# Pilot Based Time Delay Estimation for KSP-OFDM Systems in a Multipath Fading Environment

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***Abstract*— We propose two time delay estimators for known symbol padding (KSP) orthogonal frequency division multiplexing (OFDM) in a multipath fading environment. Both estimators make use of pilot symbols in the guard interval and known pilot carriers and take the frequency selectivity of the channel into account. The performance of the estimators is illustrated by means of simulation results for the mean squared error (MSE) and the bit error rate (BER). There is a degradation in performance compared with a receiver with perfect synchronization, especially for high $E_s/N_0$, but KSP-OFDM systems with the proposed estimators outperform a cyclic prefix OFDM system with the time delay estimator from [1].**

## I. Introduction

The number of wired and wireless services has increased a lot during the last years. This increase has created the need for a technique that combines high data rates with a high reliability. Orthogonal frequency division multiplexing (OFDM) is a strong candidate as it is a flexible technique that can support high data rates, and is able to combat frequency selective channels [2]. These advantageous properties have made OFDM a hot research topic and the OFDM technique has already been applied in various standards like digital audio broadcasting (DAB) [3], digital video broadcasting (DVB) [4], in modems for digital subscriber lines (xDSL) [5], in wireless local area networks (WLAN) [6], ...

An OFDM system can be efficiently implemented by the usage of fast Fourier transforms (FFT), which is a great advantage. Before the transmission, an inverse FFT (IFFT) is applied to the information to be transmitted, in order to convert the data that are modulated in the frequency domain on the different carriers into a time domain signal. Further, a guard interval is inserted to avoid inter block interference (IBI) between successively transmitted OFDM blocks. In the literature, there exist different types of guard intervals. The two most popular guard interval techniques are the cyclic prefix (CP) and the zero padding (ZP) techniques [7]. In the cyclic prefix technique, the guard interval is transmitted before each OFDM block and consists of the last samples of the OFDM block. In ZP-OFDM, the guard interval is filled with zeros, i.e. during the guard interval no signal is transmitted. In this paper however, we will consider a third guard interval technique, i.e. the known symbol padding (KSP) technique [8]. In this technique, the guard interval is filled with known samples or pilots.

Synchronization of the OFDM receiver with the OFDM transmitter requires to find the starting point of the OFDM symbol: time offsets can cause inter carrier interference (ICI) and IBI [9], [10]. For CP-OFDM, several time delay estimation algorithms have been proposed in the literature. The authors of [1] derive the maximum likelihood (ML) estimator for a time delay in the presence of additive white Gaussian noise (AWGN). The redundancy of the cyclic prefix and pilot symbols on the carriers are exploited. The blind estimator of [11] is a special case of the previous estimator and only makes use of the correlation of the cyclic prefix and the last samples of the transmitted OFDM block. A time delay estimator that makes use of a specially designed training symbol is proposed in [12] for the AWGN channel. However, as it does not employ all available information, the estimator is suboptimal. In [13], the ML time delay estimator is derived in the case of dispersive channels under the assumption of perfect channel knowledge. The estimator uses the cyclic prefix only. However, as it is in practice very difficult to obtain a channel estimate without knowledge about the time delay, the performance of this estimator can be seen as a lower bound on the performance of an estimator which does not assume any knowledge about the channel.

Common to the time synchronization algorithms proposed for CP-OFDM is the non-negligible degradation caused by the residual timing error at high signal-to-noise ratios (SNR) in the presence of a fading channel. In [14], it is shown that CP-OFDM and KSP-OFDM have essentially the same performance when the guard interval length is much smaller than the number of carriers. As this is the case in all practical situations, it motivated us to consider the timing synchronization problem for KSP-OFDM, where the pilots are spread both in the time and the frequency domain. To our knowledge, no research has been done about time delay estimation algorithms for KSP-OFDM. Both the pilot symbols in the guard interval and the pilot symbols on the pilot carriers are exploited by our estimator. The performance of the proposed estimator is compared with the estimator for CP-OFDM from [1] in terms of the mean squared error (MSE) of the time delay estimate, and in terms of the bit error rate (BER).

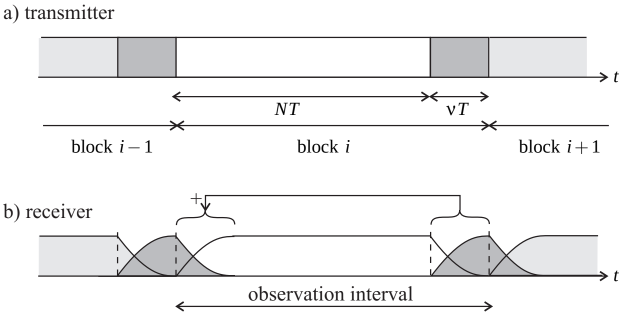


Fig. 1. Time-domain signal of a KSP-OFDM block a) transmitted signal b) received signal and observation interval

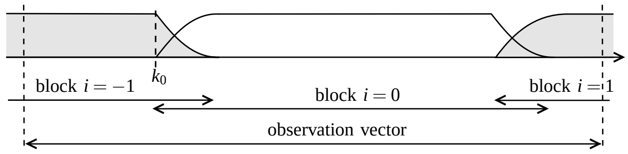


Fig. 2. Definition of the received signal vector

## II. System Model

Consider a KSP-OFDM system with $N$ carriers and a guard interval of length $\nu$. $M$ is defined as the total number of transmitted pilot symbols of which $\nu$ are transmitted during the guard interval and $M-\nu$ on the carriers. On the different carriers, we transmit blocks of symbols $\mathbf{a}_i = (a_i(0), \ldots, a_i(N-1))^T$ consisting of $M-\nu$ pilot symbols denoted as $\mathbf{b}_c = (b_c(0), \ldots, b_c(M-\nu-1))^T$ and $N+\nu-M$ data symbols denoted as $\mathbf{a}_d^{(i)} = \left(a_d^{(i)}(0), \ldots, a_d^{(i)}(N+\nu-M-1)\right)^T$. The guard interval consists of $\nu$ pilot symbols denoted as $\mathbf{b}_g = (b_g(0), \ldots, b_g(\nu-1))^T$. We define $E_s$ as the transmitted energy per symbol: $E_s = \mathrm{E}\left[|a_i(n)|^2\right] = \mathrm{E}\left[|b_g(k)|^2\right]$. The transmitted symbol vector $\mathbf{a}_i$ is modulated on the different carriers using the $N$-point IFFT. The guard interval is inserted after the $N$ IFFT outputs. The samples of the transmitted time domain signal $\mathbf{s}_i = (s_i(0), \ldots, s_i(N+\nu-1))^T$ are given by

$$\mathbf{s}_i = \sqrt{\frac{N}{N+\nu}} \begin{pmatrix} \mathbf{F}^H \mathbf{a}_i \\ \mathbf{b}_g \end{pmatrix} \tag{1}$$

where $\mathbf{F}$ denotes the $N \times N$ FFT matrix with elements $(\mathbf{F})_{k,l} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{kl}{N}}$; $k,l = 0, \ldots, N-1$. Figure 1 shows the time domain signal. We define the vectors $\mathbf{s}_p$ and $\mathbf{s}_d^{(i)}$ as

$$\mathbf{s}_p = \sqrt{\frac{N}{N+\nu}} \mathbf{F}_p \mathbf{b}_c \tag{2}$$

$$\mathbf{s}_d^{(i)} = \sqrt{\frac{N}{N+\nu}} \mathbf{F}_d \mathbf{a}_d^{(i)} \tag{3}$$

where $\mathbf{F}_p$ consists of the $M-\nu$ columns of $\mathbf{F}^H$ which correspond to the pilot carriers and $\mathbf{F}_d$ is given by the $N+\nu-M$ columns of $\mathbf{F}^H$ that correspond to the data carriers. So $\mathbf{s}_p$ and $\mathbf{s}_d^{(i)}$ can be seen as the pilot and data signal in the time domain respectively. We define $\mathbf{b}$ as the total transmitted pilot signal, so $\mathbf{b}$ collects the contribution from the pilot carriers and the pilot symbols in the guard interval

$$\mathbf{b} = \begin{pmatrix} \mathbf{s}_p \\ \sqrt{\frac{N}{N+\nu}} \mathbf{b}_g \end{pmatrix}. \tag{4}$$

The samples $\mathbf{s}_i$ are transmitted over a frequency selective channel with an impulse response of length $L$ denoted as $\mathbf{h} = (h(0), \ldots, h(L-1))^T$. In order to avoid inter block interference, the length of the guard interval $\nu$ is chosen so that the guard interval exceeds the duration of the channel impulse response: $\nu \geq L-1$.

The receiver takes a block of samples $\mathbf{r} = (r(0), \ldots, r(2(N+\nu)+L-3))^T$. Every transmitted OFDM block, which has a duration of $N+\nu$ samples, contributes to $N+\nu+L-1$ successive samples of the received signal after transmission over a channel with an impulse response of $L$ samples. The vector $\mathbf{r}$ contains the total contribution from only one OFDM block (along with partial contributions from adjacent blocks) because of its length. We assume that this block has the index $i=0$ without loss of generality. The starting point $k_0$ of this block in the received signal vector $\mathbf{r}$ is not known and has to be estimated (see figure 2).

For the detection of the data symbols transmitted in block $i$, we take the $N+\nu$ received samples from the observation interval corresponding to block $i$ as can be seen in figure 1. The contributions from the pilot symbols of the guard intervals (dark gray areas on figure 1) are first subtracted from the received signal. The resulting system can be seen as a ZP-OFDM system. Now for data detection in a ZP-OFDM system ([7]), the last $\nu$ samples of the observation interval are added to the first $\nu$ samples of the OFDM symbol (see figure 1b). The resulting block of $N$ samples is then applied to the FFT. Finally per carrier symbol detection is performed.

## III. Time Delay Estimation

In this section we derive the estimator for $k_0$ starting from the joint likelihood function of $k_0$ and $\mathbf{h}$ for the observation $\mathbf{r}$. We drop the block index $i=0$ for notational convenience. To keep things simple, we assume that $\mathbf{r}$ only contains noise besides the contribution of the considered transmitted OFDM block $\mathbf{s}$[1]. We define $\mathbf{r}_0$ as the subvector of $\mathbf{r}$ that collects the contributions from $\mathbf{s}$: $\mathbf{r}_0 = (r(k_0), \ldots, r(k_0+N+\nu+L-2))^T$. Because of the already mentioned assumption, the vector $\mathbf{r}_0$ can be written as

$$\mathbf{r}_0 = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{5}$$

where $\mathbf{s}$ is defined in (1) (with $i=0$), $\mathbf{H}$ is the $(N+\nu+L-1) \times (N+\nu)$ Toeplitz channel matrix whose entries are defined as $(\mathbf{H})_{l:l+L-1,l} = \mathbf{h}$; $l = 0, \ldots, N+\nu-1$ and $\mathbf{w} = (w(k_0), \ldots, w(k_0+N+\nu+L-2))^T$ is

[1] We only use this assumption to derive the estimator, for the simulations we will consider a continuous transmission of OFDM blocks.

the noise vector, where $w(k)$ is white additive Gaussian noise with variance $N_0$ and zero mean. The contribution of the useful signal in (5) can be written as the sum of the contribution of the data symbols and the pilot symbols:

$$\mathbf{Hs} = \mathbf{Bh} + \mathbf{Ah} \tag{6}$$

where $\mathbf{B}$ and $\mathbf{A}$ are the $(N+\nu+L-1)\times L$ Toeplitz matrices with respective entries $(\mathbf{B})_{l:l+N+\nu-1,l} = \mathbf{b}$ and $(\mathbf{A})_{l:l+N-1,l} = \mathbf{s}_d;\ l = 0,\ \ldots,\ L-1$.

The distribution of the received signal vector $\mathbf{r}$ given $k_0$, the channel impulse response $\mathbf{h}$, and the data symbol vector $\mathbf{a}_d$ is given by

$$\begin{aligned} p(\mathbf{r}|k_0,\ \mathbf{h},\ \mathbf{a}_d) = \\ C\exp\left\{-\frac{1}{N_0}\left(\sum_{k=0}^{k_0-1}|r(k)|^2 + \sum_{k=k_0+N+\nu+L-1}^{2(N+\nu+L-2)}|r(k)|^2\right)\right\}. \\ \exp\left\{-\frac{1}{N_0}\left[\mathbf{r}_0 - (\mathbf{B}+\mathbf{A})\mathbf{h}\right]^H\left[\mathbf{r}_0 - (\mathbf{B}+\mathbf{A})\mathbf{h}\right]\right\} \end{aligned} \tag{7}$$

where $C$ is some irrelevant constant. This expression still depends on the unknown data symbols $\mathbf{a}_d$ and has to be averaged over the unknown data symbols in order to be useful for our estimation problem. This averaging is rather complicated so we have to simplify (7) first. For small values of $x$, $\exp(x)$ can be approximated by the first two terms of its Taylor series, i.e. $\exp(x) \simeq 1 + x$ for $|x| \ll 1$. So for low $E_s/N_0$, expression (7) can be approximated by

$$\begin{aligned} p(\mathbf{r}|k_0,\ \mathbf{h},\ \mathbf{a}_d) = \\ C - \frac{C}{N_0}\left(\sum_{k=0}^{k_0-1}|r(k)|^2 + \sum_{k=k_0+N+\nu+L-1}^{2(N+\nu+L-2)}|r(k)|^2\right) \\ - \frac{C}{N_0}\left[\mathbf{r}_0 - (\mathbf{B}+\mathbf{A})\mathbf{h}\right]^H\left[\mathbf{r}_0 - (\mathbf{B}+\mathbf{A})\mathbf{h}\right]. \end{aligned} \tag{8}$$

Averaging (8) over the unknown data symbols is easy now as we only need to compute the averages of $\mathbf{A}$ and $\mathbf{A}^H\mathbf{A}$: $\mathrm{E}[\mathbf{A}] = \mathbf{0}$ and $\mathrm{E}\left[\mathbf{A}^H\mathbf{A}\right] = \mathbf{R_A}$ (See appendix for the computation of $\mathbf{R_A}$). This yields for $p(\mathbf{r}|k_0,\ \mathbf{h})$

$$\begin{aligned} p(\mathbf{r}|k_0,\ \mathbf{h}) = \\ C\left\{1 - \frac{1}{N_0}\left[\mathbf{r}^H\mathbf{r} - \mathbf{r}_0^H\mathbf{Bh} - \mathbf{h}^H\mathbf{B}^H\mathbf{r}_0\right]\right. \\ \left. - \frac{1}{N_0}\mathbf{h}^H\left(\mathbf{B}^H\mathbf{B} + \mathbf{R_A}\right)\mathbf{h}\right\}. \end{aligned} \tag{9}$$

The ML estimates of $k_0$ and $\mathbf{h}$ can be obtained by maximizing (9) with respect to $k_0$ and $\mathbf{h}$. The estimate of $\mathbf{h}$ given $k_0$ is obtained by deriving (9) with respect to $\mathbf{h}$ and results in

$$\hat{\mathbf{h}}(k_0) = \left(\mathbf{B}^H\mathbf{B} + \mathbf{R_A}\right)^{-1}\mathbf{B}^H\mathbf{r}_0 \tag{10}$$

When we substitute this estimate of $\mathbf{h}$ in (9) we obtain the function $\Gamma_1(k_0)$ which only depends on $k_0$:

$$\Gamma_1(k_0) = \frac{1}{N_0}\mathbf{r}_0^H\mathbf{B}\left(\mathbf{B}^H\mathbf{B} + \mathbf{R_A}\right)^{-1}\mathbf{B}^H\mathbf{r}_0. \tag{11}$$

The estimate of $k_0$ is then given by

$$\hat{k}_0 = \arg\max_{k_0}\{\Gamma_1(k_0)\}. \tag{12}$$

A second estimator can be obtained by totally neglecting the contributions of the unknown data symbols in (8). This means that we neglect $\mathbf{A}$ in (8) and $\mathbf{R_A}$ in (9). In that case, the estimate of $\mathbf{h}$ given $k_0$ is given by

$$\hat{\mathbf{h}}(k_0) = \left(\mathbf{B}^H\mathbf{B}\right)^{-1}\mathbf{B}^H\mathbf{r}_0 \tag{13}$$

and the estimate of $k_0$ is then given by

$$\hat{k}_0 = \arg\max_{k_0}\{\Gamma_2(k_0)\} \tag{14}$$

with

$$\Gamma_2(k_0) = \frac{1}{N_0}\mathbf{r}_0^H\mathbf{B}\left(\mathbf{B}^H\mathbf{B}\right)^{-1}\mathbf{B}^H\mathbf{r}_0. \tag{15}$$

Although we derive the joint estimate of $\mathbf{h}$ and $k_0$ in this algorithm, only the estimate for $k_0$ is used. Indeed, the estimate for $\mathbf{h}$ will perform badly at high $E_s/N_0$, as the contributions from the data symbols in (8) and (9) have been either neglected or replaced by their means, resulting in an error floor in the MSE of $\mathbf{h}$ and the BER (see [15] and [14]). The derivation of the estimate of $\mathbf{h}$ is only needed to remove its contribution from (9) in order to obtain a simple expression for the estimate of $k_0$. For channel estimation, better estimators are available in the literature, e.g. [16], [17], having better performance at high $E_s/N_0$ than the estimators (10) and (13).

If we take a closer look at (11) and (15), we see that the functions $\Gamma_1(k_0)$ and $\Gamma_2(k_0)$ compute the correlation between the received signal and the pilot vector $\mathbf{b}$ at $L$ successive time instants as can be seen from the matrix product $\mathbf{B}^H\mathbf{r}_0$:

$$\begin{aligned} \left(\mathbf{B}^H\mathbf{r}_0\right)_l = \sum_{k=0}^{N-1} r(k_0+l+k)\,(s_p(k))^* \\ + \sqrt{\frac{N}{N+\nu}}\sum_{k=0}^{\nu-1} r(k_0+l+N+k)\,(b_g(k))^* \end{aligned} \tag{16}$$

where $l = 0,\ \ldots,\ L-1$. Both the estimators (12) and (14) try to find the $\hat{k}_0$ that maximizes a function of the $L$ successive correlations between the received signal and the pilot vector.

## IV. Simulation Results

In this section the performance of our time delay estimators is evaluated by means of simulations. We compare the performance of the estimators with the ML time delay estimation algorithm for CP-OFDM from [1]. We consider $N = 1024$ carriers and a guard interval of length $\nu = 100$ for KSP-OFDM and CP-OFDM respectively. To make a fair [2] comparison between CP-OFDM and KSP-OFDM, we assume that the number of pilot symbols transmitted on the carriers in the CP-OFDM signal is equal to $M-\nu$. The transmitted symbols consist of randomly generated QPSK symbols. Although we derived the estimator for $k_0$ under the assumption that only one OFDM block is transmitted, we simulate a

[2]By taking $N$, $\nu$ and the number $M-\nu$ of pilot carriers the same for both CP-OFDM and KSP-OFDM, we obtain the same data throughput and, assuming perfect synchronization and channel knowledge, essentially the same BER.

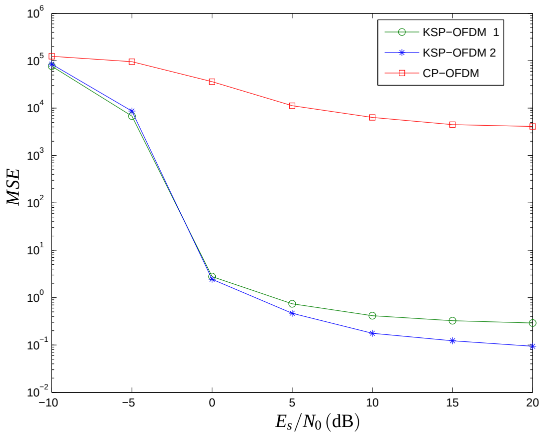


Fig. 3. MSE results for a frequency selective channel, $L = 50$, $N = 1024$, $\nu = 100$, $M = 200$

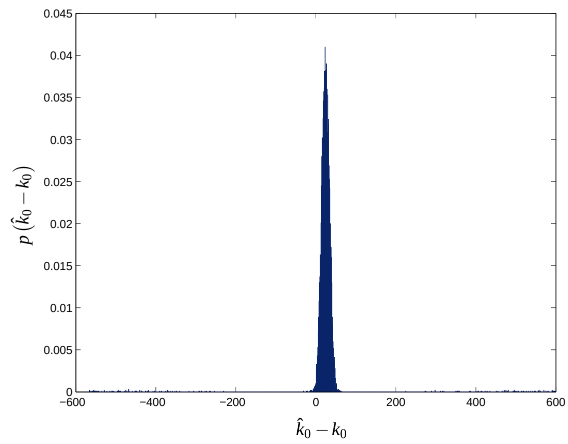


Fig. 5. Histogram of the time delay estimation error for the CP-OFDM estimator, $E_s/N_0 = 20$ dB, 100 pilot carriers

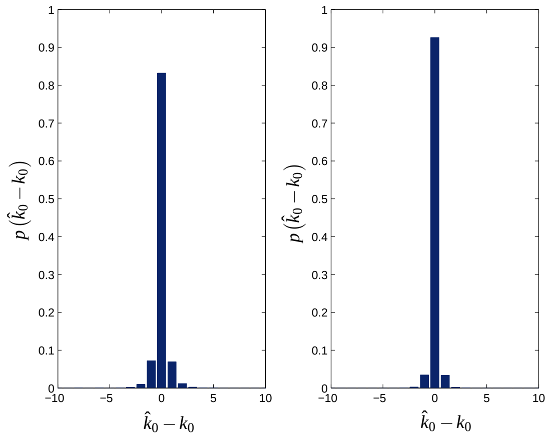


Fig. 4. Histogram of the time delay estimation error for the KSP-OFDM estimator 1 (left) and 2 (right), $E_s/N_0 = 20$ dB, $M = 200$

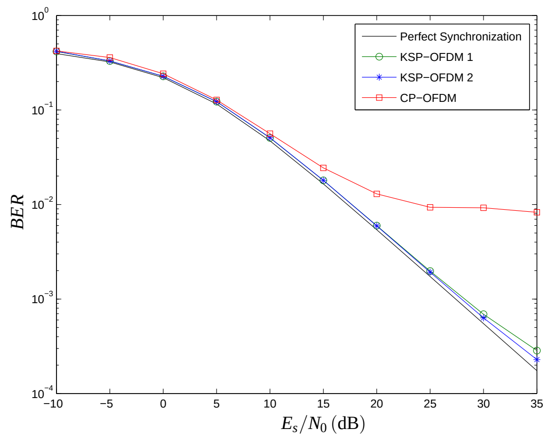


Fig. 6. BER results for a frequency selective channel, $L = 50$, $N = 1024$, $\nu = 100$, $M = 200$

continuous transmission of OFDM symbols. As we want to focus on the impact of time delay estimation errors, it is assumed for the simulation of the BER that possible phase rotations of the symbol constellation, caused by time delay estimation errors, are perfectly compensated and that the channel is perfectly estimated after the time delay estimation. For KSP-OFDM, these assumptions mean that the contributions from the pilot symbols from the guard interval can be perfectly removed from the received signal. In the figures and in the accompanying text, the KSP-OFDM estimator from (12) which takes the unknown data symbols in to account, is called '*KSP-OFDM estimator 1*', while the estimator from (14) which totally neglects the contributions from the unknown data symbols, is called '*KSP-OFDM estimator 2*'.

The performance of the estimators in a dispersive channel is shown in figures 3-6. We consider a frequency selective Rayleigh fading channel consisting of $L = 50$ channel taps. Figure 3 shows the results for the MSE on the time delay estimate. The KSP-OFDM estimators outperform the estimator for CP-OFDM as could be expected: our estimators take the dispersive nature of the channel into account while the estimator from [1] was designed for an AWGN channel and so this estimator is not robust to a dispersive channel as opposed to our estimator. The KSP-OFDM estimator 2 outperforms the first KSP-OFDM estimator for higher $E_s/N_0$.

Figures 4 and 5 show a histogram of the estimation error $\hat{k}_0 - k_0$ for the KSP-OFDM estimators and the CP-OFDM estimator respectively for $E_s/N_0 = 20$ dB and 100 pilot carriers ($M = 200$). The first KSP-OFDM estimator finds the true $k_0$ in more than 80% of all simulated cases. The second KSP-OFDM estimator performs even better and finds the real $k_0$ in more than 90% of all simulated cases. For both KSP-OFDM estimators, the estimation error $\left|\hat{k}_0 - k_0\right|$ is smaller than or equal to 2 samples in more than 99% of all simulated cases. The performance of the CP-OFDM estimator is much worse: the true $k_0$ is almost never found and less than 1% of all cases results in $\left|\hat{k}_0 - k_0\right| \leq 2$ samples.

The BER results for a dispersive channel are shown in figure 6. The BER curves confirm the results from the other figures. We see that KSP-OFDM systems with the proposed estimators exhibit a lower BER than the CP-OFDM system with the time delay estimator from [1]. The performance of receivers with the considered estimators is close to a receiver with perfect synchronization for low to middle high $E_s/N_0$. For higher $E_s/N_0$, the KSP-OFDM systems will also exhibit an error floor for the BER but CP-OFDM has a significantly higher error floor. The error floors of the proposed estimators are caused by the assumptions made in the derivation of these estimator, i.e. that only one OFDM symbol is transmitted whereas in the simulations continuous transmission is considered, and by assuming that the data symbols can be neglected or replaced by their averages. KSP-OFDM estimator 2 results in a lower error floor than KSP-OFDM estimator 1, so totally neglecting the contribution of the unknown data symbols for the estimation of the time delay gives better results than averaging first over the unknown data symbols.

## V. Conclusion

We have derived two time delay estimators for KSP-OFDM in multipath fading environments. Both estimators are based on the correlation between the received signal and the pilot symbols in the guard interval and the correlation between the received signal and the time domain contribution from the pilot carriers. The first estimator is derived after averaging the likelihood function of the received signal over the unknown data symbols. The second estimator just neglects the contribution of the unknown data symbols. We compared the proposed time delay estimators with the ML time delay estimator for a CP-OFDM system [1] in terms of MSE and BER. The KSP-OFDM systems with our time delay estimators outperform the considered CP-OFDM system. , as they result in a lower BER. The KSP-OFDM estimator which neglects the unknown data symbols, gives better performance than the estimator which averages the likelihood function of the received signal first over the unknown data symbols.

## Appendix

In this appendix we compute $\mathbf{R_A}$ which is the average of $\mathbf{A}^H\mathbf{A}$. Note that $\mathbf{A}^H\mathbf{A}$ is a Hermitian symmetric matrix, so it is sufficient to only consider the elements $(k,l)$ with $l \geq k$. The elements of $\mathbf{A}^H\mathbf{A}$ are given by

$$\left(\mathbf{A}^H\mathbf{A}\right)_{k,l} = \sum_{m=0}^{N-1-(l-k)} \left(s_d\left(m+l-k\right)\right)^* s_d\left(m\right) \qquad l \geq k, k = 0, \ldots, L-1 \quad (17)$$

where $s_d(m)$ are the elements of the vector $\mathbf{s}_d$, defined in (3). Averaging those elements over the unknown data symbols yields for the elements of $\mathbf{R_A}$

$$\left(\mathbf{R_A}\right)_{k,l} = \left(N-l+k\right)\frac{E_s}{N+\nu}\sum_{m=0}^{N-M+\nu-1} e^{-j2\pi\frac{n_m(l-k)}{N}} \qquad l \geq k, k = 0, \ldots, L-1 \quad (18)$$

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