PAPR Reduction by Symbol Nulling

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***Abstract*—As a multicarrier signal can exhibit large peaks in the time domain, the amplifier used to transmit the multicarrier signal must be highly linear, and thus expensive, to avoid non-linear signal distortion. To reduce the high peak-to-average power ratio, and hence to allow cheaper amplifiers, several techniques are described in the literature. However, the drawback of these techniques is that they cause non-linear in-band distortion and out-of-band radiation, reduce the system throughput or require side-information. To avoid these drawbacks, we propose a PAPR reduction technique that has none of the abovementioned disadvantages. In the proposed technique, we replace some of the transmitted data symbols by nulls, i.e we introduce errors in the transmitted signal. To counteract the effect of the introduced symbol errors, the transmitted information is encoded. At the receiver, an iterative decoder is used to correct the transmitter and channel errors. The performance of the proposed technique is compared with the clipping technique. Although the clipping technique slightly outperforms the proposed technique with respect to the obtainable PAPR reduction and corresponding BER degradation, the proposed technique does not suffer from non-linear in-band distortion and out-of-band radiation.**

I. INTRODUCTION

Multicarrier transmission has been selected as the physical layer for a large number of applications, as it combines a high bandwidth efficiency with the robustness to channel dispersion [1]. However, as the data stream to be transmitted is split into a large number of lower rate streams that are transmitted in parallel on different subcarriers, the time domain signal of the multicarrier system consists of the sum of a large number of contributions; as a result the system exhibits a large peak-to-average power ratio (PAPR). As the multicarrier system is highly sensitive to non-linear distortions [2], the high PAPR introduces the need for a highly linear amplifier at the transmitter to avoid the peaks in the signal to be distorted. In mass-produced systems, however, cheaper amplifiers are preferred, to keep the cost of the product as low as possible. One way to deal with the effect of the non-linear amplifiers is to reduce the average signal power, such that the effects of the amplifier non-linearities on the peak values of the signal is reduced. However, this involves a power efficiency reduction. Hence, other techniques to solve the PAPR problem are preferred.

In the literature, several techniques to reduce the PAPR have been investigated [3], [4]. The technique with the lowest complexity is the clipping technique [5]–[8]. As in this technique the amplitude of the signal is cut off at a predetermined level, the signal is subjected to non-linear distortion, causing strong out-of-band radiation caused by the spectral regrowth and in-band distortion. As the out-of-band radiation can introduce severe interference with signals in other frequency bands, clipping is combined with filtering to mitigate the spectral regrowth; this however comes at the cost of peak regrowth. Therefore, the clipping-filtering operation is repeated several times to reach the desired PAPR level and to limit the out-of-band radiation. The practical use of this technique is however limited by the difficulty to reconstruct the signal at the receiver.

A second type of PAPR reduction techniques is based on the selection between different possible sequences related to the data sequence to be transmitted, in order to minimize the PAPR [9]–[12]. The partial transmit sequences (PTS) technique [9], [10] groups the data symbols in subblocks; each of the subblocks is then weighted with its own phase which is selected such that the PAPR is minimal. The selective mapping (SLM) technique [11] represents each data sequence by a number of possible sequences by selecting one phase vector out of a predetermined set of phase vectors; the phase vector that minimizes the PAPR is selected. The SLM technique has lower complexity than the PTS technique, but the search for the optimal sequences is very complex in both cases. Although these techniques do not suffer from the disadvantages of the clipping technique, i.e. in-band distortion and out-of-band radiation, side information about the used phases is required to reconstruct the data sequence at the receiver. In the dummy carriers technique [13]–[15], some of the carriers are not used for data transmission, but are selected such that the PAPR is minimized. Although this technique does not require side-information at the receiver, it reduces the data throughput.

Coding is an essential ingredient in present standards. It is shown in the literature that coding cannot only correct errors that occur in the channel, but can also be used for PAPR reduction [16]–[18]. At the transmitter, some of the data symbols are replaced by other symbols to reduce the PAPR. The errors introduced by this technique can then be corrected at the receiver by the error correcting code: part of the error correcting capability is used for PAPR reduction. In [16] and [17], a linear block code with hard decoding is considered. However, the computational complexity of this technique strongly increases with the number of carriers, because of the decoding complexity, and the PAPR reduction comes at the cost of a rather large BER degradation. Recent developments in iterative decoding (e.g. turbo codes and LDPC codes) allow long codewords to be decoded with reasonable complexity. In [18], we introduced a symbol switching technique that is decoded using an LDPC code. However, this technique

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Algorithm I: Symbol Nulling Algorithm
for $i = 1 : M$
    for $j = 1 : N$
        $\mathbf{a}_{null} = \mathbf{a}$, replace symbol $a_{null,j} = 0$, compute corresponding $PAPR_j$
    end
    $q = \arg\min_j (PAPR_j)$
    change $a_q = 0$
end
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TABLE I
ALGORITHM I: THE SYMBOL NULLING ALGORITHM.

introduces a large BER degradation. To avoid the large BER degradation encountered in the previous code-based PAPR reduction techniques, we propose in this paper a new technique that reduces the PAPR in a coded OFDM system by replacing some of the data symbols by nulls; an iterative LDPC decoder is used at the receiver to correct the introduced errors. It turns out that the proposed symbol nulling technique outperforms the symbol switching technique from [18], and is only slightly outperformed by the clipping technique with respect to the PAPR reduction and BER performance. However, the proposed technique does not suffer from the drawbacks from the clipping technique, does not reduce the data throughput and requires no side information.

II. SYSTEM DESCRIPTION

In a coded OFDM system, the bit sequence to be transmitted is first split in information words of length k and encoded using a code with code rate $R_c = k/n$ into code words of length n. Each code word is then mapped on a sequence of $N = n/m$ data symbols $\mathbf{a} = \{a_0, \ldots, a_{N-1}\}$, where the data symbols belong to a constellation of size 2^m. The energy per data symbol equals E_s. Without loss of generality, we assume that the number of carriers equals N. The data symbols are modulated on the carriers yielding the time-domain samples $\mathbf{s} = \{s_0, \ldots, s_{N-1}\}$:

$$s_\ell = \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} a_q e^{j2\pi \frac{q\ell}{N}}. \tag{1}$$

Defining the PAPR reduction operator $Q(\cdot)$, the transmitted time-domain samples yield $\bar{\mathbf{s}} = \{\bar{s}_0, \ldots, \bar{s}_{N-1}\}$:

$$\bar{\mathbf{s}} = Q(\mathbf{s}). \tag{2}$$

In the symbol nulling technique, the PAPR reduction operator $Q(\cdot)$ is a vector operator, where M of the data symbols $\mathbf{a}$ contained in $\mathbf{s}$ are replaced by a null in a systematic way, as described in Algorithm I, in order to minimize the PAPR. Note that as M of the data symbols are set to 0, the total energy of the signal is reduced. In order to keep the total transmitted energy constant, we increase the energy per symbol of the non-nulled data symbols with a factor $N/(N-M)$; this will have no effect on the PAPR.

The symbol nulling technique is compared with the clipping technique without filtering. We consider clipping with preservation of the phase content of the signal. The clipping is performed on each time-domain sample separately, such that the PAPR reduction operator $Q(\cdot)$ is given by

$$\bar{s}_\ell = Q_{clip}(s_\ell) = \begin{cases} s_\ell & \text{if } |s_\ell| \leq \alpha \\ \alpha e^{j \arg(s_\ell)} & \text{if } |s_\ell| > \alpha \end{cases} \tag{3}$$

where $\arg(s_\ell)$ is the phase of s_ℓ and α is the clipping level.

In order to concentrate on the effect of the PAPR reduction operation, we consider the transmission of the sequence $\bar{\mathbf{s}}$ (2) over an AWGN channel with noise spectral density σ^2. At the receiver, the signal is sampled and the resulting samples $\mathbf{r}$ are applied to an FFT. The iterative decoder first computes the prior probabilities that a coded bit equals $x = 0$ or $x = 1$, based on the FFT outputs $\mathbf{z} = \{z_0, \ldots, z_{N-1}\}$:

$$P(b_i = x) = \frac{\sum_{a:b_i=x} e^{-\frac{1}{2\sigma^2}|z_q - a|^2}}{\sum_a e^{-\frac{1}{2\sigma^2}|z_q - a|^2}}, i = 0, \ldots, n-1. \tag{4}$$

where the sample z_q in (4) corresponds to the sample in which the bit b_i contributes. The sum in the numerator ranges over the constellation points a for which $b_i = x$ only, whereas the sum in the denominator ranges over all constellation points. Based on the prior probabilities, the decoder tries to determine the transmitted bit sequence in an iterative way. Note that as no side information is available, the decoder cannot use knowledge on the PAPR reduction in the decoding.

III. NUMERICAL RESULTS

The code used in the simulations is a systematic low-density parity check (LDPC) code. The number of carriers equals $N = 512$, and the 2048 code bits from a (1025,2048) LDPC code are mapped on a 16QAM constellation. Hence, one code word corresponds to one OFDM symbol. The performance of the symbol nulling technique is considered in three cases (see Table II). In the first case, the bits from the systematic LDPC code are Gray mapped on the data symbols, and all data symbols can be selected to be nulled by the symbol nulling technique. To investigate the effect of the mapping of the bits on the constellation points, we interleave the code bits before they are mapped on the constellation points in the second case, similarly as in the first case, all symbols can be nulled. This can be of importance for the decoder: in many situations, bit interleaved coded modulation (BICM), which makes use of an interleaver between the encoder and the symbol mapper, has better performance than a simple Gray mapper. In the third case, we use Gray mapping, as in the first case, but only data

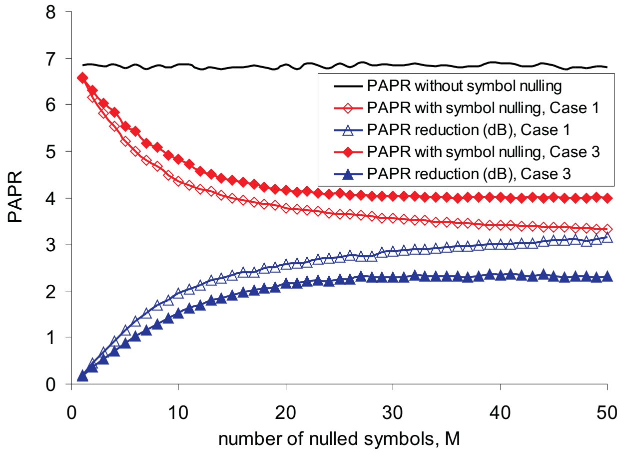

Fig. 1. PAPR reduction by symbol nulling, $N = 512$ carriers, 16QAM.

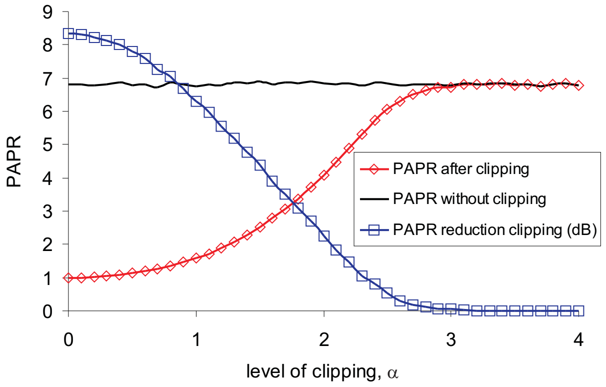

Fig. 2. PAPR reduction by clipping, $N = 512$ carriers, 16QAM.

symbols corresponding to the parity bits can be nulled. In the clipping technique that is considered for the comparison with the symbol nulling technique, the same LDPC code is considered as for the symbol nulling technique.

The average PAPR and average PAPR reduction (in dB) for the symbol nulling technique are shown in figure 1 as function of the number of symbols that are nulled. The results in these figures are obtained by averaging out over 1000 randomly generated data sequences. As the interleaver will on the average have no influence on the positions of the nulled carriers, the average PAPR does not depend on the presence of interleaver. We compare the average PAPR (reduction) in the case that all symbols can be nulled (Case 1), and the case that only the data symbols corresponding to parity bits can be nulled (Case 3). The PAPR reduction that can be obtained in Case 3 is smaller than in Case 1. This can be explained as in Case 3, the symbol nulling technique has less degrees of freedom in selecting the positions of the data symbols that can be nulled. It can be observed in figure 1 that when the number M of nulled symbols is increased, first the average PAPR drops sharply, but when the number of nulled symbols is further increased, the gain in PAPR reduction becomes small. This indicates that the strongest peaks in the time-domain signal are caused by a limited number of data symbols. To compare, the average PAPR and average PAPR reduction (dB) for the clipping technique are shown in figure 2.

	Case 1	Case 2	Case 3
parity symbols only	no	no	yes
interleaver	no	yes	no

TABLE II
SIMULATION SETS.

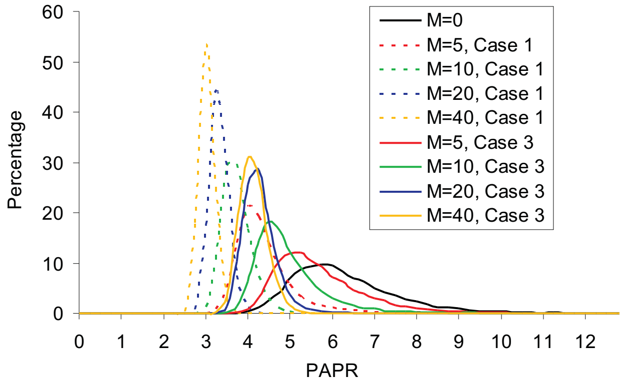

Fig. 3. PAPR distribution after symbol nulling, $N = 512$ carriers, 16QAM.

It is not only important to know how the average PAPR can be reduced by a PAPR reduction technique, also the distribution of the PAPR after PAPR reduction is of importance. In figure 3, the distribution of the PAPR after symbol nulling is shown, and in figure 4 we show the distribution of the PAPR after clipping. The results are obtained by simulating 10^4 OFDM blocks. As expected from figure 1, the top of the distribution, which is close to the average of the distribution, moves to lower values of the PAPR when M increases. Further, increasing M also results in a smaller variance, i.e. the width of the distribution becomes narrower. As explained in the previous paragraph, the presence of the interleaver will have no effect on the PAPR distribution. Hence, only Cases 1 and 3 are shown in figure 3. When all symbols can be nulled (Case 1), the peak of the distribution is located at lower values than for the case when only the parity symbols can be nulled (Case 3). This was also observed in figure 1. Further, the width of the peaks in Case 1 is narrower than in Case 3. As explained in the previous paragraph, this effect is caused by the reduced degrees of freedom in Case 3 as compared to Case 1.

Based on figure 3, the complementary cumulative distribution function (CCDF) can be obtained (see figure 5). From this figure, one can determine what is the probability that the PAPR is larger than a predetermined value. Above a certain threshold, the CCDF decreases essentially log-linearly with the PAPR. It can be observed that the CCDF becomes steeper when M increases and the decay starts at lower PAPR levels. Case 1 gives better results than Case 3. Hence, the probability

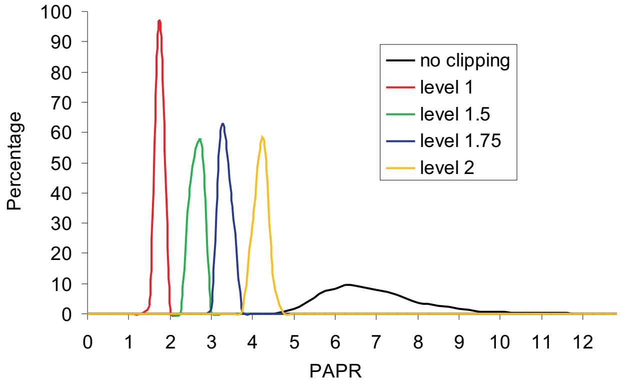

Fig. 4. PAPR distribution after clipping, $N = 512$ carriers, 16QAM.

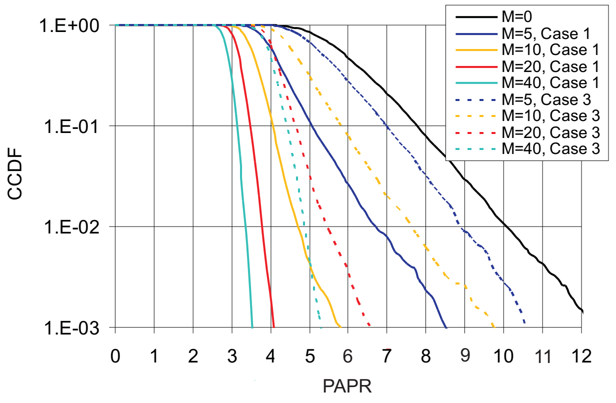

Fig. 5. Complementary cumulative distribution function, $N = 512$ carriers, 16QAM.

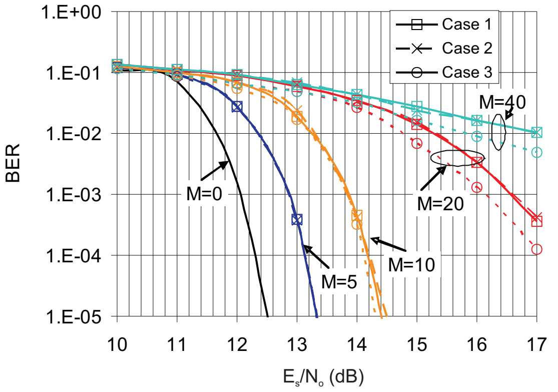

Fig. 6. BER after symbol nulling, $N = 512$ carriers, 16QAM.

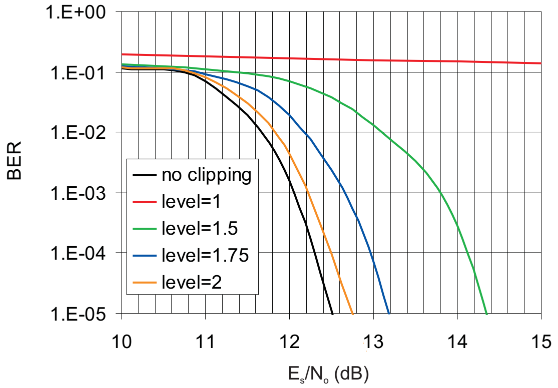

Fig. 7. BER after clipping, $N = 512$ carriers, 16QAM.

that the PAPR will be larger than a given value is much lower in Case 1 than in Case 3. Based on these simulation results, an empirical model for the CCDF can be derived.

The BER corresponding to the different cases is shown in figure 6. It can be observed in the figure that there is essentially no difference between the BER performance in Cases 1 and 2. Hence, it can be concluded that the mapping of the bits on the data symbols has no influence on the symbol nulling technique performance. Further, the BER degradation in Case 3, when only the parity symbols can be nulled, shows a smaller BER degradation than Case 1, where all symbols can be nulls. The difference in BER degradation between Cases 1 and 3 increases when the number M of nulled symbols increases. However, for small BER degradation, i.e. when M is sufficiently small, the difference is small. As the PAPR reduction in Case 1 is larger in Case 3, we can conclude that for practical situations Case 1 gives the best overall performance. For a comparison, the BER degradation for the clipping technique is shown in figure 7. It can be observed that the BER degradation for comparable PAPR reduction is slightly lower than in the symbol nulling technique. This can be explained as in the clipping method, the distortion caused by clipping is spread over all data symbols such that at the receiver, the deviation between the transmitted and the received symbols is small and within the error correcting capability of the code, whereas in the symbol nulling method, the deviations are concentrated on a few symbols. However, in contrast with the clipping method, the symbol nulling method does not suffer from out-of-band radiation and in-band distortion.

IV. CONCLUSIONS AND REMARKS

In this paper, we have proposed a new technique for the reduction of the PAPR. In this technique, a predetermined amount of data symbols are replaced by nulls in order to minimize the PAPR. To cope with the errors introduced in this technique, we encode the transmitted information. In this paper, we used an LDPC code, but the technique can also be applied to other iteratively decodable codes like e.g. turbo codes. The performance of the symbol nulling technique is evaluated through simulations. When the number M of nulled symbols increases, the PAPR first drops sharply, but further increasing M yields only a small further improvement. This indicates that nulling a limited amount of data symbols can already give a considerable PAPR reduction. Increasing M not only decreases the average PAPR, but also reduces the width of the PAPR distribution or the variance. The gain in PAPR reduction and variance reduction is smaller in the case where only parity symbols can be replaced by nulls, as the degrees of freedom in this case are reduced as compared to the case where all symbols can be nulled.

Comparing the BER degradation for the three cases considered in this paper gives rise to the following conclusions. As the interleaver has no influence on the PAPR reduction nor on the BER, it can be stated that the mapping of the bits on the data symbols has no influence on the performance of the symbol nulling technique. The case where only parity symbols are nulled has lower BER degradation than the case where all symbols can be nulled. However, the difference is small when the number of nulled symbols is sufficiently small, such that the BER degradation is small. As the latter case gives a larger PAPR reduction than the former case, it can be concluded that the symbol nulling technique where all symbols can be nulled has better overall performance in practical situations, where the tolerable BER degradation is limited.

The performance of the proposed technique is also compared with the clipping technique. Although the clipping technique slightly outperforms the proposed technique with respect to the obtainable PAPR reduction and corresponding BER degradation, the proposed technique does not suffer from non-linear in-band distortion and out-of-band radiation.

The results in this paper are shown for a 16QAM constellation, but similar results were obtained with other constellations.

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