The Sensitivity of MC-CDMA to Synchronisation Errors

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Abstract. In this contribution, we investigate the sensitivity of MC-CDMA systems to synchronisation errors. We show that the signal-to-noise ratio (SNR) of the MC-CDMA system at the input of the decision device in the presence of a carrier frequency offset or a clock frequency offset is a rapidly decreasing function of the number of carriers. For a maximal load, carrier phase jitter and timing jitter give rise to a degradation that is independent of the spectral content of the jitter; moreover, the degradation caused by carrier phase jitter is independent of the number of carriers. A constant carrier phase offset and a constant timing offset cause no degradation of the MC-CDMA system performance.

1 INTRODUCTION

Recently, different combinations of orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA) have been investigated in the context of high data rate communication over dispersive channels [1-8]. One of these systems is multicarrier CDMA (MC-CDMA), which has been proposed for downlink communication in mobile radio. In MC-CDMA the data symbols are multiplied with a higher rate chip sequence and then modulated on orthogonal carriers.

The use of a large number of carriers makes a multicarrier system very sensitive to some types of synchronisation errors [9-13]. Synchronisation errors can be classified in two classes : carrier phase errors and timing errors. The influence of carrier phase errors between the carrier oscillators at transmitter and receiver, has been investigated in [9-10]. When using a free-running local oscillator, exhibiting a frequency offset and Wiener phase noise, simulations in [9] show that MC-CDMA performance rapidly degrades for an increasing number of carriers. In [10], a phase-locked local oscillator was used to get rid of a carrier frequency offset and of phase noise components that fall within the loop bandwidth of the PLL. For a maximal load, the MC-CDMA performance degradation caused by the resulting phase jitter was shown to be independent of the number of carriers and of the spectral content of the jitter.

The influence of sampling time errors on the performance of OFDM has been investigated in [11-13]. In [11-12], it was shown that for a large number of carriers, the OFDM system severely suffers from a clock frequency offset between the transmitter clock and the sampling clock of the receiver. In [12-13], it was proposed to use a phase-locked loop for the timing recovery, to eliminate the clock frequency offset. The timing jitter resulting from the PLL exhibits a performance degradation independent of the number of carriers.

In this contribution, we investigate the sensitivity of the MC-CDMA system to synchronisation errors by incorporating the carrier phase errors and timing errors in the end-to-end transfer function, obtaining a *time-varying* equivalent filter. This unified approach allows us to present a single analysis, valid for both carrier phase errors and timing errors. In section 2, we present the description of the MC-CDMA system in the presence of synchronisation errors. In section 3, we consider the sensitivity of MC-CDMA to a constant carrier phase offset, a carrier frequency offset and carrier phase jitter. In section 4, we focus on the influence of timing errors on the MC-CDMA performance, more specifically a constant timing offset, a clock frequency offset and timing jitter. Conclusions are drawn in section 5.

2 SYSTEM DESCRIPTION

The conceptual block diagram of a downlink MC-CDMA system is shown in figure 1. The data symbols $\{a_{i,m}\}$, transmitted to the user m during the ith symbol interval, are multiplied with the corresponding CDMA chip sequences $\{c_{n,m}|n=0,...,N-1\}$, $c_{n,m}$ denoting the nth chip of the sequence belonging to the user m, resulting in the samples $\{b_{i,n}|n=0,...,N-1\}$:

$$b_{i,n} = \sum_m \sqrt{E_{sm}}\, a_{i,m} c_{n,m} \qquad (1)$$

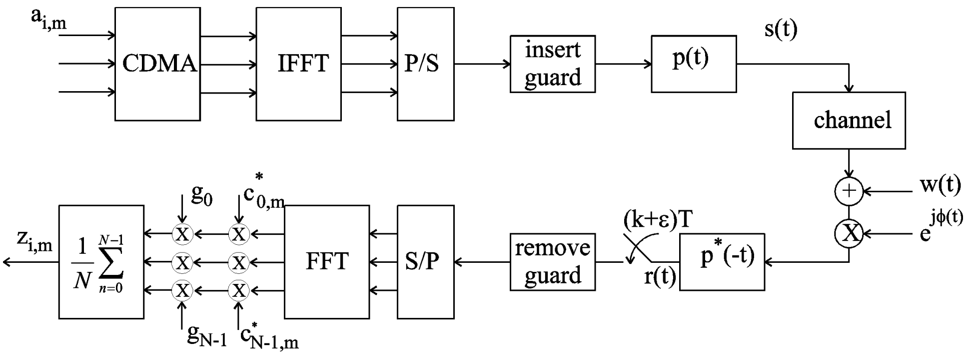

Figure 1 : MC-CDMA transceiver

where E_{sm} is the mth user's energy per symbol. Sequences belonging to different users are assumed to be orthogonal (e.g. Walsh-Hadamard codes). The sample $b_{i,n}$ is modulated on the nth carrier of a set of N orthogonal carriers, and all modulated carriers are summed to obtain the transmitted time-domain samples $s_{i,k}$:

$$s_{i,k} = \frac{1}{N}\sum_{n=0}^{N-1} b_{i,n} e^{j2\pi\frac{kn}{N}} \quad (2)$$

As in conventional OFDM, the intersymbol interference can be avoided by cyclically extending the transmitted signal with a guard interval νT : the transmitted samples during the ith symbol interval are $\{s_{i,k}|k=-\nu,...,N-1\}$, where the first ν samples are a duplication of the last ν samples, $\{s_{i,k}|k=-\nu,...,-1\}=\{s_{i,k}|k=N-\nu,...,N-1\}$. The resulting symbol rate per carrier is given by $R_{ca}=1/((N+\nu)T)$. The samples $s_{i,k}$ are applied to a transmit filter with transfer function $P(f)$, yielding the transmitted signal $s(t)$. The transmit filter is a unit energy square root Nyquist filter.

The transmitted signal is applied to the dispersive channel with channel transfer function $H_{ch}(f)$ and is disturbed by additive white Gaussian noise $w(t)$, with uncorrelated real and imaginary parts, each having a power spectral density of $N_0/2$. At the receiver, the signal disturbed by a carrier phase error $\phi(t)$ is applied to the receiver filter, which is matched to the transmit filter. The resulting signal $r(t)$ is sampled at the instants $t_{i,k}=i(N+\nu)T+kT+\varepsilon_{i,k}T$, $\varepsilon_{i,k}$ denoting the normalised timing error of the kth sample in the ith symbol interval ($|\varepsilon_{i,k}|<1/2$). When the carrier phase error is slowly varying as compared to T, the samples $r_{i,k}$ can be written as :

$$r_{i,k} = \sum_{m} h_{eq}\left(kT - mT; t_{i,k}\right) s_{i,m} + w_{i,k} \quad (3)$$

In (3), $w_{i,k}$ is the value of the matched filter output noise at the instant $t_{i,k}$; and $h_{eq}(t;t_{i,k})$ is an impulse response of an equivalent time-varying filter; its Fourier transform (with respect to the variable t) $H_{eq}(f;t_{i,k})$ is given by :

$$H_{eq}\left(f; t_{i,k}\right) = e^{j\phi_{i,k}} H\left(f\right) e^{j2\pi f\varepsilon_{i,k}T} \quad (4)$$

where $H(f)$ consists of the cascade of the transmit filter, the channel transfer function and the receiver filter, i.e. $H(f)=|P(f)|^2H_{ch}(f)$, and $\phi_{i,k}$ is the carrier phase error at the instant $t_{i,k}$. Assuming the duration of $h_{eq}(t;t_{i,k})$ does not exceed the duration of the guard interval, the transients at the edges of a symbol will affect the signal only during the guard interval, such that the adjacent symbols will have no influence on the observed symbol. It can be verified that the samples $r_{i,k}$ *outside* the guard interval are given by :

$$r_{i,k} = \frac{1}{N}\sum_{n=0}^{N-1} b_{i,n} H_n\left(t_{i,k}\right) e^{j2\pi\frac{nk}{N}} + w_{i,k} \qquad k = 0,..., N-1 \quad (5)$$

where

$$H_n\left(t_{i,k}\right) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} H_{eq}\left(\frac{n}{NT} + \frac{m}{T}; t_{i,k}\right) \quad (6)$$

The receiver selects the N samples $r_{i,k}$ outside the guard interval for further processing and disregards the other ν samples. The selected samples are demodulated using a fast Fourier transform (FFT); the nth output of the FFT is multiplied with the chip $c_{n,m}$ of the considered user m and applied to a (possibly time-varying) one-tap equaliser with coefficient $g_{n,i}$, with the index i referring to the considered symbol interval. The one-tap equaliser rotates and scales the corresponding FFT output. The resulting samples are summed yielding the samples at the input of the decision device :

$$\begin{aligned} z_{i,m} &= \frac{1}{N}\sum_{n=0}^{N-1} g_{n,i} c^{*}_{n,m} \sum_{k=0}^{N-1} r_{i,k} e^{-j2\pi\frac{kn}{N}} \\ &= \sqrt{E_{sm}}\, a_{i,m} I_{i,m,m} + \sum_{\ell\neq m} \sqrt{E_{s\ell}}\, a_{i,\ell} I_{i,\ell,m} + W_{i,m} \end{aligned} \quad (7)$$

where $W_{i,m}$ is a zero-mean complex-valued Gaussian noise term and

$$I_{i,\ell,m} = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{n,n'=0}^{N-1} g_{n',i} c_{n,\ell} c^{*}_{n',m} e^{j2\pi\frac{k(n-n')}{N}} H_n\left(t_{i,k}\right) \quad (8)$$

For $\ell\neq m$, the quantity $I_{i,\ell,m}$ denotes the multi-user interference (MUI) at the decision device input of the mth user, originating from the ℓth user during the ith symbol interval. The equaliser affects the useful component, the MUI and the statistics of the Gaussian noise. The equaliser coefficients are selected to maximise the signal-to-noise ratio at the input of the decision device.

When the carrier phase error or the timing error can be modelled as random processes, the quantities $I_{i,\ell,m}$ are random variables. The useful component can be decomposed as

$$I_{i,m,m} = E\left[I_{i,m,m}\right] + \left(I_{i,m,m} - E\left[I_{i,m,m}\right]\right) \tag{9}$$

where the first contribution denotes the average useful component and the second term the fluctuation of the useful component. Defining the signal-to-noise ratio (SNR) at the input of the decision device as the ratio of the power of the average useful component to the power of the remaining components, the SNR of the mth user is given by

$$SNR_m = E_{sm}\left|E\left[I_{i,m,m}\right]\right|^2 \Big\{ E\left[\left|W_{i,m}\right|^2\right] + E_{sm}E\left[\left|I_{i,m,m} - E\left[I_{i,m,m}\right]\right|^2\right] + \sum_{\ell \neq m} E_{s\ell} E\left[\left|I_{i,\ell,m}\right|^2\right]\Big\}^{-1} \tag{10}$$

In the absence of synchronisation errors, the equivalent filter $H_{eq}(f;t_{i,k})$ reduces to

$$H_{eq}\left(f;t_{i,k}\right) = H(f) \tag{11}$$

and the quantities $I_{i,\ell,m}$ yield

$$I_{i,\ell,m} = \frac{1}{N}\sum_{n=0}^{N-1} c_{n,\ell}\, g_{n,i} H_n c_{n,m}^* \tag{12}$$

where H_n is related to $H(f)$ through (6). The equaliser that maximises the resulting SNR at the input of the decision device is determined by

$$g_{n,i} = C\frac{H_n^*}{1 + \left(\frac{1}{N-1}\sum_{\ell \neq m}\frac{E_{s\ell}}{N_0}\right)\left|H_n\right|^2} \tag{13}$$

where C is an arbitrary constant. If we select

$$C = \left(\frac{1}{N}\sum_{n=0}^{N-1}\frac{\left|H_n\right|^2}{1 + \left(\frac{1}{N-1}\sum_{\ell \neq m}\frac{E_{s\ell}}{N_0}\right)\left|H_n\right|^2}\right)^{-1} \tag{14}$$

the useful signal component is normalised to one. Limiting values are $g_{n,i}=H_n^*$ (for $N_0 \rightarrow \infty$) and $g_{n,i}=1/H_n$ (for $N_0 \rightarrow 0$). In the former case, the power of the additive noise (which is the main disturbance) is minimised; in the latter case, the power of the MUI (which is the main disturbance) is made equal to zero. Hence, the coefficients (13) are a compromise between minimising the noise power and MUI power.

In the case of an ideal channel ($H_{ch}(f)=1$) and in the absence of synchronisation errors, the equivalent filter becomes

$$H_{eq}\left(f;t_{i,k}\right) = \left|P(f)\right|^2 \tag{15}$$

As the ideal channel yields $H_n=1$, no scaling or rotation of the FFT outputs by the equaliser are necessary (e.g. $g_{n,i}=1$); the channel introduces no attenuation of the useful component nor any multi-user interference. In the case of an ideal channel and in the absence of synchronisation errors, (10) yields $SNR_m=E_{sm}/N_0$. Hence, the degradation caused by a non-ideal channel and/or in the presence of synchronisation errors is defined as $D_m=10\log((E_{sm}/N_0)/SNR_m)$ (dB).

3 Carrier Phase Errors

In this section, we investigate the sensitivity of the MC-CDMA system to carrier phase errors in the absence of timing errors. In downstream communication, all transmitted carriers exhibit the same carrier phase error, as they are up-converted by the same oscillator. In this case, (6) and (4) yield

$$H_n\left(t_{i,k}\right) = e^{j\phi_{i,k}} H_n \tag{16}$$

In the following, we separately consider a constant carrier phase offset, a carrier frequency offset and carrier phase jitter.

3.1 Constant Carrier Phase Offset

In the case of a constant carrier phase offset $\phi(t)=\phi$, (16) reduces to

$$H_n\left(t_{i,k}\right) = e^{j\phi} H_n \tag{17}$$

It can be verified that the outputs of the FFT are rotated over an angle ϕ as compared to the case of a zero carrier phase offset. As a rotation of the FFT outputs has no influence on the noise power, a constant phase offset can be compensated without loss of performance, by an additional rotation of the outputs of the FFT over (an estimate of) $-\phi$, i.e. $g_{n,i}(\phi)=g_{n,i}(\phi=0)e^{-j\phi}$.

3.2 Carrier Frequency Offset

A frequency offset ΔF between the carrier oscillators at the transmitter and the receiver yields a carrier phase error $\phi(t)=2\pi\Delta Ft$. For a constant carrier frequency offset, the quantities $I_{i,\ell,m}$ are deterministic, so there is no fluctuation of the useful component. Assuming a slowly varying phase error as compared to T, i.e. $\Delta FT<<1$, the quantities $I_{i,\ell,m}$ are given by

$$I_{i,\ell,m} = \frac{1}{N}\sum_{n,n'=0}^{N-1} c_{n,\ell} H_n G\left(\frac{n-n'}{N}+\Delta FT\right) e^{j2\pi\Delta FTi(N+\nu)} g_{n',i} c^*_{n',m} \tag{18}$$

where

$$G(x) = \frac{1}{N}\sum_{k=0}^{N-1} e^{j2\pi kx} \tag{19}$$

In the following, we consider the case of an ideal channel, so the degradation originates only from the carrier frequency offset. Assuming the receiver can estimate the carrier frequency offset ΔF, the equaliser that maximises the SNR (10) is determined by $g_{n,i}=\exp(-j2\pi\Delta FTi(N+\nu))/G(\Delta FT)$: the equaliser eliminates the attenuation and rotation of the useful component at the FFT outputs. Under these assumptions, the useful power, the multi-user interference power and the noise power are given by

$$\begin{aligned} &E_{sm}\left| E\left[I_{i,m,m}\right]\right|^2 = E_{sm} \\ &\sum_{\ell\neq m} E_{s\ell} E\left[\left|I_{i,\ell,m}\right|^2\right] = \frac{1}{N-1}\sum_{\ell\neq m} E_{s\ell}\left(\frac{1}{\left|G(\Delta FT)\right|^2}-1\right) \\ &E\left[\left|W_{i,m}\right|^2\right] = \frac{N_0}{\left|G(\Delta FT)\right|^2} \end{aligned} \tag{20}$$

We observe in (20) that the equaliser is not able to eliminate the MUI ($|I_{i,\ell,m}|\neq 0$). As $|G(\Delta FT)|<1$ for $\Delta F\neq 0$, the equaliser yields an increase of the noise power. Assuming all users have the same energy per symbol E_s and the number of users is M, it follows from (15) and (20) that all users exhibit the same SNR and degradation

$$\begin{aligned} D_m &= -10\log\left|\frac{\sin \pi N\Delta FT}{N\sin \pi\Delta FT}\right|^2 \\ &\quad +10\log\left(1+\frac{E_s}{N_0}\frac{M-1}{N-1}\left(1-\left|\frac{\sin \pi N\Delta FT}{N\sin \pi\Delta FT}\right|^2\right)\right) \end{aligned} \tag{21}$$

In figure 2, the degradation (21) is shown for the maximum load, i.e. $M=N$. The degradation shown in figure 2 yields an upper bound for the degradation for $M<N$ users. We observe a high sensitivity of the MC-CDMA system to the carrier frequency offset. In order to obtain small degradations, only small frequency offsets are allowed, i.e. $\Delta F<<1/NT$.

3.3 CARRIER PHASE JITTER

In order to get rid of the frequency offset, a phase-locked local oscillator can be used for IF to baseband conversion. The phase-locked loop (PLL) also eliminates the phase noise components that fall within the bandwidth f_A of the PLL. The residual phase jitter can be modelled as a zero-mean stationary process with jitter spectrum $S_\phi(f)$ (which is typically of the shape shown in figure 3) and jitter variance σ_ϕ^2. Assuming slowly varying phase jitter, the bandwidth f_B of the jitter spectrum $S_\phi(f)$ needs to be $f_BT<<1$. The quantities $I_{i,\ell,m}$ are stationary processes and, assuming small jitter variances, i.e. $\sigma_\phi^2<<1$, they can be approximated by

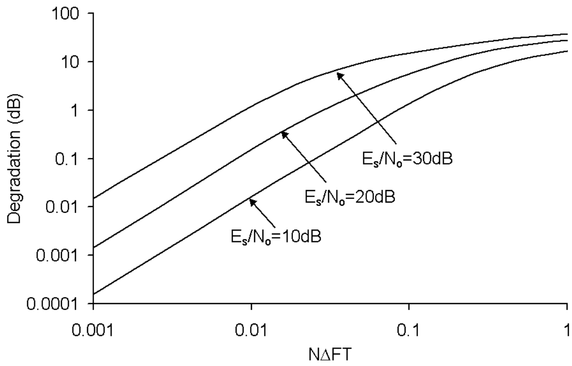

Figure 2 : Carrier frequency offset, M=N

$$I_{i,\ell,m} \cong \frac{1}{N}\sum_{n,n'=0}^{N-1} c_{n,\ell} H_n g_{n',i} c^*_{n',m} \left(\delta_{n,n'} + \frac{1}{N}\sum_{k=0}^{N-1} j\phi(kT) e^{j2\pi\frac{k(n-n')}{N}}\right) \tag{22}$$

For small jitter variances, the equaliser coefficients are essentially the same as in the absence of synchronisation errors (see (13)). Restricting our attention to an ideal channel ($H_n=1$), we obtain $g_{n,i}=1$. The resulting powers of the average useful component, the fluctuation of the useful component, the multi-user interference and the noise are given by

$$\begin{aligned} &E_{sm}\left| E\left[I_{i,m,m}\right]\right|^2 = E_{sm} \\ &E_{sm}E\left[\left| I_{i,m,m} - E\left[I_{i,m,m}\right]\right|^2\right] = E_{sm}\int_{-\infty}^{+\infty} S_\phi(f)\left|G(fT)\right|^2 df \\ &\sum_{\ell\neq m} E_{s\ell}E\left[\left| I_{i,\ell,m}\right|^2\right] = \frac{1}{N-1}\sum_{\ell\neq m} E_{s\ell}\int_{-\infty}^{+\infty} S_\phi(f)\left(1-\left|G(fT)\right|^2\right) df \\ &E\left[\left| W_{i,m}\right|^2\right] = N_0 \end{aligned} \tag{23}$$

From (23) and figure 4 it follows that the fluctuation of the useful component and the multi-user component mainly consists of the low frequency components ($<1/NT$) and the high frequency components ($>1/NT$) of the jitter spectrum, respectively. For the highest load ($M=N$) and all users exhibiting the same energy per symbol E_s, the sum of the fluctuation of the useful component and the multi-user component is independent of the spectral content of the jitter spectrum and of the number of OFDM tones, and only depends on the jitter variance, given by

$$\sigma_\phi^2 = \int_{-\infty}^{+\infty} S_\phi(f)df \qquad (24)$$

For N active users, the degradation is given by $D_m=10\log(1+(E_{sm}/N_0)\sigma_\phi^2)$, shown in figure 5, yields an upper bound for $M<N$ users.

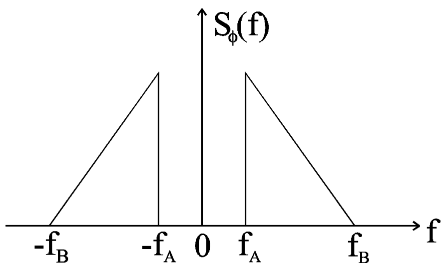

Figure 3 : Typical jitter spectrum

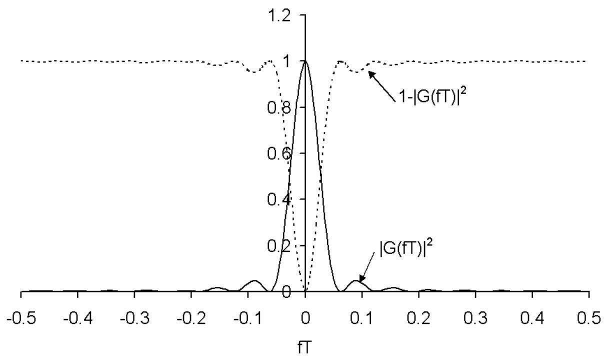

Figure 4 : The weight functions |G(fT)|^2 *and* 1-|G(fT)|^2, *N=16*

4 TIMING ERRORS

In this section, we investigate the sensitivity of the MC-CDMA system to timing errors. The deviation from the correct sampling instant by an amount $\varepsilon_{i,k}T$ influences the quantities $H_n(t_{i,k})$ as follows

$$H_n(t_{i,k}) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} H\left(\frac{n}{NT}+\frac{m}{T}\right) e^{j2\pi\varepsilon_{i,k}\left(\frac{n}{N}+m\right)} \qquad (25)$$

In downstream communication, all carriers exhibit the same timing error as the signals are synchronised at the base station. In the following, we consider a constant timing offset, a clock frequency offset and timing jitter in the case of an ideal channel ($H_{ch}(f)=1$); the transmit and receiver filters are square root raised cosine filters with rolloff α.

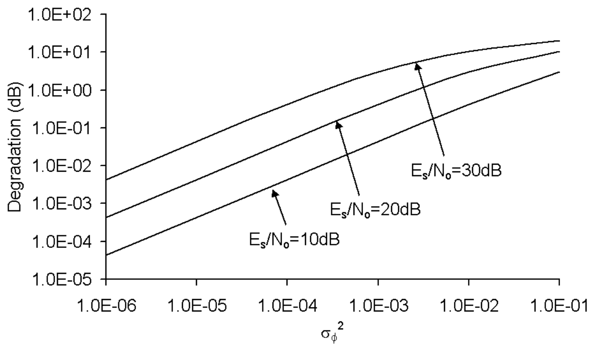

Figure 5 : Carrier phase jitter, M=N

4.1 CONSTANT TIMING OFFSET

In the case of a constant timing offset $\varepsilon_{i,k}=\varepsilon$, the quantities $I_{i,\ell,m}$ are given by

$$I_{i,\ell,m} = \frac{1}{N}\sum_{n=0}^{N-1} c_{n,\ell} H_n(t_{i,k}) g_{n,i} c_{n,m}^* \qquad (26)$$

The equivalent filter is bandwidth-limited : $H_{eq}(f;0)=0$, $|f|>(1+\alpha)/2T$, $0\leq\alpha\leq1$. For frequencies n/T outside the rolloff area, the sum (26) reduces to one contribution

$$H_n(t_{i,k}) = H_n e^{j2\pi\varepsilon\frac{\mathrm{mod}(n;N)}{N}} \qquad \frac{n}{T} \notin \left(\frac{1-\alpha}{2T},\frac{1+\alpha}{2T}\right) \qquad (27)$$

where $mod(x;N)$ denotes the modulo-N reduction of x, yielding a result in $(-N/2,N/2)$. For frequencies n/T inside the rolloff area, the sum (26) consists of two contributions for which it can be verified that

$$\left|H_n(t_{i,k})\right| < \left|H_n\right| \qquad \frac{n}{T} \in \left(\frac{1-\alpha}{2T},\frac{1+\alpha}{2T}\right) \qquad (28)$$

In figure 6, we observe that for frequencies n/T outside the rolloff area, the nth output of the FFT exhibits a constant amplitude $|H_n|=1$ and is rotated over an angle $2\pi\varepsilon mod(n;N)/N$. For frequencies n/T inside the rolloff area, the outputs of the FFT are rotated over some angle and the amplitudes are less than 1.

The equaliser multiplies the outputs of the FFT with

$$g_{n,i} = C\frac{H_n^*(t_{i,k})}{1+\left(\frac{1}{N-1}\sum_{\ell\neq m}\frac{E_{s\ell}}{N_0}\right)\left|H_n(t_{i,k})\right|^2} \qquad (29)$$

where the normalisation constant is given by

$$C=\left(\frac{1}{N}\sum_{n=0}^{N-1}\frac{\left|H_n\left(t_{i,k}\right)\right|^2}{1+\left(\frac{1}{N-1}\sum_{\ell\neq m}\frac{E_{s\ell}}{N_0}\right)\left|H_n\left(t_{i,k}\right)\right|^2}\right)^{-1} \tag{30}$$

This equalisation performs a rotation over an (estimate of the) angle $-2\pi\varepsilon mod(n;N)/N$ and a carrier-independent scaling for FFT outputs corresponding to carriers outside the rolloff area, and a carrier-dependent scaling and rotation for FFT outputs corresponding to carriers inside the rolloff area. The equaliser compensates for the attenuation and rotation of the FFT outputs, yielding the degradation

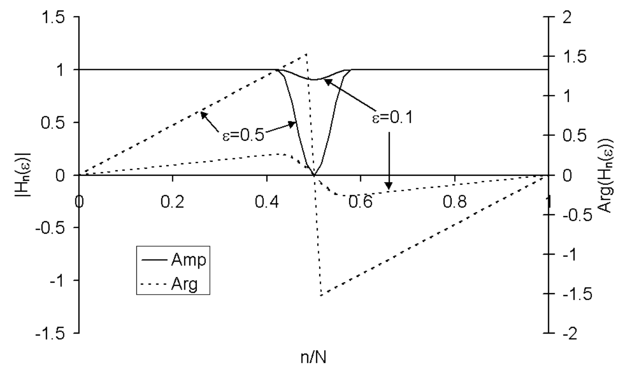

Figure 6 : Dependence of $|H_n(\varepsilon)|$ and $Arg(H_n(\varepsilon))$ on the carrier index n, α=0.15

$$\begin{aligned}D_m=&10\log\left(\frac{1}{N}\sum_{n=0}^{N-1}\frac{1}{1+\frac{1}{N-1}\sum_{\ell\neq m}\frac{E_{s\ell}}{N_0}\left|H_n\left(t_{i,k}\right)\right|^2}\right)\\&-10\log\left(\frac{1}{N}\sum_{n=0}^{N-1}\frac{\left|H_n\left(t_{i,k}\right)\right|^2}{1+\frac{1}{N-1}\sum_{\ell\neq m}\frac{E_{s\ell}}{N_0}\left|H_n\left(t_{i,k}\right)\right|^2}\right)\end{aligned} \tag{31}$$

Considering (27) and (28), (31) yields a non-zero degradation as compared to the case of a zero timing offset. For $\alpha=0$, a constant timing offset introduces no degradation. For $\alpha\neq 0$, the sensitivity of the MC-CDMA system to constant timing offsets can be eliminated by not using the carriers in the rolloff area. When using $N_c=(1-\alpha)N$ carriers, N FFT-blocks are needed for the transmission of the N_cN chips corresponding to N_c successive symbols. In the following, performance degradations will be investigated for N used carriers in the case $\alpha=0$, so that all these N carriers are outside the rolloff area.

4.2 Clock Frequency Offset

When sampling is performed by means of a free-running clock with a relative clock frequency offset of $\Delta T/T$, the normalised timing error is given by $\varepsilon_{i,k}=(k+i(N+\nu))\Delta T/T$. Assuming $\Delta T/T<<1$ the quantities $I_{i,\ell,m}$ are given by

$$\begin{aligned}I_{i,\ell,m}=&\frac{1}{N}\sum_{n,n'=0}^{N-1}c_{n,\ell}\,\frac{1}{T}\sum_{k=-\infty}^{+\infty}H\left(\frac{n}{NT}+\frac{k}{T}\right)\\&G\left(\frac{n-n'}{N}+\left(\frac{n}{N}+k\right)\frac{\Delta T}{T}\right)e^{j2\pi\left(\frac{n}{N}+k\right)i(N+\nu)\frac{\Delta T}{T}}g_{n',i}c_{n',m}^*\end{aligned} \tag{32}$$

where $G(x)$ is defined in (19). The deterministic character of the clock frequency offset causes no fluctuation of the useful power but only yields an attenuation of the useful power and gives rise to multi-user interference.

For $\alpha=0$, the quantities $I_{i,\ell,m}$ reduce to

$$\begin{aligned}I_{i,\ell,m}=&\frac{1}{N}\sum_{n,n'=0}^{N-1}c_{n,\ell}G\left(\frac{n-n'}{N}+\frac{\operatorname{mod}(n;N)}{N}\frac{\Delta T}{T}\right)\\&e^{j2\pi\frac{\operatorname{mod}(n;N)}{N}i(N+\nu)\frac{\Delta T}{T}}g_{n',i}c_{n',m}^*\end{aligned} \tag{33}$$

Assuming each user exhibits the same energy per symbol E_s, the degradation of the SNR at the input of the decision device is user independent and is shown in figure 7. It follows that the MC-CDMA system performance strongly depends on the clock frequency offset. In order to obtain small degradations, we need a small relative frequency offset $\Delta T/T<<1/N$. We can avoid the degradation of the MC-CDMA system caused by a static clock frequency offset by using a PLL-like timing correction mechanism in front of the FFT.

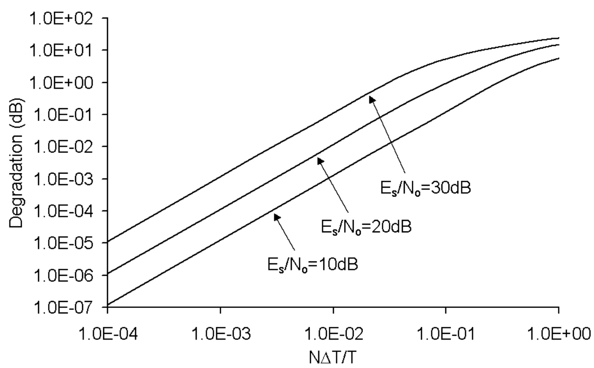

Figure 7 : Clock frequency offset, M=N

4.3 TIMING JITTER

In order to get rid of a constant timing offset and a clock frequency offset, we can perform synchronised sampling, e.g. by means of a PLL. The normalised timing error $\varepsilon_{i,k}$ resulting from the PLL can be modelled as zero-mean stationary Gaussian noise with spectrum $S_\varepsilon(f)$ and jitter variance σ_ε^2. Assuming slowly varying timing jitter, the bandwidth f_B of the jitter spectrum is limited, i.e. $f_BT<<1$. For small variances $\sigma_\varepsilon^2<<1$, we obtain the quantities

$$I_{i,\ell,m} \cong \frac{1}{N}\sum_{n,n'=0}^{N-1} c_{n,\ell}\, g_{n'} c^*_{n',m} \left(\delta_{nn'} + j2\pi\tilde{H}_n \frac{1}{N}\sum_{k=0}^{N-1}\varepsilon_{i,k} e^{j2\pi\frac{k(n-n')}{N}}\right) \quad (34)$$

where

$$\tilde{H}_n = \frac{1}{T}\sum_{m=-\infty}^{+\infty} H\left(\frac{n}{NT}+\frac{m}{T}\right)\left(\frac{n}{N}+m\right) \quad (35)$$

from which it follows that for $\ell=m$ the useful component consists of an average useful component and a fluctuation about its average; for $\ell\neq m$, (34) indicates the presence of MUI. For $\sigma_\varepsilon^2<<1$, the equaliser is essentially the same as for zero timing jitter, i.e. $g_{n,i}=1$. When all users have the same jitter spectrum $S_\varepsilon(f)$ and the same energy per symbol E_s, the powers of the average useful component, the fluctuation of the useful component and the multi-user interference become

$$\begin{aligned}
&E_{sm}\left|E\left[I_{i,m,m}\right]\right|^2 = E_s \\
&E_{sm}E\left[\left|I_{i,m,m}-E\left[I_{i,m,m}\right]\right|^2\right] = (2\pi)^2 E_s \left|\sum_{n=0}^{N-1}\frac{1}{N}\tilde{H}_n\right|^2 \int_{-\infty}^{+\infty} S_\varepsilon(f)\left|G(fT)\right|^2 df \\
&\sum_{\ell\neq m} E_{s\ell}E\left[\left|I_{i,\ell,m}\right|^2\right] = \frac{M-1}{N-1}(2\pi)^2 E_s \left\{N\sum_{n=0}^{N-1}\left|\frac{1}{N}\tilde{H}_n\right|^2 \int_{-\infty}^{+\infty} S_\varepsilon(f)df - \left|\sum_{n=0}^{N-1}\frac{1}{N}\tilde{H}_n\right|^2 \int_{-\infty}^{+\infty} S_\varepsilon(f)\left|G(fT)\right|^2 df\right\} \\
&E\left[\left|W_{i,k}\right|^2\right] = N_0
\end{aligned} \quad (36)$$

From (36) it follows that the sum of the powers of the fluctuation of the useful component and the multi-user interference linearly increases with the number of users M. For the maximal load M=N, the sum of the powers is independent of the spectral contents of the jitter but only depends on the jitter variance σ_ε^2

$$\sigma_\varepsilon^2 = \int_{-\infty}^{+\infty} S_\varepsilon(f)df \quad (37)$$

For the maximal load $M=N$, for large N ($N\rightarrow\infty$) and for $\alpha=0$, the powers of the fluctuation of the useful component and the MUI (36) reduce to

$$\begin{aligned}
&E_{sm}E\left[\left|I_{i,m,m}-E\left[I_{i,m,m}\right]\right|^2\right] = 0 \\
&\sum_{\ell\neq m} E_{s\ell}E\left[\left|I_{i,\ell,m}\right|^2\right] = E_s\frac{\pi^2}{3}\sigma_\varepsilon^2
\end{aligned} \quad (38)$$

which indicates that for large N the degradation becomes essentially independent of the number of carriers and originates from the MUI only. The degradation of the SNR, resulting from (38), is shown in figure 8.

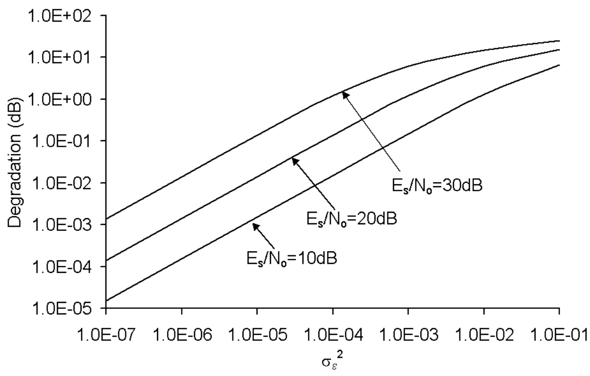

Figure 8 : Timing jitter, M=N

5 CONCLUSIONS

In this contribution, we have investigated the effect of synchronisation errors on the performance of a MC-CDMA system. A constant phase offset and a constant timing offset can be compensated without loss of performance. The MC-CDMA performance degrades for time-varying timing and carrier phase errors. In the case of a carrier frequency offset or a clock frequency offset, the MC-CDMA performance rapidly degrades when the number of carriers increases. The sensitivity of the MC-CDMA system to a carrier frequency offset or a clock frequency offset can be avoided by using phase-locked loops for carrier and timing recovery, yielding a residual carrier phase jitter and timing jitter. For the maximum load, the degradations caused by this carrier phase jitter and timing jitter are independent of the spectral content of the jitter, and essentially independent of the number of carriers. Table I summarises the effect of the various synchronisation errors on the MC-CDMA performance for the maximal load, i.e. $M=N$. In order to avoid the performance degradation caused by a fixed timing offset, we have proposed not to use the carriers in the rolloff area. In this case, each of the FFT outputs experiences less interference than when all carriers are used. Consequently, the resulting degradations caused by carrier frequency offset, carrier phase jitter, clock frequency offset and timing jitter are expected to be slightly less than the degradations presented in Figures 2,5,7 and 8, respectively.

Table I :Effect of synchronisation errors

constant carrier phase offset	-no degradation
constant timing offset	-no degradation
carrier frequency offset	-strong degradation, increasing with N -degradation avoided by correction in front of FFT
clock frequency offset	-strong degradation, increasing with N -degradation avoided by correction in front of FFT
carrier phase jitter	-degradation independent of N and spectral contents jitter
timing jitter	-degradation independent of spectral contents jitter -degradation independent of N for large N

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