The Effect of Carrier Phase Jitter on the Performance of Orthogonal Frequency-Division Multiple-Access Systems

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***Abstract*—We investigate the sensitivity to carrier phase jitter of an orthogonal frequency-division multiple-access (OFDMA) system. When all OFDMA carriers have the same power level and jitter spectrum, the degradation caused by the jitter is shown to be equal to the degradation of an OFDM system. Also, traditional FDMA is found to be slightly more robust than OFDMA.**

***Index Terms*— Carrier phase jitter, multiple-access systems, orthogonal frequency-division multiplexing.**

I. System Description

ORTHOGONAL frequency-division multiple access (OFDMA) is closely related to orthogonal frequency-division multiplexing (OFDM), which is well documented in the literature (e.g., [1]–[3]). In the case of OFDM, the received signal can be viewed as an OFDM signal, but with each carrier generated by a different user instead of all carriers generated by the same user. In [4] OFDMA has been proposed as an access technique for the return channel in a community antenna television system (CATV) network, i.e., for the communication from the users to the head-end.

In an OFDMA scenario the receiving station (e.g., the head-end in a CATV network) transmits network synchronization signals to the different users, from which these derive the appropriate carrier frequency, symbol rate, and time alignment needed for the orthogonality of the modulated carriers. For instance, each user generates its sinusoidal carrier from the received synchronization signals by means of a phase-locked loop (PLL). However, each of these sinusoidal carriers is affected by phase jitter.

The complex envelope $r(t)$ of the received OFDMA signal is given by [4]

$$r(t) = \sum_m \sum_{n=0}^{N-1} a_{m,n} \sqrt{\frac{E_{s,n}}{N}} \sum_{\ell=0}^{N-1} \exp\left(j2\pi \frac{n\ell}{N}\right) \cdot p\left[t - \left(mN+\ell\right)\frac{T}{N}\right] \exp[j\phi_n(t)] + n(t) \quad (1)$$

where $a_{m,n}$ denotes the mth data symbol (with unit energy) transmitted by the nth user on the carrier with frequency n/T, N is the number of orthogonal carriers, $1/T$ is the symbol rate per carrier, and N/T is the total symbol rate. Data symbols generated by different users are uncorrelated. The pulse $p(t)$ is a real-valued unit energy square-root Nyquist pulse with respect to the interval T/N

$$\int_{-\infty}^{+\infty} p(t)p\left(t + n\frac{T}{N}\right) dt = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0. \end{cases} \quad (2)$$

$E_{s,n}$ denotes the energy per symbol for the nth user and $n(t)$ represents the additive noise. In a CATV environment this additive noise consists of ingress noise [5], which is characterized by a strongly frequency-dependent power spectral density (PSD). The process $\varphi_n(t)$ denotes the phase jitter from the nth user and is modeled as a stationary zero-mean process whose bandwidth is much smaller than N/T. The phase jitter processes related to different users are uncorrelated for OFDMA. Fig. 1 shows the conceptual block diagram of the receiver. In order to detect the symbol $a_{k,n}$, the receiver feeds to the decision device the quantity $z_n(kT)$, which is obtained by evaluating at the frequency n/T the discrete Fourier transform of the matched filter output samples taken at the instants $(kN+\ell)T/N$ with $\ell = 0, \cdots, N-1$. The matched filter has impulse response $p(-t)$.

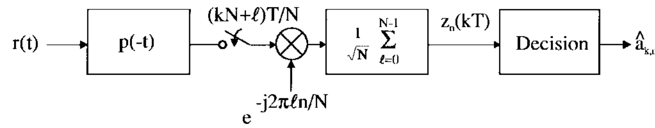

Fig. 1. Conceptual block diagram of receiver for OFDMA.

II. Sensitivity to Carrier Phase Jitter

In this section we compute the degradation (in decibels) of the signal-to-noise ratio (SNR) at the input of the decision device when carrier phase jitter is present.

Let us concentrate on the detection of the symbol $a_{0,n}$. The variation of the phase jitter over the impulse response duration of the matched filter (which is in the order of T/N) can be neglected, because of the small bandwidth of the jitter. The input to the decision device is given by

$$z_n(0) = \sqrt{E_{s,n}}\, a_{0,n} E[I_{n,0}] + \sqrt{E_{s,n}}\, a_{0,n}[I_{n,0} - E(I_{n,0})] + \sum_{\substack{m=0\\ m\neq n}}^{N-1} \sqrt{E_{s,m}} a_{0,m} I_{m,n-m} + W_n(0) \quad (3)$$

where

$$I_{m,k} = \frac{1}{N}\sum_{\ell=0}^{N-1} \exp\left[j\varphi_m\left(\frac{\ell T}{N}\right)\right] \exp\left(-j2\pi\frac{k\ell}{N}\right) \quad (4)$$

$$W_n(0) = \frac{1}{\sqrt{N}} \sum_{\ell=0}^{N-1} \exp\left(-j2\pi\frac{n\ell}{N}\right) \int_{-\infty}^{+\infty} n(t)p \cdot \left(t - \ell\frac{T}{N}\right) dt. \quad (5)$$

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The quantity $I_{m,k}$ is the discrete Fourier transform of $\{\exp[j\varphi_m(\ell T/N)] | \ell = 0, \cdots, N-1\}$, evaluated at the frequency k/T. Note that for $\varphi_m(t) = 0$ we obtain $I_{m,0} = 1$ and $I_{m,k} = 0$ for $k \neq 0$.

The first term of (3) containing the mean value of $I_{n,0}$ is the useful component, the second term is a zero-mean disturbance caused by the fluctuation of $I_{n,0}$ with respect to its mean value $E[I_{n,0}]$, the third term is the zero-mean intercarrier interference (ICI), and the fourth term is caused by the additive noise. As all jitter contributions contain a different data symbol, these contributions are uncorrelated. For small phase jitter, the approximation $\exp[j\varphi_m(\ell T/N)] \cong 1 + j\varphi_m(\ell T/N)$ is valid.

In the absence of phase jitter, the SNR at the input of the decision device corresponding to the nth user equals $E_{s,n}/E[|W_n(0)|^2]$. In the presence of phase jitter, the SNR is reduced. This degradation, expressed in decibels, is given by

$$D_n = 10 \log \left\{ 1 + \frac{E_{s,n}}{E[\,|W_n(0)|^2]} E[|I_{n,0} - E(I_{n,0})|^2] + \sum_{\substack{m=0 \\ m \neq n}}^{N-1} \frac{E_{s,m}}{E[\,|W_n(0)|^2]} E[|I_{m,n-k}|^2] \right\}. \quad (6)$$

The degradation (6) can be computed when the energies $E_{s,m}$ and the phase jitter spectra $S_{\varphi_m}\{\exp[j2\pi f(T/N)]\}$ of user m for $m = 0, \cdots, N-1$ and the additive noise spectrum are known.

In the following we consider the degradation (6) under the assumption that the energy per symbol and the jitter spectrum equal E_s and $S_\varphi[\exp(j2\pi fT/N)]$, respectively, for all N users. In this case a similiar analysis as in (6) shows that the degradation reduces to

$$D_n = 10 \log \left\{ 1 + \frac{E_s \sigma_\varphi^2}{E[|W_n(0)|^2]} \right\} \quad (7)$$

where

$$\sigma_\varphi^2 = \frac{T}{N} \int_{-(N/2T)}^{N/2T} S_\varphi \left[\exp\left(j2\pi f \frac{T}{N} \right) \right] df \quad (8)$$

is the jitter variance. Hence, when the signals received by the different users have the same energy per symbol and the same jitter spectrum, the degradation depends on the jitter variance but not on the specific shape of the jitter spectrum.

In the case of OFDM the received signal is again given by (1), but with $E_{s,n} = E_s$ and $\varphi_n(t) = \varphi(t)$ for $n = 0, \cdots, N-1$: the power level of all carriers is the same and all carriers exhibit identical phase jitter as they are generated by the same oscillator. Following the same reasoning, we obtain that the degradation of the SNR is given by (7). This is in agreement with the result from [6], where the degradation for OFDM has been computed assuming an additive white Gaussian noise (AWGN) channel.

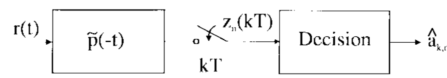

Fig. 2. Conceptual block diagram of receiver for FDMA.

It might be surprising that OFDMA (with *uncorrelated* phase noise processes having the same PSD $S_\varphi[\exp(j2\pi fT/N)]$) and OFDM (where all carriers exhibit *identical* phase jitter with PSD $S_\varphi[\exp(j2\pi fT/N)]$) yield the same degradation (7) of the SNR. However, as the individual contributions to the disturbance in (3) are uncorrelated when the data symbols are uncorrelated, the degradation of the SNR is not affected by the presence or absence of correlation between the phase jitter processes.

In the case of traditional FDMA the modulated carriers occupy nonoverlapping frequency bands. As far as the detection of the data symbols from the nth user is concerned, the complex envelope of the received signal is given by

$$r(t) = \sqrt{E_{s,n}} \sum_m a_{m,n} \tilde{p}(t - mT) \exp[j\varphi_n(t)] + n(t) \quad (9)$$

where $\tilde{p}(t)$ is a real-valued square-root Nyquist pulse with respect to the interval T and the other quantities have the same meaning as in (1).

Fig. 2 shows the conceptual block diagram of the receiver for FDMA. In order to detect the symbol $a_{k,n}$ the receiver feeds to the decision device the quantity $z_n(kT)$, which is obtained by sampling at the instant kT the output of the matched filter (with impulse response $\tilde{p}(-t)$) which is driven by $r(t)$.

Following a similar analysis, the resulting degradation D_n (in decibels) of the SNR at the input of the decision device is given by

$$D_n = 10 \log \left\{ 1 + \frac{E_{s,n}}{E[\,|W(0)|^2]} \sum_{m=-\infty}^{+\infty} \iint R_{\varphi_n}(u - v) \tilde{p}(u) \cdot \tilde{p}(v) \tilde{p}(u - mT) \tilde{p}(v - mT)\, du\, dv \right\} \quad (10)$$

where $R_{\varphi_n}(.)$ is the autocorrelation function of the phase noise (i.e., the inverse Fourier transform of the phase noise spectrum). When the bandwidth of the phase noise is much smaller than $1/T$, the degradation (10) reduces to (7), in which case OFDMA and traditional FDMA suffer the same degradation.

III. Numerical Results

Fig. 3 shows the degradation D_n from (7) as a function of $E_{s,n}/[|W_n(0)|^2]$ for different values of the phase jitter variance $\sigma_{\varphi_n}^2$. The degradation (7) holds for both OFDMA and OFDM, provided that all carriers have the same power level and the same jitter spectrum.

We now compare the degradations (7) and (10) for OFDMA and traditional FDMA, respectively. We have assumed that $\tilde{p}(t)$ is a square-root cosine rolloff pulse, and the jitter PSD

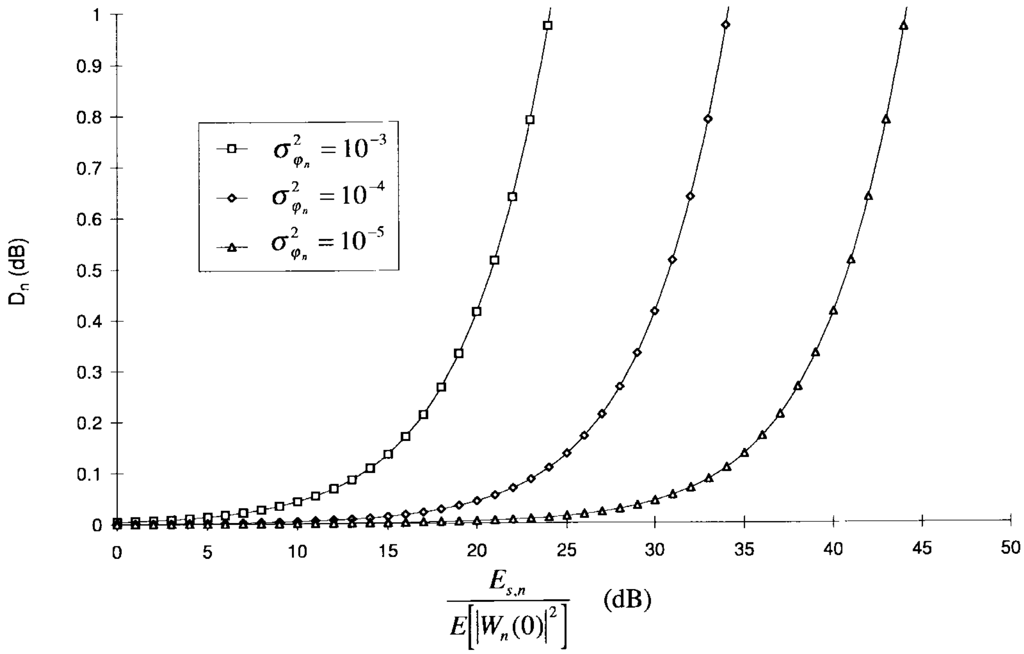

Fig. 3. Degradation for OFDMA.

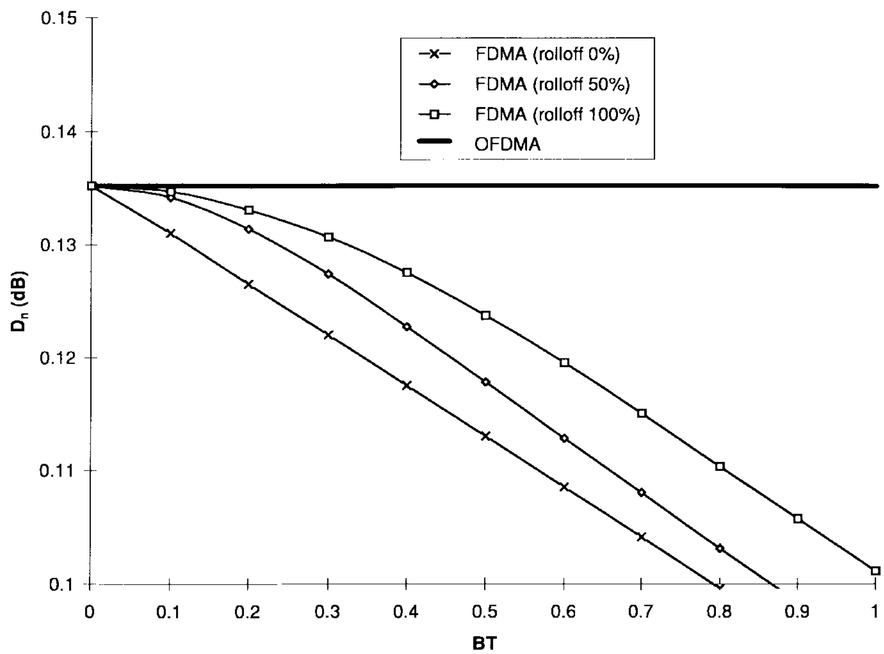

Fig. 4. Comparison of degradations for OFDMA and FDMA.

is given by

$$S_{\varphi_n}(f) = \begin{cases} \sigma^2_{\varphi_n} \dfrac{1}{B}\left(1 - \dfrac{|f|}{B}\right), & |f| < B \\ 0, & |f| \geq B \end{cases} \tag{11}$$

with B denoting the jitter bandwidth. For $\sigma^2_{\varphi_n} = 10^{-4}$ and $E_{s,n}/E[|W_n(0)|^2] = 25$ dB, Fig. 4 shows the degradations (7) and (10) as a function of the jitter bandwidth B, normalized to the symbol rate $1/T$. For OFDMA, the degradation does not depend on the shape of the jitter spectrum. For FDMA, the degradation decreases with increasing jitter bandwidth and with decreasing rolloff. When the jitter bandwidth is much smaller than the symbol rate, the degradation for FDMA converges to the degradation for OFDMA. When the jitter bandwidth is less than the symbol rate, we observe that FDMA is only slightly more robust against phase jitter than OFDMA.

IV. Conclusions and Remarks

In this letter we have investigated the degradation, caused by carrier phase jitter, of the SNR at the input of the decision device for OFDMA, OFDM, and traditional FDMA. Our results can be summarized as follows.

- Assuming that all carriers have the same power level and the same jitter spectrum, the degradation for OFDMA depends on the jitter variance but not on the shape of the jitter spectrum.
- The degradation for OFDMA is not affected by the correlation between the phase jitter processes on different carriers. As OFDM can be considered as OFDMA with identical phase jitter on the different carriers, the degradation for OFDM is the same as for OFDMA with the same jitter spectrum on all carriers.
- The degradation for traditional FDMA depends on the shape of the jitter spectrum. When the jitter bandwidth is much smaller than the symbol rate, the degradation for FDMA is essentially the same as for OFDMA. When the jitter bandwidth increases, the degradation for FDMA decreases. When the jitter bandwidth does not exceed the symbol rate, the degradation for FDMA is only slightly less than the degradation for OFDMA.

REFERENCES

[1] S. B. Weinstein and P. M. Ebert, "Data transmission by frequency division multiplexing using the discrete Fourier transform," *IEEE Trans. Commun.*, vol. COM-19, pp. 628–634, Oct. 1971.

[2] M. Alard and R. Lassalle, "Principles of modulation and coding for digital broadcasting for mobile receivers," *EBU Rev.*, no. 224, pp. 3–25, Aug. 1987.

[3] J. S. Chow, J. C. Tu, and J. M. Cioffi, "A discrete multitone transceiver for HDSL applications," *IEEE J. Select. Areas Commun.*, vol. 9, pp. 895–908, Aug. 1991.

[4] H. Sari, Y. Lévy, and G. Karam, "Orthogonal frequency-division multiple access for the return channel on CATV networks," in *Proc. Int. Conf. Telecommunications ICT'96*, Istanbul, Turkey, Apr. 1996, pp. 602–607.

[5] C. A. Eldering, N. Himayat, and F. M. Gardner, "CATV return path characterization for reliable communications," *IEEE Commun. Mag.*, vol. 33, pp. 62–69, Aug. 1995.

[6] T. Pollet, M. Moeneclaey, I. Jeanclaude, and H. Sari, "The effect of carrier phase jitter on single-carrier and multi-carrier QAM systems," in *Proc. IEEE Int. Conf. Communications ICC'95*, Seattle, WA, June 1995, pp. 1046–1050.