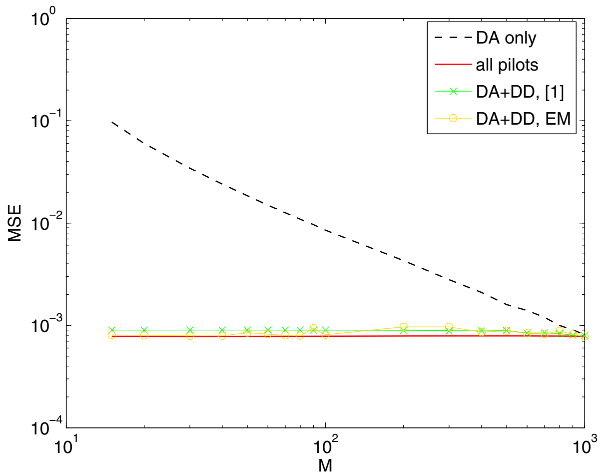

Figure 3. Influence of M on the MSE, $\nu = 7$, $N = 1024$, $E_s/N_0 = 10$ dB

Figure 3 shows the influence of the number of pilot symbols M on the performance of the EM algorithm. Again, the performances of the DA estimator, the all pilot estimator and the iterative joint DA/DD estimator from [1] are added to the figure. As mentioned above, the MSE of the all pilot estimator coincides with $\frac{L}{N+\nu}SNR^{-1}$ and does not depend on the number of pilot symbols M. We observe that the MSE of the EM algorithm is very close to the MSE of the all pilot estimator and is almost independent of M for a large range of M. The EM algorithm gives a small performance gain compared with the iterative joint DA/DD estimator from [1].

Figure 4 shows the BER performance when using the EM algorithm, the iterative joint DA/DD estimator from [1] and the DA channel estimator [8] respectively. The transmitted data symbols consist of BPSK symbols. The performance of a receiver with perfect channel knowledge is also added. The BER of a receiver with perfect channel knowledge was

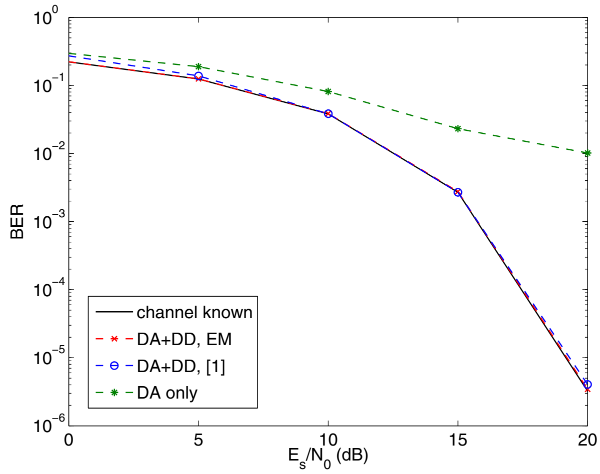

Figure 4. Influence of channel estimation errors on the BER, BPSK, $N = 1024$, $\nu = 7$, $M = 40$

computed analytically and is given by

$$BER = \sum_{n=0}^{N-1} \frac{1}{N} Q\left(\sqrt{2\left(\frac{N}{N+\nu}\right)^2 |H(n)|^2 \frac{E_s}{N_0}}\right)$$

with $H(n) = \sum_{l=0}^{L-1} h(l) e^{-j2\pi\frac{nl}{N}}$. The BER curves corresponding to the EM algorithm and the iterative joint DA/DD estimator [1] are close to the BER of the receiver with perfect channel knowledge for the considered range of E_s/N_0. Both iterative channel estimation algorithms result in a significantly lower BER compared to the DA channel estimator [8]. Only for high E_s/N_0, the EM algorithm performs slightly better than the iterative joint DA/DD estimator [1].

Finally we compare the EM algorithm and the iterative joint DA/DD estimator from [1] in terms of computational complexity. The initial estimate is the same for both estimation methods. Every iteration, the EM algorithm has to compute (15) and (16) while the iterative joint DA/DD estimator only has to compute an estimate of the matrix $\mathbf{C}$ by making hard decisions on the unknown data symbols $\mathbf{a}_d$. In other words the iterative joint DA/DD estimator has to compute only the first order statistics of the unknown data symbols $\mathbf{a}_d$ while the EM algorithm needs to estimate both the first and the second order statistics. This results in a higher computational complexity for the EM algorithm.

V. Conclusions

In this paper we have proposed an iterative channel estimation algorithm for KSP-OFDM based on the EM algorithm. The initial estimate needed to start up the EM algorithm is obtained by means of the DA channel estimator from [8]. Then the EM algorithm is applied. Every step the received signal is averaged over the unknown data symbols by using the channel estimate from the previous step and a new estimate of the channel is obtained. This process is repeated until the EM algorithm has reached convergence. At high SNR, the MSE of the proposed algorithm coincides with the MSE of the all pilot estimator. Simulation results also show that the performance of algorithm is almost independent of the number of pilot symbols for sufficiently high SNR. The EM algorithm outperforms the iterative joint DA/DD estimator from [1] in terms of MSE but has a higher computational complexity.

Appendix
Computation of the E-step

We have to average the log likelihood $\log p(\mathbf{r}|\mathbf{a}_d, \mathbf{b}_c, \mathbf{b}_g, \mathbf{h})$ over the unknown data vector $\mathbf{a}_d$ given the observation $\mathbf{r}$, the pilot vectors $\mathbf{b}_c$ and $\mathbf{b}_g$ and last obtained estimate of the channel vector $\hat{\mathbf{h}}_{k-1}$. The vector of received samples $\mathbf{r}$ (4) given the channel vector $\mathbf{h}$, the pilot symbol vectors $\mathbf{b}_c$ and $\mathbf{b}_g$, and the data symbols $\mathbf{a}_d$, has a Gaussian distribution with mean $\mathbf{Ch}$ and autocorrelation matrix $N_0\mathbf{I}_{N+\nu}$ so the log likelihood $\log p(\mathbf{r}|\mathbf{a}_d, \mathbf{b}_c, \mathbf{b}_g, \mathbf{h})$ is given by

$$\log p(\mathbf{r}|\mathbf{a}_d, \mathbf{b}_c, \mathbf{b}_g, \mathbf{h}) = -\frac{1}{N_0}(\mathbf{r}-\mathbf{Ch})^H(\mathbf{r}-\mathbf{Ch}). \quad (14)$$

The averaging of (14) over the unknown data vector requires the computation of the expected value of $\mathbf{C}$

$$\begin{aligned} \tilde{\mathbf{C}} &= \mathrm{E}_{\mathbf{a_d}}\left[\mathbf{C}\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right] \\ &= \mathbf{B} + \mathrm{E}_{\mathbf{a_d}}\left[\mathbf{A}\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right] \end{aligned} \quad (15)$$

and of $\mathbf{C}^H\mathbf{C}$

$$\begin{aligned} \tilde{\mathbf{R}}_{\mathbf{C}} &= \mathrm{E}_{\mathbf{a_d}}\left[\mathbf{C}^H\mathbf{C}\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right] \quad (16) \\ &= \mathbf{B}^H\mathbf{B} + \mathrm{E}_{\mathbf{a_d}}\left[\mathbf{B}^H\mathbf{A} + \mathbf{A}^H\mathbf{B} + \mathbf{A}^H\mathbf{A}\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right]. \end{aligned}$$

The posterior distribution of the data symbols $\mathbf{a}_d$ given the observation $\mathbf{r}$, the pilot vectors $\mathbf{b}_c$ and $\mathbf{b}_g$ and the last obtained estimate of the channel vector $\hat{\mathbf{h}}_{k-1}$, is given by

$$p\left(\mathbf{a}_d\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right) \sim p\left(\mathbf{r}\left|\mathbf{a}_d, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right) p(\mathbf{a}_d). \quad (17)$$

We assume that all data sequences are equiprobable. Using (14) where we substitute $\mathbf{h}$ by its estimate $\hat{\mathbf{h}}_{k-1}$, the posterior distribution of the data symbols (17) can be rewritten as

$$p\left(\mathbf{a}_d\left|\mathbf{r}, \mathbf{b}_c, \mathbf{b}_g, \hat{\mathbf{h}}_{k-1}\right.\right) \sim e^{(\mathbf{a}_d-\mathbf{m}_a)^H\mathbf{R}_a^{-1}(\mathbf{a}_d-\mathbf{m}_a)} \quad (18)$$

where

$$\begin{aligned} \mathbf{R_a} &= \frac{N}{N+\nu} N_0\left(\mathbf{F}_d^H\hat{\mathbf{H}}_{k-1}^H\hat{\mathbf{H}}_{k-1}\mathbf{F}_d\right) \quad (19) \\ \mathbf{m_a} &= \left(\mathbf{R_a}^H\mathbf{R_a}\right)^{-1}\mathbf{R_a}^H\left(\mathbf{r}-\mathbf{B}\hat{\mathbf{h}}_{k-1}\right). \quad (20) \end{aligned}$$

The $(N+\nu)\times N$ matrix $\hat{\mathbf{H}}_{k-1}$ mentioned in (19), is defined as

$$\left(\hat{\mathbf{H}}_{k-1}\right)_{l,m} = \hat{h}_{k-1}(l-m). \quad (21)$$

Note that $\hat{h}_{k-1}(l) = 0$ for $l < 0$ or $l \geq L$. For large N, the matrix $\mathbf{R_a}$ can be approximated by a diagonal matrix. This means that the data symbols $a_d(m)$, $m = 0, \ldots, N+\nu-M-1$, given $\mathbf{r}$, $\hat{\mathbf{h}}_{k-1}$, $\mathbf{b}_c$ and $\mathbf{b}_g$ can be considered as statistically independent.

To obtain $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{R}}_{\mathbf{C}}$ we need to compute

$$\mathrm{E}\left[a_d(m)\left|\mathbf{r},\mathbf{b}_c,\mathbf{b}_g,\hat{\mathbf{h}}_{k-1}\right.\right] = \sum_{a_d(m)} a_d(m)\, p\left(a_d(m)\left|\mathbf{r},\mathbf{b}_c,\mathbf{b}_g,\hat{\mathbf{h}}_{k-1}\right.\right) \quad (22)$$

and

$$\mathrm{E}\left[a_d(m)\,a_d^*(m')\left|\mathbf{r},\mathbf{b}_c,\mathbf{b}_g,\hat{\mathbf{h}}_{k-1}\right.\right] = \begin{cases} \sum_{a_d(m)} |a_d(m)|^2\, p\left(a_d(m)\left|\mathbf{r},\mathbf{b}_c,\mathbf{b}_g,\hat{\mathbf{h}}_{k-1}\right.\right) & m=m' \\ \mathrm{E}\left[a_d(m)\left|\mathbf{r},\mathbf{b}_c,\mathbf{b}_g,\hat{\mathbf{h}}_{k-1}\right.\right]\mathrm{E}\left[a_d(m')\left|\mathbf{r},\mathbf{b}_c,\mathbf{b}_g,\hat{\mathbf{h}}_{k-1}\right.\right] & m\neq m' \end{cases} \quad (23)$$

for $m,m'=0,\ldots,N+\nu-M-1$. Finally, substituting $a_d(m)$ and $a_d(m)\,a_d^*(m')$ by their respective expected values (22) and (23) in $\mathbf{C}$ and $\mathbf{C}^H\mathbf{C}$ yields $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{R}}_{\mathbf{C}}$.

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