Cooperative hybrid localization using Gaussian processes and belief propagation

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***Abstract*—Wireless localization using signal strength has been very popular in commercial applications due to the wide availability of 802.11 WiFi networks. However, signal strength information alone provides very rough location estimates. In this paper we consider supplementing the receiver of each user with a ranging unit required for accurate positioning. By allowing range-based cooperation between the users, it becomes possible to increase the positioning accuracy without the need of a fully deployed network of ranging anchors. To this end, we propose a fully distributed localization algorithm that uses belief propagation for fusing signal strength and ranging information. Extensive simulations, using 3D ray tracing to provide accurate radio maps, show that the proposed fusion of measurements results in a very scalable localization solution, where the localization performance smoothly transitions in accuracy, depending on the available infrastructure.**

***Index Terms*—Wireless localization, RSS, Gaussian process, cooperation, message passing**

I. Introduction

Accurate positioning information in harsh propagation environments, as inside buildings, is becoming increasingly important for a number of emerging applications such as commercial, automotive, public service, and military systems [1]. However, an accurate large scale localization system does not exist yet due to the lack of infrastructure to support localization. In general, position information can be obtained by exchanging signals with a number of fixed transmitters (possibly at a known position). The user position is then estimated using one or more of the received signal properties such as received signal strength (RSS), time-of-arrival or angle-of-arrival.

RSS-based localization is favored by most commercial systems because it can rely on the wide availability of 802.11 WiFi networks. However, the unpredictability of signal propagation in an indoor environment makes RSS-based localization a challenging task. In general, accurately predicting the signal strength requires a full description of the RF environment. Approximate RSS models such as the path loss model are therefore too crude, resulting in a very low positioning accuracy. More advanced methods make use of a calibration procedure where the signal strength is measured in a (small) number of positions. In a method called fingerprinting [2], the position is estimated by searching for calibration data with a similar signal strength signature. In areas where calibration data is sparse, however, this method breaks down. In [3] and [4], this problem is alleviated by employing a Gaussian process (GP) that predicts the signal strength at any given location, even in areas with sparse calibration data. With the GP, an accuracy of 2.12m was reported in [4]. For many indoor applications, however, this level of accuracy is still insufficient.

Alternatively, centimeter accuracy can be achieved by adopting time-of-arrival (ToA) measurements, for example by using ultra-wideband transmitters. To circumvent the requirement of a large number of anchors, which are network nodes at a known position, users can be allowed to make ranging measurements with their neighbors such that cooperation between users becomes possible. In [5], it is shown that cooperative localization in UWB networks indeed increases positioning accuracy while requiring a lower amount of anchors within the area. However, even with a large number of neighboring users, which is not always guaranteed, this method still requires at least one anchor within range for good reliability. In an attempt to completely circumvent the requirement of a range based infrastructure, cooperative RSS based localization was proposed in several works, such as [6] and [7]. However, these methods use the inaccurate path loss model to transform RSS measurements into distances resulting in a low accuracy.

In this paper we present a novel hybrid localization algorithm that uses GPs to model the RSS measurements, and ToA-based ranging to allow for cooperation between users. With this approach, the positioning accuracy significantly increases without the need for a range based infrastructure. Furthermore, our algorithm still allows us to incorporate range measurements with anchors to further increase the positioning accuracy whenever this is required. This results in a localization method that seamlessly adapts its accuracy depending on the available infrastructure. Because it is hard to obtain large amounts of *accurate* ground truth for the evaluation of our algorithm, we have chosen to use 3D ray tracing to simulate a large realistic wireless environment. The use of ray tracing, which can compute the signal strength with great accuracy [8], allows us to produce very large data sets with exact ground truth[1] required for the statistically correct evaluation of the algorithm. This is in contrast with most experiments where only a small data set is available and the ground truth is subject to errors.

[1]The radio maps generated by 3D ray tracing are available online at http://telin.ugent.be/~slvdveld/rayTrace.zip

II. Problem Formulation

Consider a wireless network consisting of N mobile users capable of connecting with access points and making range measurements with other users and fixed anchors. We assume the position of the anchors is known to all users. Each user i with coordinates $\mathbf{x}_i \in \mathbb{R}^{2,3}$ can make a set of independent range measurements $\{z_{ij}\}_{j\in\mathcal{N}_i,\mathcal{A}_i}$ where $\mathcal{N}_i$ and $\mathcal{A}_i$ denote the set of neighboring users and ranging anchors, respectively. Similarly, each user i can make a set of signal strength measurements $\{y_{ij}\}_{j\in\mathcal{C}_i}$ where $\mathcal{C}_i$ is the set of connected WiFi access points. All the RSS and range measurements are collected in $\mathcal{Y}$ and $\mathcal{Z}$, respectively. For the calibration of the GP, we assume that some calibration data $\mathcal{D}_\mathrm{c}$ is available for the access points. The goal is to estimate the marginal posterior of each user given by $p_i(\mathbf{x}_i|\mathcal{D}_\mathrm{c}, \mathcal{Y}, \mathcal{Z})$.

III. Statistical description

A. Global description

In order to describe the estimation problem of the unknown users' positions $\mathbf{x}_{1:N}$, we construct the joint posterior distribution of $\mathbf{x}_{1:N}$ which takes into account the different measurements $\mathbf{y}$ and $\mathbf{z}$, and the calibration data $\mathcal{D}_\mathrm{c}$. Using Bayes' theorem and the independence of the measurements we can factorize the joint posterior distribution of $\mathbf{x}_{1:N}$ as follows

$$
\begin{aligned}
&p\left(\mathbf{x}_{1:N}|\mathcal{D}_\mathrm{c}, \mathcal{Y}, \mathcal{Z}\right)\\
&\quad\propto \quad p(\mathcal{Y}, \mathcal{Z} \mid \mathcal{D}_\mathrm{c}, \mathbf{x}_{1:N})\, p_0(\mathbf{x}_{1:N})\\
&\quad= \quad \underbrace{\prod_{j\in\mathcal{C}_{1:N}} p_j\left(\mathbf{y}_j \mid \mathcal{D}_c, \mathbf{x}_{1:N}\right)}_{\text{RSS}} \prod_{i=1}^{N} \Bigg[\underbrace{\prod_{j\in\mathcal{A}_i} p\left(z_{ij} \mid \mathbf{x}_i\right)}_{\text{ToA anchors}}\\
&\qquad\quad \times \underbrace{\prod_{j\in\mathcal{N}_i} p\left(z_{ij} \mid \mathbf{x}_i, \mathbf{x}_j\right)}_{\text{ToA cooperation}} \underbrace{p_0(\mathbf{x}_i)}_{\text{prior}} \Bigg], \qquad (1)
\end{aligned}
$$

where $\mathbf{y}_j$ are all the RSS measurements made by the different users with the jth access point and $p_0(\mathbf{x}_i)$ the prior distribution of the ith user position. For example, in a tracking scenario, the prior is related to the user position at a previous time step and can incorporate measurements from an inertial measurement unit (IMU). In this paper, no prior information is considered and the corresponding factors can be neglected. The remaining factors present in (1) depend on the measurement models and will be described in detail in the following subsections. Once the joint posterior distribution is fully described, a distributed method to obtain the users' position estimates from the factorized distribution in (1) will be described in Section IV.

B. Gaussian process for the RSS likelihood model

In an indoor environment, no simple mathematical model exists to accurately predict the received signal strength from the receiver's position. This is illustrated in Fig. 1, where the unpredictable behavior of the signal strength is shown. In [4], [3], a Gaussian process (GP) was proposed that offers a non-parametric prediction of the signal strength at an arbitrary position using a small amount of calibration points. Additionally, the GP also returns the uncertainty of the prediction, which takes into account the measurement noise and the density of calibration data within the vicinity of the point of interest. The key assumption of the GP is that for points $\mathbf{x}_m$ and $\mathbf{x}_n$ that are close in space, the corresponding signal strengths y_m and y_n must be correlated. This is expressed with the following covariance function

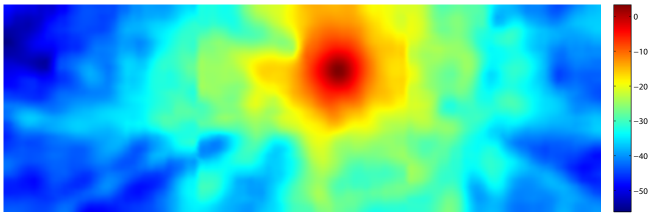

Figure 1. Radio map of a WiFi access point, generated by 3D ray tracing. The transmit power of the access point is 100mW. The signal strength values are given in dBm.

$$\mathrm{cov}(y_m, y_n) = k(\mathbf{x}_m, \mathbf{x}_n) + \sigma^2_{\mathrm{RSS}}\delta_{mn}, \qquad (2)$$

where σ^2_{RSS} is the variance of a signal strength measurement, δ_{mn} is the Kronecker delta function[2], and $k(\mathbf{x}_m, \mathbf{x}_n)$ an arbitrary covariance function or *kernel*. In this article we employ the commonly used Gaussian kernel[3]:

$$k(\mathbf{x}_m, \mathbf{x}_n) = \sigma_f^2 \exp(-\frac{1}{2l^2}\|\mathbf{x}_m - \mathbf{x}_n\|^2), \qquad (3)$$

with l and σ_f^2 the length scale and signal variance, respectively. Both parameters control how much the signal strength of nearby points is correlated and thus control the smoothness of the radio map estimated by the GP. These parameters can be estimated from the calibration data and are different for every access point. See [4] for more information on the hyperparameter estimation of l and σ_f^2. Consider the calibration data for a single access point which consists of L positions[4] $\tilde{\mathbf{x}}_\ell$ for $\ell = 1..L$, for which the corresponding signal strength measurements to the access point are given by $\tilde{\mathbf{y}} = [\tilde{y}_1, \tilde{y}_2, .., \tilde{y}_L]^\mathrm{T}$. For the entire set of calibration points $\{\tilde{\mathbf{x}}_{1:L}, \tilde{\mathbf{y}}\}$, we can calculate the covariance which yields $\mathrm{cov}(\tilde{\mathbf{y}}) = \tilde{\mathbf{K}} + \sigma^2_{\mathrm{RSS}}\mathbf{I}_L$ where $\tilde{\mathbf{K}}$ is the $L \times L$ kernel matrix with elements $\tilde{\mathbf{K}}[m, n] = k(\tilde{\mathbf{x}}_m, \tilde{\mathbf{x}}_n)$, and $\mathbf{I}_L$ the $L\times L$ identity matrix.

For a single access point, the Gaussian process models the conditional distribution of the measured signal strengths $\tilde{\mathbf{y}}$, given the positions $\tilde{\mathbf{x}}_{1:L}$, as a zero mean Gaussian distribution $p(\tilde{\mathbf{y}}|\tilde{\mathbf{x}}_{1:L}) = \mathcal{N}(\mathbf{0}, \mathrm{cov}(\tilde{\mathbf{y}}))$. Using the same model, we can

[2] $\delta_{mn} = 1$ if $m = n$, and zero otherwise.

[3] Another kernel used for modeling the correlation of signal strenghts is the Matérn kernel [3].

[4] Notice that different access points can share the same known positions for calibration.

describe the joint distribution of calibration signals $\tilde{\boldsymbol{y}}$ and the signals $\mathbf{y}$ measured by the N users, this yields

$$p\left(\left[\begin{array}{c}\tilde{\mathbf{y}}\\ \mathbf{y}\end{array}\right]|\tilde{\mathbf{x}}_{1:L},\mathbf{x}_{1:N}\right)=\mathcal{N}\left(\mathbf{0},\left[\begin{array}{cc}\mathrm{cov}(\tilde{\mathbf{y}}) & \mathrm{cov}(\tilde{\mathbf{y}},\mathbf{y})\\ \mathrm{cov}(\mathbf{y},\tilde{\mathbf{y}}) & \mathrm{cov}(\mathbf{y})\end{array}\right]\right) \tag{4}$$

with $\mathbf{C} = \mathrm{cov}(\tilde{\mathbf{y}},\mathbf{y}) = [\mathbf{k}_1,\mathbf{k}_2,...,\mathbf{k}_N]$ and $\mathbf{k}_i = [k(\tilde{\mathbf{x}}_1,\mathbf{x}_i),k(\tilde{\mathbf{x}}_2,\mathbf{x}_i),\ldots,k(\tilde{\mathbf{x}}_L,\mathbf{x}_i)]^{\mathrm{T}}$. Using the formula for the conditional distribution of a joint Gaussian [9] on (4), we obtain $p\left(\mathbf{y}|\tilde{\mathbf{y}},\tilde{\mathbf{x}}_{1:L},\mathbf{x}_{1:N}\right) = \mathcal{N}(\boldsymbol{\mu},\boldsymbol{\Sigma})$ with mean $\boldsymbol{\mu} = [\mu_1,\mu_2,...,\mu_N]^{\mathrm{T}}$ and $\boldsymbol{\Sigma}$ the $N\times N$ covariance matrix with elements $\boldsymbol{\Sigma}[i,k]=\sigma^2_{ik}$, where μ_i and σ^2_{ik} are given by:

$$\mu_i \;=\; \mathbf{k}_i^T(\tilde{\mathbf{K}}+\sigma^2_{\mathrm{RSS}}\mathbf{I}_L)^{-1}\tilde{\mathbf{y}} \tag{5}$$
$$\sigma^2_{ik} \;=\; k(\mathbf{x}_i,\mathbf{x}_k)+\sigma^2_{\mathrm{RSS}}\delta_{ik}-\mathbf{k}_i^T(\tilde{\mathbf{K}}+\sigma^2_{\mathrm{RSS}}\mathbf{I}_L)^{-1}\mathbf{k}_k. \tag{6}$$

We can now use the above formulas to obtain the likelihood function $p_j\left(\mathbf{y}_j|\mathcal{D}_{\mathrm{c}},\mathbf{x}_{1:N}\right)$ for the jth access point, using the appropriate calibration data, required for estimating the user's positions through (1). In a sense, the multi-user GP approach already exhibits some cooperation between users. This can be understood by considering the covariance function, which states that it is very unlikely that two users are close to each other, while measuring significantly different signal strengths. This 'cooperation', however, is very weak and greatly complicates estimation. Because of this we make the approximation where we do not impose the correlation constraint for measurements of different users. This results in:

$$p_j(\mathbf{y}_j|\mathcal{D}_{\mathrm{c}},\mathbf{x}_{1:N}) \;\approx\; \prod_{i=1}^{N} p\left(y_{ij}|\mathcal{D}_{\mathrm{c}},\mathbf{x}_i\right) \tag{7}$$
$$=\; \prod_{i=1}^{N}\mathcal{N}\left(\mu_j(\mathbf{x}_i),\sigma^2_j(\mathbf{x}_i)\right), \tag{8}$$

where $\mu_j(\mathbf{x}_i)$ and $\sigma^2_j(\mathbf{x}_i)$ are given by μ_i in (5) and σ^2_{ii} in (6), respectively, using the calibration data for the jth access point.

C. Ranging and cooperation

We assume all range measurements to be in line-of-sight (LOS). This is a reasonable assumption whenever non-LOS mitigation or detection techniques are used [10]. Furthermore, we assume a standard Gaussian noise model resulting in $p\left(z_{ij}\mid\mathbf{x}_i,\mathbf{x}_j\right) = \mathcal{N}(\|\mathbf{x}_i-\mathbf{x}_j\|,\sigma^2_{\mathrm{d}})$ and $p\left(z_{ij}\mid\mathbf{x}_i\right) = \mathcal{N}(\|\mathbf{x}_i-\mathbf{x}_j^{\mathrm{ref}}\|,\sigma^2_{\mathrm{d}})$, where σ^2_{d} is the noise variance of the range measurement and $\mathbf{x}_j^{\mathrm{ref}}$ the known position of the jth anchor.

IV. Belief propagation for sensor fusion

The factorization of (1) can be represented graphically by a factor graph on which message passing algorithms, such as belief propagation (BP), can be applied [11]. The factor graph is created by drawing square vertices for every factor of the joint probability function and circular vertices for every variable. Every variable vertex is connected by an edge to a factor vertex when the variable is an argument of the particular factor. The factor graph corresponding to the posterior likelihood in (1) in case of $N=3$ users, 1 WiFi access point and 1 ranging anchor[5] is shown in Figure 2 on the left. The factor graph on the right uses the approximation in (7). It is clear that this approximation results in a graph with less cycles. In general, too much cycles in the factor graph should be avoided as they increase the complexity of the algorithm and may have a negative effect on the accuracy of the estimation [12]. Because of these considerations, we will only consider the factor graph that uses the approximation (7).

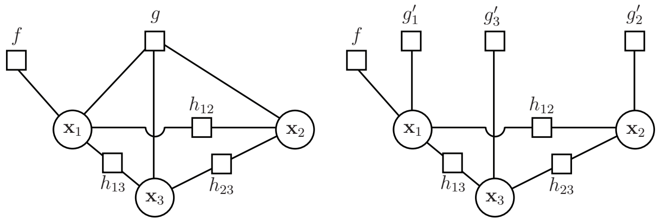

Figure 2. Factor graph representation of the posterior likelihood function for $N=3$ users. Here, $f(\mathbf{x}_i)=p\left(\mathbf{z}_i\mid\mathbf{x}_i\right)$ is the information from the ranging of the user with the anchors, $g\left(\mathbf{x}_{1:N}\right)=p\left(\mathbf{y}\mid\mathbf{x}_{1:N}\right)$ is the information from the RSS measurements, $h_{ij}\left(\mathbf{x}_i,\mathbf{x}_j\right)=p\left(z_{ij}\mid\mathbf{x}_i,\mathbf{x}_j\right)$ is the information from ranging between users i and j, and $g'_i(\mathbf{x}_i)=p(y_i|\mathbf{x}_i)$ is the RSS approximation in (7).

Using BP, we can efficiently compute the belief $b_i(\mathbf{x}_i)$ for each user, which is an approximation of the desired marginal distribution $p_i(\mathbf{x}_i|\mathcal{D}_{\mathrm{t}},\mathcal{Y},\mathcal{Z})$. Because all factor nodes are connected with two variable nodes at most in (1), and because z_{ij} is independent from z_{ji}, we can use the SPAWN [5] implementation of BP to compute the beliefs. Let f_{ij} be a factor connected to the variable $\mathbf{x}_i$; then the formulas for messages $m_{f_{ij}\to i}(\mathbf{x}_i)$, going from factor node f_{ij} to variable node $\mathbf{x}_i$, and for the belief $b_i(\mathbf{x}_i)$ are given by:

$$m_{f_{ij}\to i}(\mathbf{x}_i) \;=\; \int f_{ij}(\mathbf{x}_i,\mathbf{x}_j)b_j(\mathbf{x}_j)\,\mathrm{d}\mathbf{x}_j \tag{9}$$
$$b_i(\mathbf{x}_i) \;\propto\; \prod_{\forall f_{ij}} m_{f_{ij}\to i}(\mathbf{x}_i). \tag{10}$$

For factors f_j that are connected to variable node $\mathbf{x}_i$ only, equation (9) simply revert to $m_{f_{ij}\to i}(\mathbf{x}_i)=f_{ij}(\mathbf{x}_i)$. Because (9) requires the beliefs of neighboring nodes, which are unknown at the start of the algorithm, we iteratively solve these equations. This is referred to as loopy BP. Using formulas (9) and (10) for the posterior in (1), the belief for the ith user is given by:

$$b_i(\mathbf{x}_i) \;\propto\; p_0(\mathbf{x}_i)\prod_{j\in\mathcal{C}_i}p_j\left(y_{ij}\mid\mathbf{x}_i\right)\prod_{j\in\mathcal{A}_i}p\left(z_{ij}\mid\mathbf{x}_i\right) \times\prod_{j\in\mathcal{N}_i}\int p(z_{ij}|\mathbf{x}_i,\mathbf{x}_j)b_j(\mathbf{x}_j)\,\mathrm{d}\mathbf{x}_j. \tag{11}$$

[5] Here, the ranging anchor is within radio range with the first user only.

It can be seen from equation (11) that the belief $b_i(\mathbf{x}_i)$ of the ith user is expressed by the measurements that are locally available to the user. Hence, the belief can be calculated by the user itself, resulting in a fully distributed algorithm. In general, there is no closed form expression for the computation of the belief given by (11), such that we must rely on approximate methods. In this article we will use a non-parametric particle-based approach where the belief $b_i(\mathbf{x}_i)$ of the ith user is represented by a set of K particles $\mathbf{x}_i^{(k)}$ and corresponding weights $w_i^{(k)}$, i.e., $b_i(\mathbf{x}_i) \leftrightarrow \{\mathbf{x}_i^{(k)}, w_i^{(k)}\}_{k=1}^K$. In [13], a particle based method for the computation of the beliefs was devised where every message was individually represented by particles and multiplied by using kernel density estimation (KDE) required to evaluate the messages in an arbitrary point. In this work we do not represent the messages by means of particles but rather compute the belief in (11) directly, resulting in a more accurate and computational less demanding algorithm. In order for the particles to give a good representation of the belief, we sample them from a proposal distribution $q_i(\mathbf{x}_i)$ which we choose to be a) easy to sample from and b) closely resembling the belief $b_i(\mathbf{x}_i)$. A possible proposal distribution[6] can be the prior distribution p_0 or if present, a factor $p\left(z_{ij} \mid \mathbf{x}_i\right)$ with small z_{ij}[7]. With the proposal distribution $q_i(\mathbf{x}_i)$ selected, the weights are given by $w_i^{(k)} = \frac{b_i(\mathbf{x}_i^{(k)})}{q_i(\mathbf{x}_i^{(k)})}$. Now, with a sufficient amount of particles, a good approximation for the minimal mean squared error (MMSE) position estimate $\hat{\mathbf{x}}_i$ can be obtained using a Monte Carlo approximation:

$$\hat{\mathbf{x}}_i = \int \mathbf{x}_i b_i(\mathbf{x}_i)\, \mathrm{d}\mathbf{x}_i \approx \frac{\sum_{k=1}^K w_i^{(k)} \mathbf{x}_i^{(k)}}{\sum_{k=1}^K w_i^{(k)}}. \tag{12}$$

For numerical stability, we compute the weights in log domain: $\bar{w}_i^{(k)} = \log(w_i^{(k)}) = \log\left(b_i(\mathbf{x}_i^{(k)})\right) - \log\left(q_i(\mathbf{x}_i^{(k)})\right)$. It can be seen from equation (11) that $\log\left(b_i(\mathbf{x}_i^{(k)})\right)$ results in a number of terms corresponding to the prior, RSS measurements, anchors and neighbors. These terms can easily be evaluated for every $\mathbf{x}_i^{(k)}$, with the exception of the terms corresponding to the cooperation with neighbors. For this, we again make a Monte Carlo approximation for $m_{j\to i}(\mathbf{x}_i^{(k)})$ from (9):

$$\int p(z_{ij}|\mathbf{x}_i^{(k)}, \mathbf{x}_j) b_j(\mathbf{x}_j)\, \mathrm{d}\mathbf{x}_j \approx \sum_{\ell=1}^K w_j^{(\ell)} p\left(z_{ij} \mid \mathbf{x}_i^{(k)}, \mathbf{x}_j^{(\ell)}\right). \tag{13}$$

The full algorithm is outlined in Algorithm 1. In this algorithm, the beliefs are calculated until a stopping criterion is reached, such as reaching a fixed number of loops or when the beliefs have converged[8].

[6]The message corresponding to the RSS measurement could also be used as a proposal distribution. However, their does not exist an efficient method to sample from the GP-based likelihood in (7). This could be a opportunity for future research.

[7]Whenever z_{ij} is small, the entropy of $p\left(z_{ij} \mid \mathbf{x}_i\right)$ will also be small, meaning that it can be represented by a small amount of particles. See [13] how to draw samples from $p\left(z_{ij} \mid \mathbf{x}_i\right)$.

[8]Usually, the beliefs converge after 2 or 3 iterations.

Algorithm 1 Cooperative hybrid localization.

Sample K **points** $\mathbf{x}_i^{(k)}$ **from** $q_i(\mathbf{x}_i)$
Calculate non-cooperative weights:

$$\bar{w}_{\mathrm{p},i}^{(k)} = \log\left(p_0(\mathbf{x}_i^{(k)})\right) \tag{14}$$

$$\bar{w}_{\mathrm{RSS},i}^{(k)} = -\sum_{j\in\mathcal{C}_i} \log\left(2\pi\sigma_j(\mathbf{x}_i^{(k)})\right) + \left(\frac{(y_{ij} - \mu_j(\mathbf{x}_i^{(k)}))^2}{2\sigma_j^2(\mathbf{x}_i^{(k)})}\right) \tag{15}$$

$$\bar{w}_{\mathrm{a},i}^{(k)} = -\sum_{j\in\mathcal{A}_i} \left(\frac{(z_{ij} - \|\mathbf{x}_i^{(k)} - \mathbf{x}_j^{\mathrm{ref}}\|)^2}{2\sigma_{\mathrm{d}}^2}\right) \tag{16}$$

$$\bar{w}_i^{(k)} = w_{\mathrm{p},i}^{(k)} + w_{\mathrm{RSS},i}^{(k)} + w_{\mathrm{a},i}^{(k)} - \log(q_i(\mathbf{x}_i^{(k)})) \tag{17}$$

Estimate $\hat{\mathbf{x}}_i$ **and broadcast belief** $\{\mathbf{x}_i^{(k)}, \bar{w}_i^{(k)}\}_{k=1}^K$.
Loop
Receive beliefs $\{\mathbf{x}_j^{(k)}, \bar{w}_j^{(k)}\}_{k=1}^K$ **from neighbors** $j \in \mathcal{N}_i$.
Calculate updated belief:

$$\bar{w}_{\mathrm{coop},i}^{(k)} = -\sum_{j\in\mathcal{N}_i} \log\left(\sum_{\ell=1}^K w_j^{(\ell)} p\left(z_{ij} \mid \mathbf{x}_i^{(k)}, \mathbf{x}_j^{(\ell)}\right)\right) \tag{18}$$

$$\bar{w}_i^{(k)} = \bar{w}_{\mathrm{coop},i}^{(k)} + \bar{w}_{\mathrm{p},i}^{(k)} + \bar{w}_{\mathrm{RSS},i}^{(k)} + \bar{w}_{\mathrm{a},i}^{(k)} - \log(q_i(\mathbf{x}_i^{(k)})) \tag{19}$$

Estimate $\hat{\mathbf{x}}_i$ **and broadcast belief** $\{\mathbf{x}_i^{(k)}, \bar{w}_i^{(k)}\}_{k=1}^K$.
until stopping criterion is reached.

V. Numerical results

A. Simulation setup and 3D ray tracing

For our simulations, we have created a 3D model of an office corridor consisting of 18 rooms covering a total area of over $900\mathrm{m}^2$. The floor plan of the corridor is shown in Fig. 3. Using this model together with the dielectric properties (ε_r, σ) of the objects, a radio map for each WiFi access point is computed by means of 3D ray tracing. In our study, we have used the commercial Ray Tracing software tool called WinProp from AWE Communications [14] which employs the method of images [15] for its calculations. We consider the radio map as the ground truth and use it to generate signal strength measurements by adding zero mean Gaussian noise with standard deviation $\sigma_{\mathrm{rss}} = 2\mathrm{dBm}$. This noise level is equal to the empirically obtained noise level in [4]. In the office area, $L = 500$ positions were randomly selected for the calibration of the GPs. In our simulations we always consider 5 access points to be available. For the ranging measurements we defined a maximum radio range of 20m and zero mean Gaussian noise with standard deviation $\sigma_{\mathrm{d}} = 10\mathrm{cm}$. The positions of the access points and ranging anchors are shown in Fig. 3. The users' positions are random within the considered area and are different for every simulation run such that the effect geometry is averaged out over the full simulation. Simulations are performed for 2D localization and $K = 800$ particles are used to represent the beliefs. Note that the number

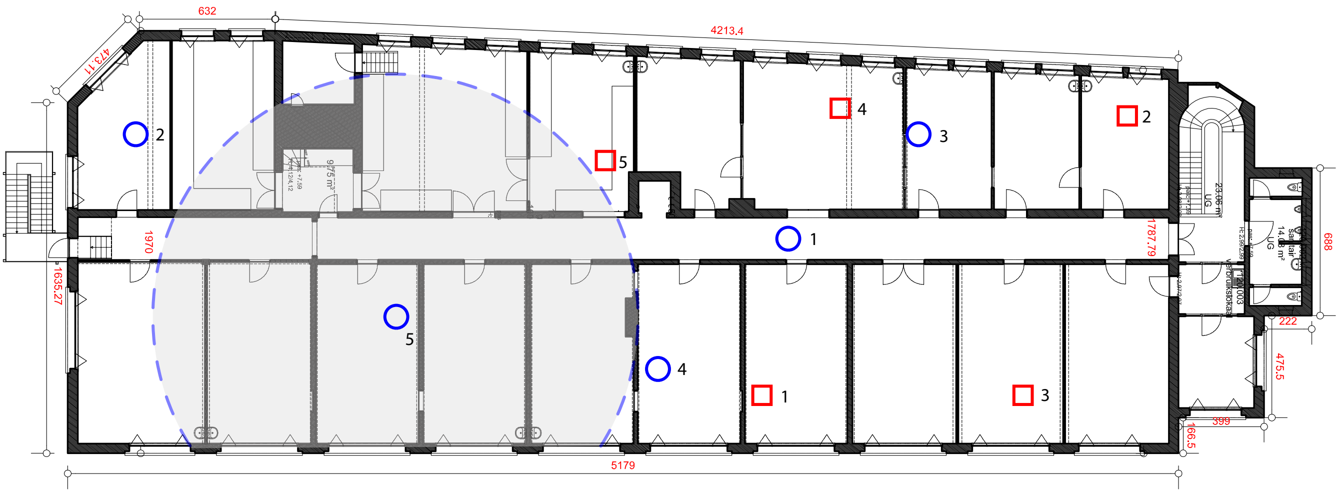

Figure 3. Floor plan of the office corridor in the university building used to model the 3D environment for ray tracing. The squares and circles represent the access points and anchors, respectively. For the fifth anchor, the radio range of 20m is shown. The access points have radio range within the entire corridor. The total area is over 900m^2.

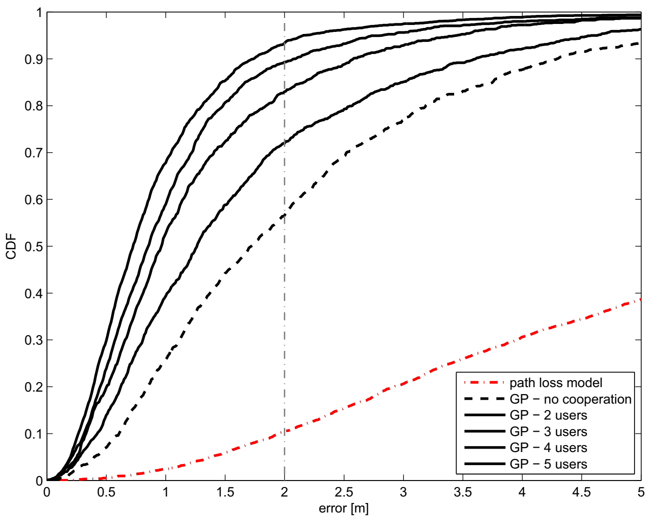

Figure 4. CDF curve comparing errors in position estimate for different number of users involved in cooperation.

of particles can be further reduced in tracking scenarios where the prior corresponds to the previous belief.

B. Results

In the first simulation, the user is connected to all five Wifi access points ($|\mathcal{C}_i| = 5$) and no range capable anchors are used, i.e., $\mathcal{A}_i = \emptyset$. Localization is performed using RSS measurements and in case of multiple users also ToA measurements between the users (cooperation). In Fig. 4, the cumulative density function (CDF) of the estimation error is shown for a varying number of users. For comparison, we also included the result obtained from estimating the user position with the path loss model, i.e., $P_{\text{dBm}} = P_{0,\text{dbm}} + 10\gamma \log_{10}(d)$ with P_{dBm} the received signal strength, $P_{0,\text{dbm}} = 20\text{dBm}$, γ the path loss exponent, and d the distance from the access point. We estimated the path loss exponent γ using the calibration data. The user position was estimated using classic multilateration given the RSS-based distances to the WiFi access points. It can be seen from the figure that estimation using this path loss model results in very poor performance. By employing a GP to model the radio map and in the absence of cooperation between users, we see that the positioning estimation is much more accurate. Furthermore, it can be seen that the proposed algorithm that allows cooperation by means of ToA measurements between users further improves the performance of estimation. For example, without cooperation, the probability of obtaining an error below 2m is 55%, as compared to a probability of 92% when 5 users are cooperating.

In Figure 5, the probability of a positioning error larger than 1m is shown for an increasing number of ToA ranging anchors, i.e., $\mathcal{A}_i \neq \emptyset$. Again all Wifi access points are used. It can be seen that every ranging anchor that is added, results in an immediate drop in outage percentage. Also, it can be seen that with cooperation, the outage percentage drops faster. This is because the users that benefit from a nearby anchor, can help a lot in locating a neighboring user that is not within range of the anchor. In other words, users with a good positioning estimate can have a positive effect on their neighbors' estimate.

Finally, we summarize our simulation results in Table I where the average positioning error is shown as function of the number of cooperating users, and the number of ranging anchors. The top-left average error value of 2.16m corresponds to the estimation without any ToA ranging. Moving down, or to the right in this table always results in a lower average positioning error which asserts the premise of a seamless algorithm.

VI. Conclusions and Future Work

The cost of infrastructure for time-of-arrival (ToA) based localization remains a major obstacle to achieve widespread accurate localization. In order to avoid the expenses of new

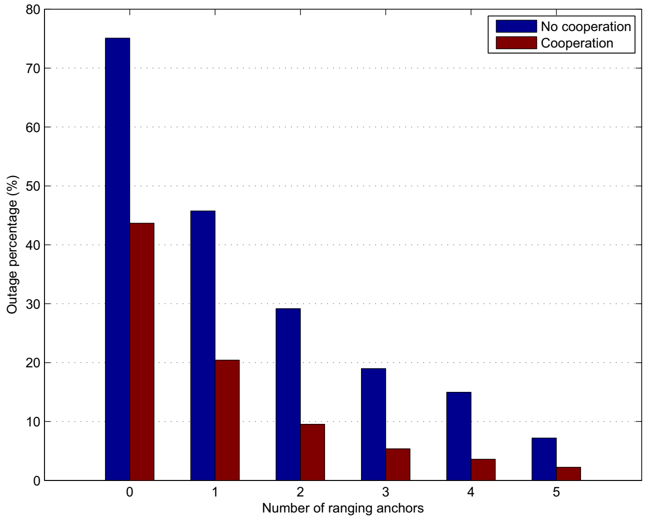

Figure 5. bar plot showing the probability of a positioning error larger than 1m for both cooperative and non-cooperative hybrid localization.

Table I
AVERAGE POSITIONING ERROR.

	1 user	2 users	3 users	4 users	5 users
0 ranging anchors	2.16m	1.71m	1.28m	1.09m	0.93m
1 ranging anchor	1.48m	1.12m	0.90m	0.77m	0.61m
2 ranging anchors	0.99m	0.72m	0.58m	0.51m	0.46m
3 ranging anchors	0.72m	0.57m	0.46m	0.40m	0.35m

infrastructure, less accurate received signal strength (RSS) based localization has been widely adopted in most commercial localization systems. In this paper we propose a novel hybrid localization method that combines both RSS and ToA measurements. This hybridization allows for a seamless transition between crude signal strength localization and accurate range-based localization. Furthermore, by allowing users to share positioning information, it is possible to augment the localization performance without requiring any new ToA-based infrastructure at all. In this article, the fusion of measurements is performed by means of Belief Propagation and the modeling of the signal strengths is governed by Gaussian processes. Extensive simulations using 3D ray tracing to realistically model a large wireless indoor environment, demonstrate the effectiveness of the proposed hybrid algorithm.

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