Clipping Versus Symbol Switching for PAPR Reduction in Coded OFDM

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***Abstract*— In multicarrier systems, the transmitted time-domain signal exhibits large amplitude peaks. This peak-to-average power ratio (PAPR) problem complicates the practical use of multicarrier systems: the amplifier used to transmit the signal saturates because of the large peaks and causes non-linear distortion. As this non-linear distortion frustrates severely the detection of the multicarrier signal, the average power of the multicarrier signal must be reduced such that the system operates in the linear part of the amplifier. However, this power reduction comes at the cost of a reduced capacity of the multicarrier system. Hence, several techniques were investigated to reduce the PAPR. In this paper, we compare two PAPR reduction techniques for coded OFDM using an iterative decoder, i.e. clipping and symbol switching. Clipping outperforms the symbol switching technique as for given PAPR reduction, a lower BER degradation is obtained. However, the clipping technique causes out-of-band radiation whereas the spectrum is not changed when using the symbol switching technique.**

I. Introduction

The last decade has witnessed an immense increase of wireless communications services, to keep pace with the ever increasing demand for higher data rates combined with higher mobility. To satisfy this demand for higher data rates, the throughput over the existing transmission media had to be increased. One of the techniques that was investigated in this context is the multicarrier transmission technique [1]. In multicarrier transmission, the data sequence to be transmitted is split into a number of lower rate data streams, each of which is modulated on a different carrier. Because the time-domain multicarrier signal consists of the sum of the contributions of the different carriers, the amplitude of the time-domain signal can exhibit large peaks. This peak-to-average power ratio (PAPR) problem hampers the proper action of the multicarrier system: if no action is taken, the amplifier used to transmit the multicarrier signal will saturate because of the large peaks in the signal, and will cause non-linear distortion. To avoid the non-linear distortion, which disturbs the detection of the multicarrier signal, one can reduce the transmit power of the multicarrier signal, such that the amplifier can work in its linear area. However, by reducing the transmit power, the capacity of the multicarrier system is reduced. Hence, the research has focused on techniques to reduce the PAPR.

In the literature, several techniques to reduce the PAPR can be found [2]-[3]. Among all techniques available, clipping is the technique with the lowest complexity [4]-[7]. In this technique, the amplitude of the time-domain signal is cut off when it exceeds a predetermined threshold. The clipping can be performed on the in-phase and quadrature component separately, causing the phase content of the signal to be changed, or on the modulus in order to maintain the phase content of the signal; the latter results in better performance results than the former. Clipping causes non-linear distortion of the multicarrier signal, resulting in out-of-band radiation. Hence, by clipping the signal, the spectral efficiency of the multicarrier signal is reduced. To avoid this out-of-band radiation problem, the clipped multicarrier signal is filtered. However, filtering then again causes a peak regrowth. Therefore, the clipping-filtering operation is repeated several times to reach the desired amplitude level and to limit the out-of-band radiation. The difficulty to reconstruct the signal at the receiver limits the practical use of this technique.

In a second class of techniques, the data sequence to be transmitted is selected from a set of possible sequences such that the PAPR is minimized [8]-[11]. In the partial transmit sequences (PTS) technique [8]-[9], the data symbols are grouped in subblocks, and each of the subblocks is weighted with its own phase which is selected such that the PAPR is minimal. The PAPR reduction improves by increasing the number of subblocks. However, the search for the optimal phases is very complex especially when the number of subblocks is large, and side information about the used phases is required to reconstruct the data sequence at the receiver. In the selective mapping (SLM) technique [10], each data sequence can be represented by a number of possible sequences by selecting one phase vector out of a predetermined set of phase vectors; the phase vector that minimizes the PAPR is selected. The complexity of this technique is lower than the PTS technique, as the set of possible phase vectors is smaller, although the PAPR reduction that can be obtained is smaller. Similarly as in the PTS technique, side information about the phase vector is necessary to reconstruct the data. In contrast with the clipping technique, the signal is not distorted and no out-of-band radiation is present, but side information is required for reconstructing the data sequence.

To avoid the necessity of side information, other PAPR reduction techniques were introduced. In one of these techniques, some of the carriers are not used for data transmission, but for PAPR reduction purposes [12]-[14]. In this technique pilots or dummy carriers that are inserted, are selected such that the PAPR is minimized. This however comes at the cost of a reduced throughput, as the carriers used for PAPR reduction can not be used for data transmission. Another technique that does not need side information makes use of coding [15]-[16]. The data is encoded using e.g. a block code. Instead of transmitting the data symbol sequence corresponding to the codeword, a different sequence is transmitted where some of the data symbols are replaced by others (i.e. symbol switching) in order to reduce the PAPR. The errors that are deliberately introduced in this way, can be corrected by the error correcting code. Hence, part of the error correcting capability of the code is sacrificed to PAPR reduction. Most of the literature on this topic deals with linear block codes (like the Golay code or Reed-Muller codes) with hard decoding. Further, the computational complexity of this technique strongly increases with the number of carriers, because of the decoding complexity and the search for which symbols need to be switched. Recent developments in iterative decoding (e.g. turbo codes and LDPC codes) allow long codewords to be decoded with reasonable complexity. To our knowledge, no work has been done on the use of iteratively decodable codes for PAPR reduction. In this paper, we present a systematic low-complexity approach to select the data symbols to be switched, and the errors that are introduced by the symbol switching are corrected by using an iteratively decodable code. The results are compared with the clipping technique.

II. System Description

The bit sequence to be transmitted is split into information words of k bits, where $\mathbf{b}_i = \{b_{i,0}, \ldots, b_{i,k-1}\}$ is the information word at time interval i. The information words $\mathbf{b}_i$ of k bits are converted into codewords $\mathbf{c}_i = \{c_{i,0}, \ldots, c_{i,n-1}\}$ of n bits, using an (n, k) code. During time interval i, the n bits of the codeword $\mathbf{c}_i$ are mapped on N data symbols $\mathbf{a}_i = \{a_{i,0}, \ldots, a_{i,N-1}\}$ selected from a 2^m-point constellation using Gray mapping, where $N = \frac{n}{m}$ and the energy per symbol equals $E_s = E[|a_{i,\ell}|^2]$. The N data symbols $\mathbf{a}_i$ are modulated on the carriers using an N-point inverse fast Fourier transform (IFFT), resulting in the time-domain sequence $\mathbf{s}_i = \{s_{i,0}, \ldots, s_{i,N-1}\}$:

$$s_{i,\ell} = \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} a_{i,q} e^{j2\pi \frac{q\ell}{N}}. \quad (1)$$

In the following, we drop the time index i for notational convenience. The peak-to-average power ratio (PAPR) of the time-domain sequence $\mathbf{s}$ is defined as

$$PAPR(\mathbf{s}) = \frac{\max_\ell |s_\ell|^2}{E_{\mathbf{s}}\left[\frac{1}{N}||\mathbf{s}||^2\right]}. \quad (2)$$

The time-domain signal $\mathbf{s}$ is applied to the PAPR reduction operator $Q(\cdot)$, resulting in the sequence $\bar{\mathbf{s}} = \{\bar{s}_0, \ldots, \bar{s}_{N-1}\}$:

$$\bar{\mathbf{s}} = Q(\mathbf{s}). \quad (3)$$

The time-domain sequence $\bar{\mathbf{s}}$ is transmitted over an AWGN channel with noise spectral density σ^2. The resulting received signal $\mathbf{r}$ is converted to the frequency domain using an FFT. The resulting FFT outputs $\mathbf{z} = \{z_0, \ldots, z_{N-1}\}$ are decoded using an iterative decoder. As no side information is available, the decoder cannot use knowledge on the PAPR reduction in the iterative decoding. The receiver computes the prior probabilities that a received bit equals $x = 0, 1$ from the received samples as follows:

$$P(b_i = x) = \frac{\sum_{a:b_i=x} e^{-\frac{1}{2\sigma^2}|z_q - a|^2}}{\sum_a e^{-\frac{1}{2\sigma^2}|z_q - a|^2}}, i = 0, \ldots, n-1. \quad (4)$$

The sample z_q in (4) corresponds to the sample in which the bit b_i contributes. The sum in the numerator ranges over the constellation points a for which $b_i = x$ only, whereas the sum in the denominator ranges over all constellation points.

A. Clipping

In this paper, we consider clipping with preservation of the phase content of the signal. As the clipping is performed on each time-domain sample separately, the PAPR reduction operator $Q(\cdot)$ is given by

$$\bar{s}_\ell = Q_{clip}(s_\ell) = \begin{cases} s_\ell & \text{if } |s_\ell| \leq \alpha \\ \alpha e^{j\arg(s_\ell)} & \text{if } |s_\ell| > \alpha \end{cases} \quad (5)$$

where α is the clipping level and $\arg(s_\ell)$ is the phase of s_ℓ.

B. Symbol Switching

To reduce the PAPR, we replace M data symbols in $\mathbf{a}$ by other constellation points. The search for the optimal sequence $\tilde{\mathbf{a}}$ where M symbols are switched, such that the PAPR is minimum, is an intractable problem when the number of switched symbols M and the number of carriers N is large. Therefore, we propose a suboptimal, systematic approach to switch the M symbols. The switching algorithm is shown in Algorithm I. For the algorithm, we use the clipping operator defined in (5). First, the $M+1$th maximum of the modulus of the time-domain signal vector $\mathbf{s}$ is determined and the clipping level α is set to this value. The time-domain signal vector $\mathbf{s}$ is clipped with the operator (5) with level α, resulting in M clipped peaks. The resulting time-domain signal $\mathbf{s}_{clip}$ is applied to an FFT, and the resulting vector $\mathbf{a}_{clip}$ is used to compute the error vector $\mathbf{e}_{clip} = \mathbf{a}_{clip} - \mathbf{a}$. Using this error vector, the M symbol positions that will be switched are determined sequentially. For each of the M symbol positions, the position q is determined that has the largest contribution to the error vector. Then it is checked if the symbol at the position q was already switched or not. If the symbol at position q was already switched before, the position corresponding to the next largest contribution to the error vector is checked. If a position q is found that was not switched before, the data symbol a_q at this position is changed into all possible constellation points. For each of the 2^m constellation points, the PAPR is computed and the data symbol is replaced by the constellation point with the smallest PAPR. In this way, the M different symbols are switched with linear complexity.

III. Numerical Results

In the simulations, we consider a low-density parity-check (LDPC) coded OFDM system [17]. We consider two different parameter sets (see table II). In the first set of parameters, a 4QAM constellation is used on $N = 512$ carriers. One

Algorithm I: Symbol Switching Algorithm

set clipping level α to $M+1$th maximum of abs($\mathbf{s}$)
$\mathbf{s}_{clip} = Q_{clip}(\mathbf{s})$ (5), $\mathbf{a}_{clip} = FFT(\mathbf{s}_{clip})$, $\mathbf{e}_{clip} = \mathbf{a}_{clip} - \mathbf{a}$
indices = ones(N,1) % vector of N symbol indices
for i=1:M
  $q = \arg\max \text{abs}(\mathbf{e}_{clip})$ % search for symbol index q with largest contribution in the error vector $\mathbf{e}_{clip}$
  while indices(q)==0 % check if symbol was already switched
    $\mathbf{e}_{clip}(q) = 0$, $q = \arg\max \text{abs}(\mathbf{e}_{clip})$
  end
  indices(q)=0
  for $a_s \in$ constellation
    change a_q into a_s, compute PAPR
  end
  replace a_q by a_s with smallest PAPR
end

TABLE I
ALGORITHM I: SYMBOL SWITCHING ALGORITHM.

	Set 1	Set 2
constellation	4QAM	16QAM
n	1024	2048
k	513	1025
N	512	512

TABLE II
SIMULATION PARAMETERS.

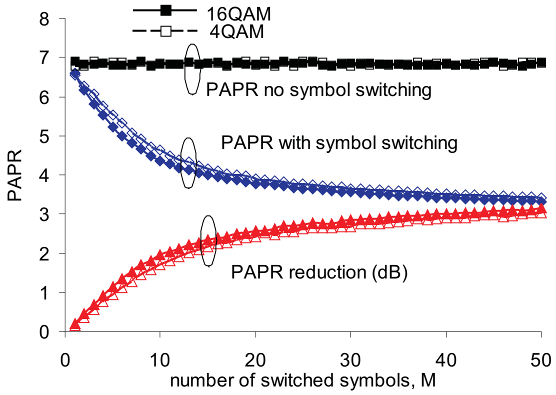

Fig. 1. PAPR with and without symbol switching, 4QAM, $N = 512$ carriers.

OFDM block contains one codeword, such that the code length equals $n = 1024$. The used LDPC code has approximately rate $1/2$ with $k = 513$. In the second set of parameters, a 16QAM constellation is used on $N = 512$ carriers. Similarly as in the first set of parameters, one OFDM block contains one codeword, such that the code length equals $n = 2048$. The code rate is approximately equal to $1/2$ with $k = 1025$. The energy per transmitted data symbol is normalized to $E_s = 1$, and the signal-to-noise ratio (SNR) is defined as $SNR = E_s/\sigma^2$. The PAPR reduction (in dB) is defined as the difference in PAPR (in dB) without PAPR reduction and the PAPR (in dB) after the PAPR reduction technique.

In figures 1 and 2, the PAPR is shown for the symbol switching technique and clipping, respectively, for both 4QAM and 16QAM. The results in these figures are obtained by averaging out over 1000 randomly generated data sequences.

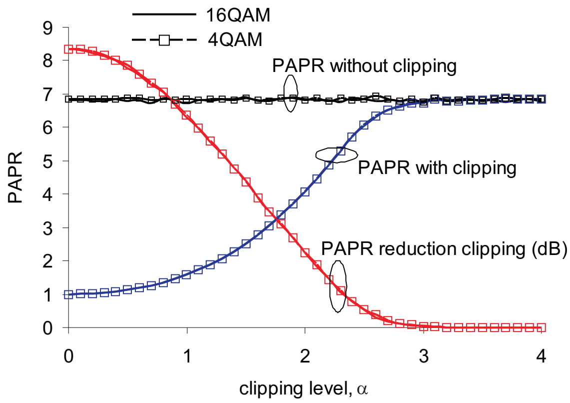

Fig. 2. PAPR with and without clipping, 16QAM, $N = 512$ carriers.

As can be observed, the results are essentially independent of the used constellation. In the symbol switching technique, the PAPR first strongly decreases by increasing the number M of switched symbols but increasing M above 10 only results in a small extra PAPR reduction. On the other hand, the PAPR reduction for clipping strongly depends on the clipping level α. This dependency of the PAPR reduction on α is strongest in the area $\alpha \in [0.5, 2.5]$, whereas the PAPR reduction is very small when α is larger than 2.5.

In figure 3, the distribution of the PAPR after symbol switching is shown for different values of the number M of switched symbols. As in figure 1, it can be observed that the average of the PAPR decreases when the number of switched symbols increases. Moreover, the width of the distribution becomes narrower, which implies that the uncertainty on the PAPR decreases. This simplifies the design of the amplifier: because the narrower distribution of the PAPR, the probability that the transmitted signal is saturated decreases. Further, because of the lower average PAPR, the signal transmission power can be increased without suffering from saturation.

The probability density function (pdf) of the number of erroneous bits that are introduced in the data sequence by using symbol switching is shown in figures 4 and 5 for 4QAM and 16QAM, respectively. The results are obtained by

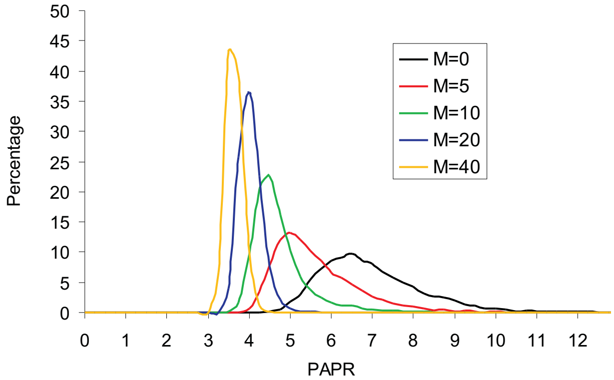

Fig. 3. PAPR distribution, 4QAM, $N = 512$ carriers.

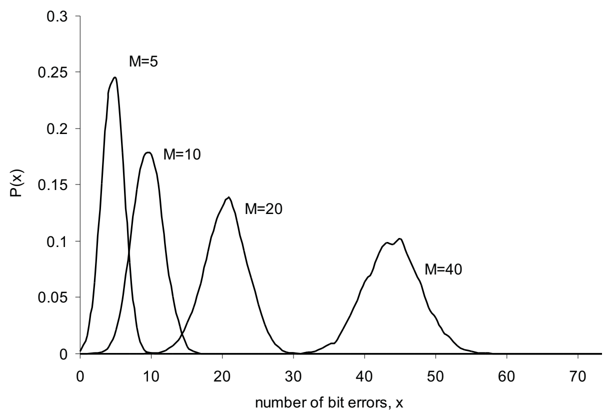

Fig. 4. Number of erroneous bits with symbol switching, 4QAM.

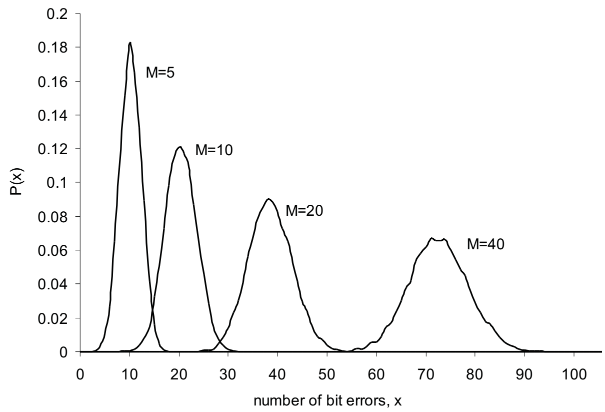

Fig. 5. Number of erroneous bits with symbol switching, 16QAM.

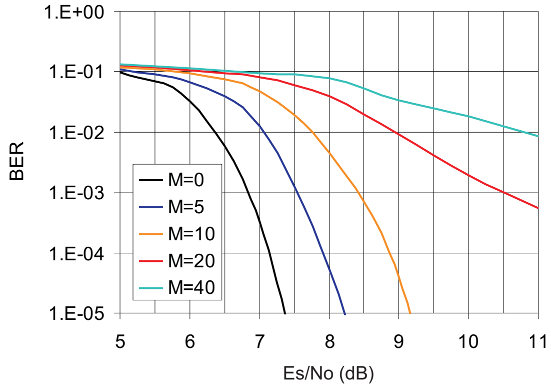

Fig. 6. BER with symbol switching, $N = 512$ carriers, 4QAM.

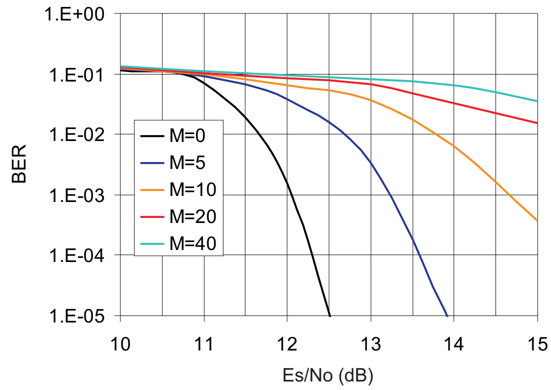

Fig. 7. BER with symbol switching, $N = 512$ carriers, 16QAM.

randomly generating 10000 data sequences. It follows from the figures that the number of erroneous bits are on the average approximately equal to M and slightly less than $2M$ for 4QAM and 16QAM, respectively. Hence, in both cases, approximately half of the bits corresponding to the switched symbols are changed. The spreading of the pdf increases with increasing M, which can be expected as the number of possible combinations of switched symbols increases with increasing M. If the error correcting code is not able to correct the M (4QAM) or $2M$ (16QAM) switched bits, the bit error rate (BER) will show an error floor at high SNR. Hence, the error correcting capacity of the code limits the number of symbols that can be switched. Further, the BER will show a degradation as compared to the no PAPR reduction case as the error correction code exchanges part of its error correcting capability with PAPR reduction.

This can be observed in figures 6 and 7, where the BER is shown for 4QAM and 16QAM, respectively, for different values of M. The corresponding BER results for clipping are shown in figures 8 and 9 for different values of the clipping level α. It can be observed that for given M, the BER degradation is larger for 16QAM than for 4QAM. This can be explained because the number of switched bits in 16QAM is larger than in 4QAM, such that the reduction

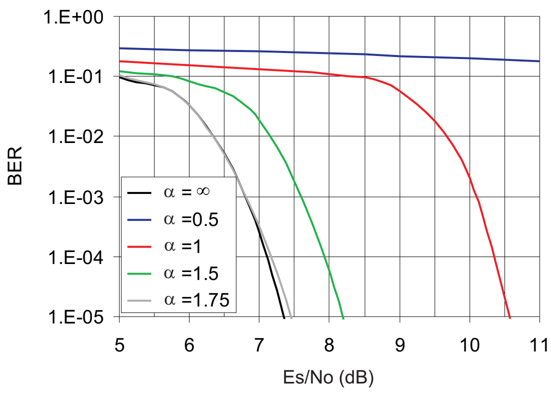

Fig. 8. BER with clipping, $N = 512$ carriers, 4QAM.

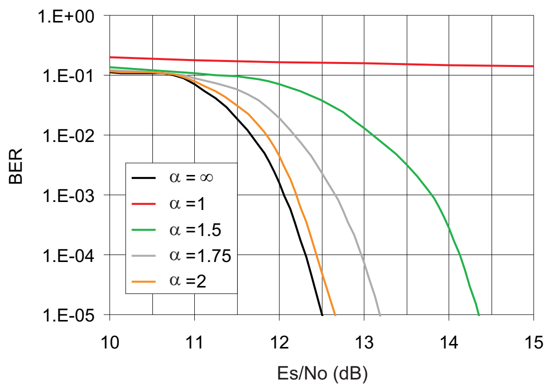

Fig. 9. BER with clipping, $N = 512$ carriers, 16QAM.

in error correction capability in 16QAM is larger than in 4QAM. Similar results can be observed for clipping: for given clipping level α, the BER degradation is larger for 16QAM than for 4QAM. Comparing clipping and symbol switching, it is clear that the BER degradation for symbol switching is larger than for clipping with comparable PAPR reduction. This can be explained as in the clipping method, the distortion caused by clipping is spread over all data symbols such that at the receiver, the deviation between the transmitted and the received symbols is small and within the error correcting capability of the code, whereas in the symbol switching method, the deviations are concentrated on a few symbols. However, in contrast with the clipping method, the symbol switching method does not suffer from out-of-band radiation.

IV. Conclusions and Remarks

In this paper, we considered PAPR reduction by using an iterative code to correct errors introduced by symbol switching. We have presented a simple algorithm for switching symbols. The results are compared with the clipping method. The PAPR shows a strong reduction when the number M of switched symbols is small, whereas for larger M, the extra PAPR reduction is small. The number of bit errors that are introduced by symbol switching of M symbols is on the average equal to $2M/m$, where m is the number of bits per symbol. The spreading of the bit errors introduced by symbol switching increases with M. Although the clipping method outperforms the symbol switching method, the latter does not suffer from out-of-band radiation.

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