The Sensitivity of Downlink MC-DS-CDMA to Carrier Frequency Offsets

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***Abstract*—In this letter, we investigate the sensitivity of a downlink MC-DS-CDMA system to carrier frequency offsets. It is shown that the resulting interference power depends only on the ratio of the frequency offset to the carrier spacing. Hence, for given frequency offset and system bandwidth, the MC-DS-CDMA performance degradation is independent of the spreading factor but increases with the number of carriers; equivalently, for given frequency offset and symbol rate, the performance degradation increases with the ratio of the number of carriers to the spreading factor.**

***Index Terms*—Carrier frequency offset, MC-DS-CDMA, multicarrier systems.**

I. Introduction

RECENTLY, some new techniques for high data rate communications, based on a combination of multicarrier modulation and CDMA, were proposed [1]. One of these techniques is multicarrier direct-sequence CDMA (MC-DS-CDMA), which is suitable for mobile radio [2].

The presence of a frequency offset between the upconverting oscillator at the transmitter and the downconverting oscillator at the receiver gives rise to a performance degradation. In [3]–[5], it has been reported that multicarrier techniques like orthogonal frequency-division multiplexing (OFDM) and multicarrier CDMA (MC-CDMA) strongly degrade in the presence of a carrier frequency offset when the number of carriers increases. In this contribution, we investigate the effect of a carrier frequency offset on downlink MC-DS-CDMA, and determine the resulting performance degradation in terms of the system parameters.

II. System Description

The conceptual block diagram of a downlink multicarrier direct-sequence CDMA (MC-DS-CDMA) system is shown in Fig. 1. The symbol sequence to be transmitted at rate R_s to user ℓ is split into N_c symbol sequences, each having rate R_s/N_c and each modulating a different carrier of the multicarrier system. We denote by $a_{i,k,\ell}$ the ith symbol sent on the kth carrier to user ℓ, with k belonging to a set I_s of N_c carrier indices. The data symbol $a_{i,k,\ell}$ is multiplied with a spreading sequence $\{c_{i,n,\ell}|n = 0, \ldots, N_s - 1\}$ with spreading factor N_s, where $c_{i,n,\ell}$ is the nth chip of the sequence that spreads the symbol $a_{i,k,\ell}$. Note that the spreading sequences are independent of the carrier index k. The N_s components of the spread data symbol $a_{i,k,\ell}$, i.e., $\{a_{i,k,\ell}c_{i,n,\ell}|n = 0, \ldots, N_s - 1\}$, are transmitted *serially* on the kth carrier of an orthogonal multicarrier system. (This is in contrast with MC-CDMA, where the chips corresponding to the same data symbol are transmitted *in parallel* over the different carriers.) The modulation of the spread data symbols on the orthogonal carriers is accomplished by means of a N_F-point inverse fast Fourier transform (inverse FFT). To avoid that channel dispersion causes interference, each FFT block at the inverse FFT output is extended with a cyclic prefix of N_p samples, that exceeds the duration of the channel impulse response. The resulting samples are applied at a rate $1/T = (N_F + N_p)N_sR_s/N_c$ to a square-root raised-cosine transmit filter $P(f)$ with rolloff α. The carrier index k corresponds to a carrier frequency $\mathrm{mod}(k, N_F)F$, where $F = 1/(N_FT)$ denotes the carrier spacing, and $\mathrm{mod}(x; N_F)$ is the modulo-N_F reduction of x, yielding a result in the interval $[-N_F/2; N_F/2]$. In the following, we assume that carriers inside the rolloff area are not modulated, i.e., they have zero amplitude. Hence, of the N_F available carriers, only N_c carriers are actually used ($N_c \leq (1-\alpha)N_F$). Assuming N_c to be odd, the set of carrier indices actually used is given by $I_c = \{0, \ldots, (N_c-1)/2\} \cup \{N_F-(N_c-1)/2, \ldots, N_F-1\}$.

In a multiuser scenario, the basestation transmits the sum of N_u different user signals. To be able to extract the reference user signal at the mobile receiver, each user is assigned a unique spreading sequence. In this contribution, we consider orthogonal sequences, that are obtained by multiplying user-dependent Walsh–Hadamard sequences with a complex-valued random scrambling sequence that is common to all N_u users. The maximum number of users that can be supported equals the spreading factor N_s.

The sum of the N_u user signals reaches the receiver of the reference user ($\ell = 0$) through a dispersive channel with transfer function $H_{ch}(f)$. The output of the channel is affected by a carrier phase error $\phi(t) = 2\pi\Delta Ft + \phi(0)$, where ΔF denotes a small carrier frequency offset ($|\Delta FT| \ll 1$). Furthermore, the received signal is disturbed by additive white Gaussian noise $w(t)$, with uncorrelated real and imaginary parts, each having a power spectral density of $N_0/2$.

The resulting signal is applied to the receiver filter, which is matched to the transmit filter, and sampled at a rate $1/T$. The receiver removes from each cyclically extended FFT block the N_p samples corresponding to the cyclic prefix, and keeps the remaining N_F samples for further processing. This involves applying an FFT of length N_F, followed by one-tap equalizers $g_{i,k,n}$ that scale and rotate the kth FFT output during the nth FFT block in the ith symbol interval. Each equalizer output is multiplied with the corresponding chip of the reference user

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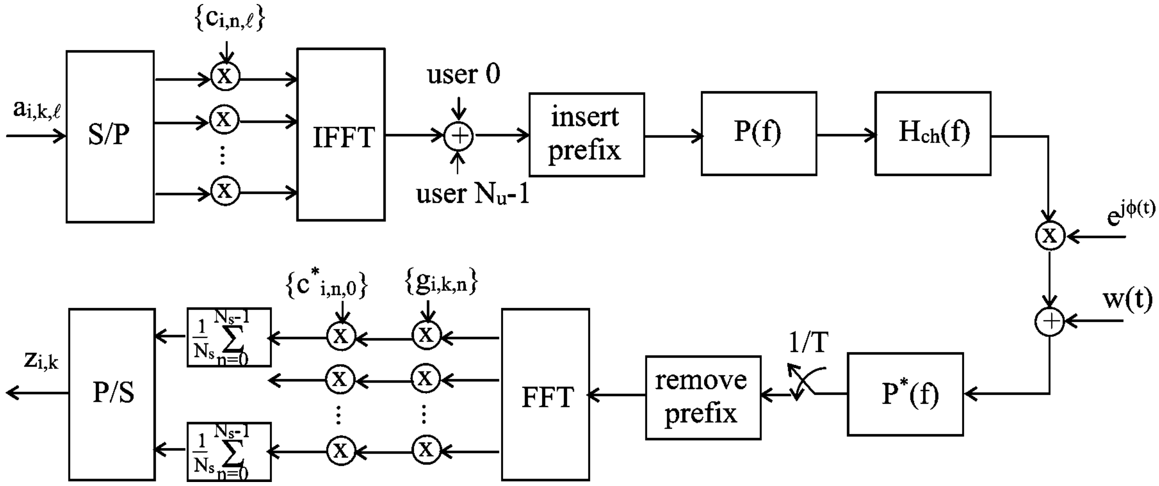

Fig. 1. Conceptual block diagram of a downlink MC–DS-CDMA system.

spreading sequence, and summed to obtain the decision variable $z_{i,\,k}$, used to detect the symbol $a_{i,\,k,\,0}$. This decision variable can be decomposed as

$$z_{i,\,k} = \sqrt{\frac{N_F}{N_F+N_P}} \sum_{\ell=0}^{N_u-1} \sum_{k' \in I_c} a_{i,\,k',\,\ell} I_{i,\,k,\,k',\,\ell} + W_{i,\,k}, \quad k \in I_c \tag{1}$$

where

$$I_{i,\,k,\,k',\,\ell} = \frac{H_{k'}}{N_s} \sum_{n=0}^{N_s-1} g_{i,\,k,\,n} c^*_{i,\,n,\,0} c_{i,\,n,\,\ell} A_{i,\,k,\,k',\,n} \tag{2}$$

$$A_{i,\,k,\,k',\,n} = e^{j\phi(0)} e^{j2\pi\Delta F(iN_s+n)(N_F+N_p)T} \cdot D\left(\frac{k'-k}{N_F} + \Delta FT\right) \tag{3}$$

$$D(x) = \frac{1}{N_F} \sum_{m=0}^{N_F-1} e^{j2\pi mx} = \frac{1-\exp(j2\pi N_F x)}{N_F(1-\exp(j2\pi x))}. \tag{4}$$

In (2), $H_{k'} = H(\mathrm{mod}(k'; N_F)/N_F T)/T$, where $H(f) = |P(f)|^2 H_{ch}(f)$. The quantity $I_{i,k,k',\ell}$ represents the contribution of the data symbol $a_{i,k',\ell}$ to the sample $z_{i,k}$ at the input of the decision device. The sample $z_{i,k}$ from (1) contains a useful contribution with coefficient $I_{i,k,k,0}$; the power of this contribution is denoted P_{U_k}. For $k' \neq k$, the quantities $I_{i,k,k',0}$ correspond to intercarrier interference from other symbols transmitted to the reference user, while for $\ell \neq 0$, the quantities $I_{i,k,k',\ell}$ correspond to multiuser interference. The sum of intercarrier and multiuser interference power is denoted P_{I_k}. The additive noise term $W_{i,\,k}$ has a power P_{N_k} given by

$$P_{N_k} = E[|W_{i,\,k}|^2] = N_0 \frac{1}{N_c} \sum_{n=0}^{N_c-1} |g_{i,\,k,\,n}|^2. \tag{5}$$

The equalizer coefficients are selected such that the coefficient $I_{i,\,k,\,k,\,0}$ of the useful component equals 1. This yields

$$g_{i,\,k,\,n} = \frac{\exp(-j\phi(0))\exp(-j2\pi\Delta FT(iN_s+n)(N_F+N_P))}{H_k D(\Delta FT)}. \tag{6}$$

The equalizer compensates the phase rotation and scaling of the useful component [assuming that accurate estimates of $\phi(0)$, ΔFT and H_k are available]. Substituting (6) into (2) reveals that the equalizer eliminates all multiuser interference ($I_{i,\,k,\,k',\,\ell} = 0$ for $\ell \neq 0$), but cannot eliminate the intercarrier interference ($I_{i,\,k,\,k',\,0} \neq 0$ for $k' \neq k$). Hence the interference power P_{I_k} is independent of the number N_u of active users.

III. Performance Degradation

We define the signal-to-noise ratio (SNR) at the input of the decision device as the ratio of the power of the useful component to the sum of the interference and noise powers, i.e., $\mathrm{SNR}_k(\Delta FT) = P_{U_k}/(P_{N_k} + P_{I_k})$. In the absence of a carrier frequency offset, we obtain $\mathrm{SNR}_k(0) = (E_{sk,\,0}/N_0)|H_k|^2(N_F/(N_F+N_P))$, where $E_{sk,\,0} = E[|a_{i,\,k,\,0}|^2]$ is the energy per symbol transmitted to the reference user on carrier k. The degradation (in decibels) caused by the frequency offset is then given by $\mathrm{Deg}_k = 10\log(\mathrm{SNR}_k(0)/\mathrm{SNR}_k(\Delta FT))$.

From the above equations, it is straightforward to derive an analytical expression for Deg_k in terms of ΔFT and $\{H_{k'}, E_{s_{k',0}} | k' \in I_c\}$. However, in order to clearly isolate the effect of the carrier frequency offset, we assume that the channel is ideal ($H_{k'} = 1$), and that the symbol energy is independent of the carrier index ($E_{sk,\,0} = E_s$ for $k \in I_c$). In this case the degradation is given by

$$\mathrm{Deg}_k = -10\log|D(\Delta FT)|^2 + 10\log\left(1+\mathrm{SNR}(0)\left(\sum_{k'\in I_c}\left|D\left(\frac{k'-k}{N_F}+\Delta FT\right)\right|^2 - |D(\Delta FT)|^2\right)\right) \tag{7}$$

with $\mathrm{SNR}(0) = (E_s/N_0)(N_F/(N_F+N_P))$ and $|D(x)| = |\sin(\pi N_F x)/(N_F \sin(\pi x))|$. The summation index k' in (7) ranges over the set I_c of the N_c *modulated* carriers. A simple upper bound on the degradation is obtained

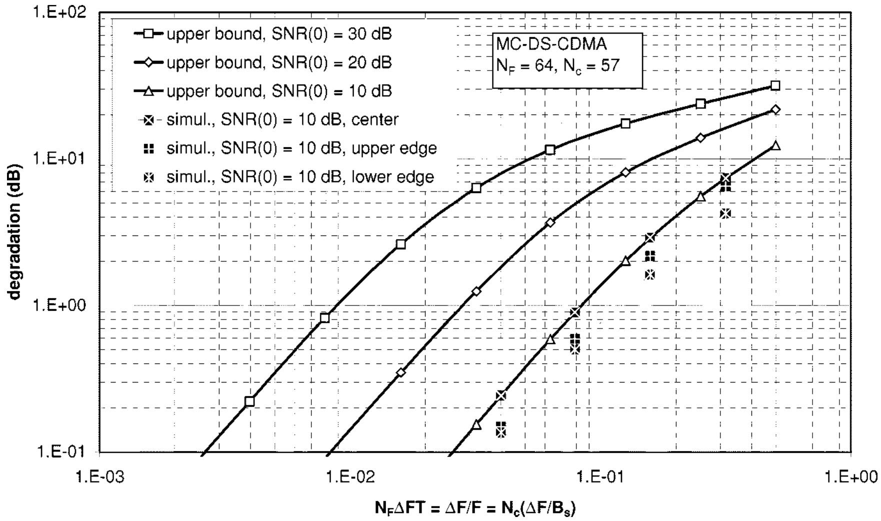

Fig. 2. Degradation caused by a carrier frequency offset ($H_k = 1,\ E_{sk,0} = E_s$).

by extending this summation over all N_F *available* carriers, i.e., $k' = 0, \ldots, N_F - 1$. This yields

$$\text{Deg} \leq -10\log|D(\Delta FT)|^2 + 10\log(1 + \text{SNR}(0)(1 - |D(\Delta FT)|^2)) \quad (8)$$

which is independent of the carrier index k. For $|\Delta FT| \ll 1$ we have $\sin(\pi\Delta FT) \cong \pi\Delta FT$, so that the upper bound (8) is only a function of SNR(0) and of $N_F\Delta FT = \Delta F/F = N_c(\Delta F/B_s)$, where $B_s = R_s N_s(1 + (N_P/N_F)) \cong R_s N_s$ denotes the system bandwidth. This is according to intuition, as the bandwidth of each individual carrier signal equals about F. The bound (8) is reached when all carriers are modulated $(N_s = N_F;\ \alpha = 0)$. For $\alpha > 0$, this upper bound yields an accurate approximation for the carriers near the center of the signal band. For given $\Delta F/B_s$, the degradation of the MC-DS-CDMA system is independent of the spreading factor N_s, but increases with the number of carriers N_c. Equivalently, for given $\Delta F/R_s$, the degradation increases with the ratio N_c/N_s.

Fig. 2 shows the bound (8), along with simulation results (assuming $N_F = 64,\ N_c = 57$) for the carriers at the center ($k = 0$) and at the upper ($k = 28$) and lower ($k = 36,\ \text{mod}\,(k,\,64) = -28$) edge of the band. We have verified that the result for $k = 0$ [which essentially coincides with the bound (8)] holds for 51 of the 57 carriers; this illustrates the importance of the simple bound (8). To obtain small degradations, it is required that $|\Delta F/F| \ll 1$, in which case the degradation is proportional to $(\Delta F/F)^2$. For BPSK (QPSK) transmission over an AWGN channel, $\text{SNR}(0) = 10$ dB yields $\text{BER} \cong 3 \times 10^{-6}$ (7×10^{-4}) when $\Delta F/F = 0$; for $\Delta F/F \cong 6 \times 10^{-2}$, or $\Delta F/B_s \cong 1 \times 10^{-3}$ when $N_c = 57$, the degradation is about 0.5 dB, yielding $\text{BER} \cong 1 \times 10^{-5}$ (1.5×10^{-3}).

The upper bound (8) for MC-DS-CDMA is exactly the same as the corresponding bounds for OFDM and MC-CDMA, derived in [3], [4] (with $1/T$ denoting for the three multicarrier systems the sampling rate at the receiver).

REFERENCES

[1] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Commun. Mag.*, vol. 35, pp. 126–133, Dec 1997.

[2] V. M. DaSilva and E. S. Sousa, "Performance of orthogonal CDMA sequences for quasisynchronous communication systems," in *Proc. IEEE ICUPC'93*, Ottawa, Canada, Oct. 1993, pp. 995–999.

[3] T. Pollet, M. Van Bladel, and M. Moeneclaey, "BER sensitivity of OFDM systems to carrier frequency offset and Wiener phase noise," *IEEE Trans. Commun.*, vol. 43, pp. 191–193, Feb./Mar./Apr. 1993.

[4] H. Steendam and M. Moeneclaey, "The effect of synchronization errors on MC-CDMA performance," in *Proc. Int. Conf. on Communications ICC'99*, Vancouver, Canada, June 6–10, 1999, paper S38.3, pp. 1510–1514.

[5] L. Tomba and W. A. Krzymien, "Effect of carrier phase noise and frequency offset on the performance of multicarrier CDMA systems," in *Proc. Int. Conf. on Communications ICC'96*, Dallas, TX, June 1996, paper S49.5, pp. 1513–1517.