The Effect of Timing Jitter on MC-DS-CDMA

Heidi Steendam, *Member, IEEE,* and Marc Moeneclaey, *Fellow, IEEE*

***Abstract*—In this paper, we consider the effect of timing jitter on the performance of a multicarrier direct-sequence code-division multiple-access system for both uplink and downlink transmission, assuming orthogonal spreading sequences. Theoretical expressions are derived for the performance degradation caused by the timing jitter, in the presence of a multipath channel. Assuming an additive white Gaussian noise channel, perfect power control, and full load, it is shown that the performance degradation for the downlink transmission is independent of the number of subcarriers, of the spreading factor, and of the spectral contents of the jitter at the receiving mobile station, but only depends on the jitter variance. Under the same assumptions, we point out that, if the jitter spectra of all transmitting mobile stations are the same, the degradation on the uplink is the same as the degradation on the downlink.**

***Index Terms*—Timing jitter, multicarrier (MC) systems, multiple-access systems.**

I. Introduction

MULTICARRIER modulations have received considerable attention in the context of high-data-rate communications, as they combine a high bandwidth efficiency with an immunity to channel dispersion [1]–[3]. Recently, some new techniques, based on a combination of the multicarrier modulation technique and the code-division multiple access (CDMA) scheme, were proposed in the literature (see [4] and the references therein). One of these combinations is the multicarrier direct-sequence CDMA (MC-DS-CDMA) technique, which has been considered for mobile radio communications [5]–[8]. In the MC-DS-CDMA technique, the serial-to-parallel converted data stream is multiplied with the spreading sequence, and then the chips belonging to the same symbol modulate the same subcarrier: the spreading is done in the time domain.

The use of a large number of subcarriers makes the multicarrier systems very sensitive to clock frequency offsets [9], [10]. To avoid the degradation associated with a clock frequency offset, it was proposed in [9] and [10] to correct the timing offset of the transmitter clock (uplink) or of the receiver clock (downlink) by means of a timing synchronization algorithm. In this case, clock frequency offsets and constant timing offsets are eliminated, such that the multicarrier system is affected only by the timing jitter resulting from the synchronizer. The effect of timing jitter on different multicarrier systems has been studied in the literature [9]–[11]. For an additive white Gaussian noise (AWGN) channel, it has been shown that the performance degradation for orthogonal frequency-division multiplexing (OFDM) [9] and multicarrier CDMA (MC-CDMA) [10], caused by timing jitter, is independent of the number of subcarriers, of the spreading factor (in MC-CDMA), and of the spectral contents of the jitter, but only depends on the jitter variance.

In this paper, we consider the effect of timing jitter on MC-DS-CDMA for both uplink and downlink transmission. We determine analytical expressions for the performance degradations in terms of the system parameters that allow us to compare the sensitivities of uplink and downlink transmission to timing jitter.

Paper approved by L. Vandendorpe, the Editor for Transmission Systems of the IEEE Communications Society. Manuscript received October 20, 2001; revised February 25, 2003 and August 29, 2003. This paper was presented in part at the Eighth Symposium on Communications and Vehicular Technology SCVT'01, Delft, The Netherlands, October 2001.

The authors are with the Department of Telecommunications and Information Processing, Ghent University, B-9000 Gent, Belgium (e-mail: Heidi.Steendam@telin.UGent.be; Marc.Moeneclaey@telin.UGent.be).

Digital Object Identifier 10.1109/TCOMM.2004.823632

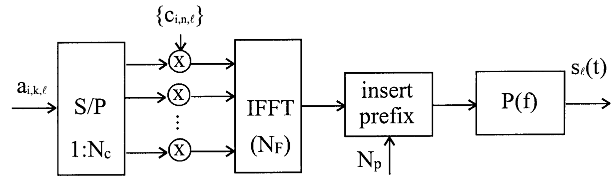

Fig. 1. MC-DS-CDMA transmitter structure for a single user.

II. System Description

A. Uplink MC-DS-CDMA

The conceptual block diagram of the transmitter of an MC-DS-CDMA system for a single mobile user is shown in Fig. 1. In MC-DS-CDMA, the complex data symbols, to be transmitted at rate R_s, are first split into N_c symbol sequences at rate R_s/N_c. Each of these lower rate symbol sequences modulates a different subcarrier of the orthogonal multicarrier system. We denote by $a_{i,k,\ell}$ the data symbol transmitted by user ℓ on subcarrier k during the ith symbol interval; k belongs to a set I_c of N_c subcarrier indexes. The data symbol $a_{i,k,\ell}$ is then multiplied with a higher rate spreading sequence $\{c_{i,n,\ell}\,|\,n=0,\ldots,N_s-1\}$ with spreading factor N_s, where $c_{i,n,\ell}$ denotes the nth chip of the sequence that spreads the data symbols from user ℓ during the ith symbol interval. Note that the spreading sequence does not depend on the subcarrier index k: all data symbols from user ℓ that are transmitted during the same symbol interval are spread with the same spreading sequence. It is assumed that $|c_{i,n,\ell}|=1$. We denote by $\{b_{i,n,k,\ell}=c_{i,n,\ell}a_{i,k,\ell}/\sqrt{N_s}\,|\,n=0,\ldots,N_s-1; k\in I_c\}$ the N_s components of the spread data symbol $a_{i,k,\ell}$. The components $b_{i,n,k,\ell}$ are serially transmitted on the kth subcarrier of an orthogonal multicarrier system, i.e., the spreading is done in the time domain. To modulate the spread data symbols on the orthogonal subcarriers, we use an N_F-point inverse fast Fourier transform (IFFT). To avoid the multipath channel causing interference between the data symbols at the receiver, each FFT block at the IFFT output is cyclically extended with a prefix of N_p samples. This results in the sequence of samples $\{s_{i,n,m,\ell} \,|\, m = -N_p, \ldots, N_F - 1\}$ determined by

$$s_{i,n,m,\ell} = \frac{1}{\sqrt{N_F + N_p}} \sum_{k \in I_c} b_{i,n,k,\ell} e^{j2\pi \frac{km}{N_F}}. \quad (1)$$

The sequence $\{s_{i,n,m,\ell} \,|\, m = -N_p, \ldots, N_F - 1\}$ is fed to a square-root raised-cosine filter $P(f)$ with rolloff α and unit-energy impulse response $p(t)$. The resulting continuous-time transmitted complex baseband signal $s_\ell(t)$ is given by

$$s_\ell(t) = \sum_{i=-\infty}^{+\infty} \sum_{m=-N_p}^{N_F-1} \sum_{n=0}^{N_s-1} s_{i,n,m,\ell} \cdot p(t - (m + (n + iN_s)(N_F + N_p))T - \tau_{i,n,m,\ell}) \quad (2)$$

where $1/T = (N_F + N_p)N_s R_s / N_c$ is the network reference clock frequency and $\tau_{i,n,m,\ell}$ is a time-varying delay representing the transmit clock phase of user ℓ. In the following, it is assumed that subcarriers inside the rolloff area of the transmit filter are not modulated, i.e., they have zero amplitude. Hence, of the N_F available subcarriers, only N_c subcarriers are actually used ($N_c \leq (1 - \alpha)N_F$). Assuming N_c to be odd, the set I_c of subcarriers actually used is given by $I_c = \{0, \ldots, N_c/2 - 1\} \cup \{N_F - (N_c/2 - 1), \ldots, N_F - 1\}$. The corresponding subcarrier spacing Δf and system bandwidth B are given by

$$\Delta f = \frac{1}{N_F T} = \frac{N_s}{N_c} R_s \frac{N_F + N_p}{N_F} \approx \frac{N_s}{N_c} R_s$$
$$B = N_c \Delta f = \frac{N_c}{N_F T} = N_s R_s \frac{N_F + N_p}{N_F} \approx N_s R_s. \quad (3)$$

The above approximations are valid for $N_p \ll N_F$.

In a multiuser scenario, each user transmits to the basestation a similar signal $s_\ell(t)$. To allow separation of the different user signals at the receiver, each user is assigned a unique spreading sequence $\{c_{i,n,\ell}\}$, with ℓ denoting the user index. In this paper, we consider orthogonal sequences, consisting of user-dependent Walsh–Hadamard (WH) sequences of length N_s, multiplied with a complex-valued random scrambling sequence that is common to all N_u active users. Hence, the maximum number of users that can be accommodated equals N_s, i.e., the number of WH sequences of length N_s. Note that the number of carriers N_c can be chosen independently of the spreading factor N_s, which, in turn, equals the maximum number of users. Without loss of generality, we focus on the detection of the data symbols transmitted by the reference user ($\ell = 0$).

The signal $s_\ell(t)$ transmitted by user ℓ reaches the basestation through a multipath channel with transfer function $H_{\text{ch},\ell}(f)$ that depends on the user index ℓ [see Fig. 2(a)]. The basestation receives the sum of the resulting user signals and an AWGN process $w(t)$. The real and imaginary parts of $w(t)$ are uncorrelated, and each have a power spectral density of $N_0/2$. The resulting signal is applied to the receiver filter, which is matched to the transmit filter. The matched-filter output is sampled at the instants $t_{i,n,m} = (m + (n + iN_s)(N_F + N_p))T$ (see Fig. 3).

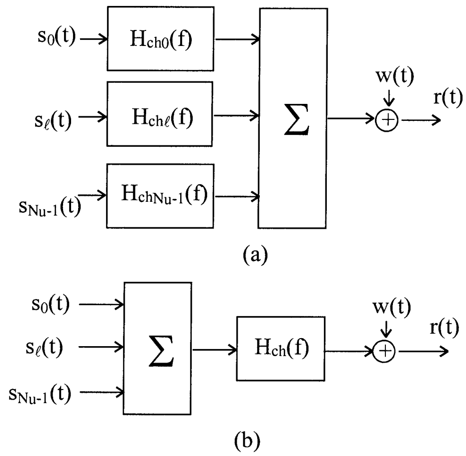

Fig. 2. Channel structure. (a) Uplink. (b) Downlink.

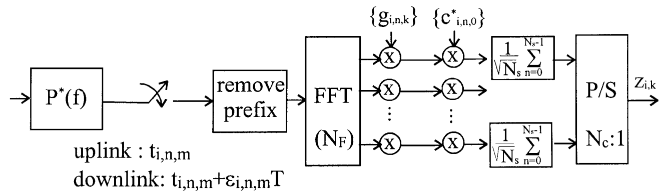

Fig. 3. MC-DS-CDMA receiver structure.

Only the N_F samples with $m = 0, \ldots, N_F - 1$ are kept for further processing. We assume that the length of the cyclic prefix is sufficiently longer than the maximum duration T_{ch} of the impulse responses of the composite channels with transfer functions $H_\ell(f) = |P(f)|^2 H_{\text{ch},\ell}(f), \ell = 0, \ldots, N_u - 1$. This allows the transmitter of each user to adapt its transmit clock phase $\tau_{i,n,m,\ell}$ to synchronize the different user signals at the basestation within a cyclic prefix, such that the samples used for further processing at the basestation are not affected by interference from other FFT blocks, since the time dispersion is absorbed by the cyclic prefix. This situation is depicted in Fig. 4(a). Whereas static timing misalignments between the received user signals are absorbed by the cyclic prefix, the adaptation of the transmit clock phase, however, introduces timing jitter $\epsilon_{i,n,m,\ell}T$. This timing jitter can be modeled as a zero-mean stationary random process with jitter spectral density $S_{\epsilon,\ell}(f)$ and jitter variance $\sigma^2_{\epsilon,\ell}$. The contribution from each user is affected by a different timing jitter process $\epsilon_{i,n,m,\ell}T$, as each user signal is generated with a different transmit clock.

The N_F samples selected by the basestation are applied to an N_F-point FFT, followed by one-tap equalizers $g_{i,n,k}$ that scale and rotate the FFT outputs. We denote by $g_{i,n,k}$ the coefficient of the equalizer, operating on the kth FFT output during the nth FFT block of the ith symbol interval. Each equalizer output is multiplied with the corresponding chip of the reference user's spreading sequence, and summed over N_s consecutive values to yield the samples $z_{i,k}$ at the input of the decision device.

The detection of the symbol $a_{i,k,0}$ is based upon the decision variable $z_{i,k}$, which can be decomposed as

$$z_{i,k} = \sqrt{\frac{N_F}{N_F + N_p}} \sum_{\ell=0}^{N_u-1} \sum_{k' \in I_c} a_{i,k',\ell} I_{i,k,k',\ell} + W_{i,k}, k \in I_c \quad (4)$$

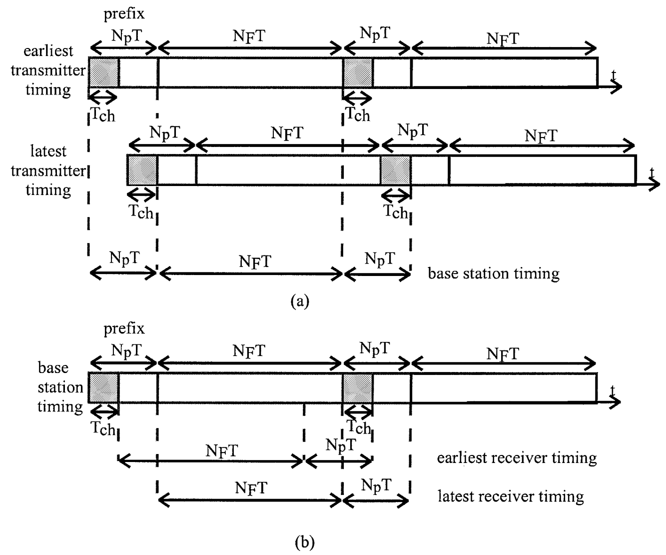

Fig. 4. Earliest and latest possible timing to avoid interference between FFT blocks. (a) Uplink. (b) Downlink.

where

$$I_{i,k,k',\ell} = \frac{1}{N_s}\sum_{n=0}^{N_s-1} c^*_{i,n,0}c_{i,n,\ell}A_{i,n,k,k',\ell} \quad (5)$$

$$\begin{aligned}A_{i,n,k,k',\ell} &= H_{k',\ell}\sum_{m=0}^{N_F-1} e^{-j2\pi\frac{mk-\text{mod}(k';N_F)(m+\epsilon_{i,n,m,\ell})}{N_F}}\\ &\approx H_{k,\ell}\delta_{k-k'} + \tilde{H}_{k',\ell}\frac{1}{N_F}\\ &\quad\cdot\sum_{m=0}^{N_F-1} j2\pi\epsilon_{i,n,m,\ell}e^{-j2\pi\frac{m(k-k')}{N_F}}. \end{aligned}\quad (6)$$

In (4), $W_{i,k}$ is the additive noise contribution, with

$$E[W_{i,k}W^*_{i',k'}] = N_0\delta_{i-i'}\delta_{k-k'}\frac{1}{N_s}\sum_{n=0}^{N_s-1}|g_{i,n,k}|^2. \quad (7)$$

In (6), $H_{k,\ell} = H_\ell(\text{mod}(k;N_F)/(N_FT))/T, \tilde{H}_{k,\ell} = (\text{mod}(k;N_F)/N_F)H_{k,\ell}, \text{mod}(x;N_F)$ is the modulo-N_F reduction of x, yielding a result in the interval $[-N_F/2, N_F/2]$. The second line in (6) results from the linearization of the exponential function around $\epsilon_{i,n,m,\ell} = 0$, which is accurate when the timing jitter variances are small.

The quantity $I_{i,k,k',\ell}$ in (4) denotes the contribution from the data symbol $a_{i,k',\ell}$ to the sample $z_{i,k}$ at the input of the decision device. The sample $z_{i,k}$ contains a useful component with coefficient $I_{i,k,k,0}$. This component can be decomposed into an average useful component $E[I_{i,k,k,0}]$ and a zero-mean fluctuation $I_{i,k,k,0} - E[I_{i,k,k,0}]$ about its average, i.e., the self-interference (SI). The quantities $I_{i,k,k',0}, (k' \neq k)$ correspond to intercarrier interference (ICI), i.e., the contribution from data symbols transmitted by the reference user on other subcarriers. For $\ell \neq 0$, the quantities $I_{i,k,k',\ell}$ correspond to multiuser interference (MUI), i.e., the contribution from data symbols transmitted by other users. The equalizer coefficients are selected such that the coefficients $E[I_{i,k,k,0}]$ of the average useful component equal one, for $k \in I_c$. This yields

$$g_{i,k,n} = \frac{1}{H_{k,0}}. \quad (8)$$

It is instructive to consider the case where all timing jitter is zero, i.e., $\epsilon_{i,n,m,\ell} = 0$ for $\ell = 0,\ldots,N_u-1$. In this case, the quantities (6) reduce to

$$A_{i,n,k,k',\ell} = H_{k,\ell}\delta_{k-k'}. \quad (9)$$

Hence, the contribution from user ℓ to the kth FFT output $y_{i,n,k}$ is proportional to $b_{i,n,k,\ell}H_{k,\ell}$, which means that SI and ICI are absent. Further, as the factor $H_{k,\ell}$ does not depend on the chip index n, the orthogonality between the contributions from different users to the same FFT output is not affected. We obtain $I_{i,k,k',\ell} = 0$ for $\ell \neq 0$, hence, MUI is absent as well. This indicates that, in the absence of timing jitter, the only effect of the multipath channel on the uplink MC-DS-CDMA signal with cyclic prefix is to multiply the symbols from each user with a factor $H_{k,\ell}$ that depends on the user index ℓ and the subcarrier index k. The presence of this factor affects the signal-to-noise ratio (SNR) at the input of the decision device, but does not give rise to interference.

B. Downlink MC-DS-CDMA

In downlink MC-DS-CDMA, the basestation synchronizes the signals to be transmitted to the N_u mobile users ($\tau_{i,n,m,\ell} = \tau$ for $\ell = 0,\ldots,N_u-1$), and broadcasts the sum of the N_u user signals $s_\ell(t)$ from (2). As shown in Fig. 2(b), this broadcast signal reaches the receiver of the reference user through a multipath channel with transfer function $H_{\text{ch}}(f)$. The output of the channel is disturbed by AWGN $w(t)$ with uncorrelated real and imaginary parts, each having a power spectral density of $N_0/2$. Similarly as in uplink MC-DS-CDMA, the resulting signal is applied to the receiver filter of Fig. 3 in order to detect the data symbols transmitted to the reference user ($\ell = 0$). Assuming that the length of the cyclic prefix is longer than the duration T_{ch} of the composite channel with transfer function $H(f) = |P(f)|^2H_{\text{ch}}(f)$, the receiver adjusts its sampling clock phase such that the N_F samples to be processed are free from interference from other blocks [see Fig. 4(b)]. The sampling instants at the mobile receiver are denoted $t_{i,n,m}+\epsilon_{i,n,m}T$, where $t_{i,n,m} = (m + (n + iN_s)(N_F + N_p))T$ and the timing jitter $\epsilon_{i,n,m}T$ is a zero-mean stationary random process with jitter spectral density $S_\epsilon(f)$ and jitter variance σ^2_ϵ; $1/T$ is the network reference clock frequency. Only the samples with indexes $m = 0,\ldots,N_F-1$ are kept for further processing by the user. As in uplink MC-DS-CDMA, the sample $z_{i,k}$ at the input of the decision device can be represented by (4). The quantities $I_{i,k,k',\ell}$ are given by (5), with $S_{\epsilon,\ell}(f)$ and $H_\ell(f)$ substituted by $S_\epsilon(f)$ and $H(f)$, respectively. The samples $z_{i,k}$ are decomposed into a useful component, ICI, MUI, and noise. The equalizer coefficients $g_{i,n,k}$, that are selected such that the coefficients $I_{i,k,k,0}$ of the useful component equal one, are given by (8), with $H_\ell(f)$ substituted by $H(f)$.

III. Performance Analysis

A. Uplink MC-DS-CDMA

The performance of the MC-DS-CDMA system is measured by the SNR, which is defined as the ratio of the power of the average useful component (P_U) to the sum of the powers of the total interference ($=\mathrm{SI}+\mathrm{ICI}+\mathrm{MUI}$)($P_I$), and the noise ($P_N$) at the input of the decision device. Note that these quantities depend on the index k of the considered subcarrier. This yields

$$\mathrm{SNR}_k(\underline{\epsilon}) = \frac{\frac{N_F}{N_F+N_p}P_{U,k}}{P_{N,k}+\frac{N_F}{N_F+N_p}P_{I,k}}. \tag{10}$$

In (10), the powers of the useful component, the total interference, and the noise are given by

$$\begin{aligned} P_{U,k} &= E_{s,k,0} \\ P_{I,k} &= \sum_{k'\in I_c} E_{s,k',0}\left|\frac{\tilde{H}_{k',0}}{H_{k,0}}\right|^2 (2\pi)^2 X_{k,k',0} \\ &\quad + \frac{1}{N_s-1}\sum_{\ell=1}^{N_u-1}\sum_{k'\in I_c, k'\neq k} E_{s,k',\ell}\left|\frac{\tilde{H}_{k',\ell}}{H_{k,0}}\right|^2 \\ &\quad \cdot (2\pi)^2 (Y_{k,k',\ell} - X_{k,k',\ell}) \\ P_{N,k} &= N_0 \frac{1}{|H_{k,0}|^2} \end{aligned} \tag{11}$$

where

$$\begin{aligned} X_{k,k',\ell} &= \int_{-\infty}^{+\infty} S_{\epsilon,\ell}(f)\left|D_{N_F}\left(\frac{k'-k}{N_F}+fT\right)\right|^2 \\ &\quad \cdot |D_{N_s}(fT(N_F+N_p))|^2\, df \\ Y_{k,k',\ell} &= \int_{-\infty}^{+\infty} S_{\epsilon,\ell}(f)\left|D_{N_F}\left(\frac{k'-k}{N_F}+fT\right)\right|^2 df \end{aligned} \tag{12}$$

$$\begin{aligned} D_M(x) &= \frac{1}{M}\sum_{m=0}^{M-1} e^{j2\pi mx} \\ &= e^{j\pi(M-1)x}\frac{\sin(\pi Mx)}{M\sin(\pi x).} \end{aligned} \tag{13}$$

For the derivation of (11) and (12), the reader is referred to [12] and [13], where similar equations have been obtained for the case of MC-CDMA. In (11), $E_{s,k,\ell} = E[|a_{i,k,\ell}|^2]$ denotes the symbol energy transmitted on subcarrier k by user ℓ. In the absence of timing jitter ($\epsilon_{i,n,m,\ell} = 0$ for $\ell = 0,\ldots,N_u-1$), the SNR (10) reduces to $\mathrm{SNR}_k(0) = (N_F/(N_F+N_p))|H_{k,0}|^2(E_{s,k,0}/N_0)$. The degradation (in decibels) caused by the timing jitter is defined as $\mathrm{Deg}_k = 10\log(\mathrm{SNR}_k(0)/\mathrm{SNR}_k(\underline{\epsilon}))$. This degradation corresponds to the increase of E_s/N_0 (or E_b/N_0) required to maintain the SNR in the presence of timing jitter the same as in the absence of timing jitter. Note that the same increase of E_s/N_0 (or E_b/N_0) keeps the bit-error rate (BER) in the presence of timing jitter the same as the BER in the case of perfect timing (provided that the sum of the interference and noise is approximately Gaussian).

In order to clearly isolate the effect of the timing jitter, we assume that the channels are ideal, the jitter spectrum is equal for all users, the load is maximum ($N_s = N_u$), and all users have the same energy per symbol on each subcarrier (i.e., $|H_{k,\ell}| = 1, S_{\epsilon,\ell}(f) = S_\epsilon(f)$ and $E_{s,k,\ell} = E_s$ for $k\in I_c, \ell = 0,\ldots,N_s-1$). In this case, the quantities $X_{k,k',\ell}$ and $Y_{k,k',\ell}$ are independent of the user index ℓ. In the following, we drop the user index. The total interference power in this case is given by

$$P_{I,k} = E_s(2\pi)^2\sum_{k'\in I_c}\beta_{k'}Y_{k,k'} \tag{14}$$

where $\beta_k = (\mathrm{mod}(k;N_F)/N_F)^2$ and the degradation is given by

$$\mathrm{Deg}_k = 10\log\left(1+\mathrm{SNR}(0)(2\pi)^2\sum_{k'\in I_c}\beta_{k'}Y_{k,k'}\right). \tag{15}$$

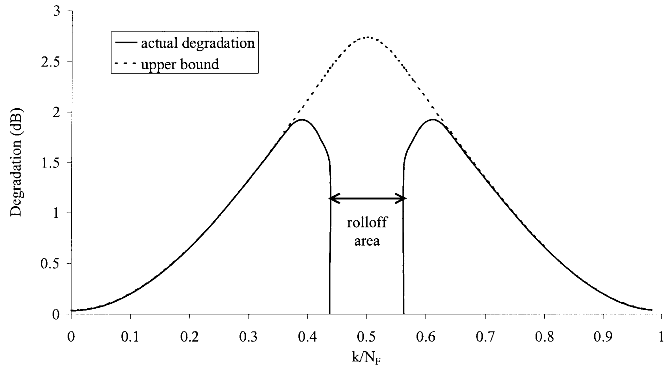

Fig. 5. Performance degradation caused by timing jitter ($N_F = 64, N_c = 57, \mathrm{SNR}(0) = 10$ dB, $\sigma_\epsilon^2 = 10^{-2}$).

In Fig. 5, the degradation (15) is shown as function of the subcarrier index, for $N_F = 64, N_c = 57, \mathrm{SNR}(0) = 10$ dB, and $\sigma_\epsilon^2 = 10^{-2}$, and a jitter spectrum $S_\epsilon(f) = \sigma_\epsilon^2/(2f_{\max}^2)(f_{\max}-|f|)$, with $f_{\max} = 2/(N_sN_FT) = 2\Delta f/N_s$. We observe that subcarriers with indexes k and N_F-k, that are located symmetrically with respect to $k = N_F/2$, exhibit the same degradation. For $k < N_F/2$, the degradation first increases with k, but then decreases with k when k approaches the vicinity of the edge of the rolloff area. For given k, the degradation depends on the number N_c of modulated subcarriers, because in (15), the summation over k' ranges over the set I_c of N_c modulated subcarriers. An upper bound on this degradation is obtained by extending in (15) this summation interval over all N_F available subcarriers, i.e., $k' = 0,\ldots,N_F-1$. This yields

$$\mathrm{Deg}_k \le 10\log\left(1+\mathrm{SNR}(0)(2\pi)^2\sum_{k'=0}^{N_F-1}\beta_{k'}Y_{k,k'}\right). \tag{16}$$

This bound is also shown in Fig. 5, for $k = 0,\ldots,N_F-1$. For given k, the upper bound (16) on the degradation is independent of the number N_c of modulated subcarriers, and becomes maximum for the subcarrier $k = N_F/2$. The upper bound is reached when all subcarriers are modulated ($N_c = N_F; \alpha = 0$). When $\alpha > 0$, the upper bound (16) yields an accurate approximation for the actual degradation on subcarriers that are not too close to the edge of the rolloff area.

Under the same assumptions that gave rise to (14), we define define an average SNR, which is obtained by replacing the in-

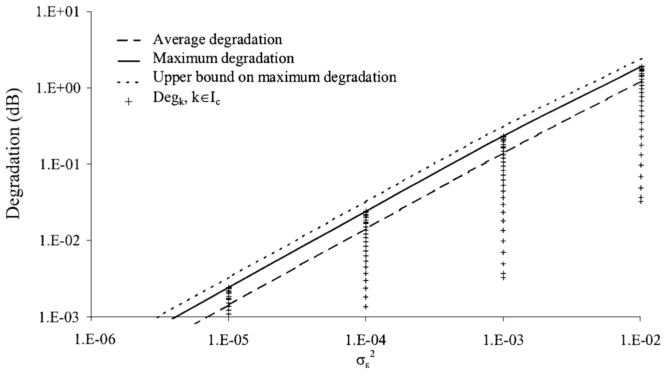

Fig. 6. Maximum and average degradation caused by timing jitter ($N_F = 64, N_c = 57, \mathrm{SNR}(0) = 10$ dB).

terference power $P_{I,k}$ from (14) by its arithmetical average over all N_F subcarriers. Note that the average interference power depends on the number N_c of modulated subcarriers. Extending in (14) the summation over k' to all N_F available subcarriers, and then performing the arithmetical average over the index k, we obtain an upper bound on the average interference power that does not depend on N_c. The corresponding upper bound on the degradation of this average SNR turns out to be independent of the jitter spectrum, of the spreading factor N_s, and for $N_F \gg 1$ becomes independent of the number of available (N_F) subcarriers (see [13])

$$\mathrm{Deg}_{\mathrm{Av}} \leq 10 \log \left(1 + \mathrm{SNR}(0) \frac{\pi^2}{3} \sigma_\epsilon^2\right) \tag{17}$$

where $\mathrm{SNR}(0) = (N_F/(N_F + N_p))E_s/N_0$ is the SNR in the absence of timing jitter, and the jitter variance is given by

$$\sigma_\epsilon^2 = \int_{-\infty}^{+\infty} S_\epsilon(f) df. \tag{18}$$

The bound on the average degradation (17) is shown in Fig. 6 as function of the jitter variance, along with the actual degradation from (15) for $k \in I_c$, the maximum (over k) of the actual degradation (15), and the upper bound (16) at the edge of the rolloff area (i.e., for $k = (N_c - 1)/2$), for $N_F = 64, N_c = 57, \mathrm{SNR}(0) = 10$ dB. As we observe, for a substantial number of modulated carriers, the actual degradation is between the average degradation (17) and the maximum degradation; the latter is close to the upper bound (16); this illustrates the importance of average degradation (17) and of the bound (16). For small jitter variance, the degradation (in decibels) is proportional to σ_ϵ^2.

B. Downlink MC-DS-CDMA

As in uplink MC-DS-CDMA, the performance is measured by the SNR, defined in (10). The powers of the average useful component, the total interference, and the noise are given by (11), with $S_{\epsilon,\ell}(f)$ and $H_\ell(f)$ substituted by $S_\epsilon(f)$ and $H(f)$, respectively, for $\ell = 0, \ldots, N_u - 1$. In this case, it follows from (12) that the quantities $X_{k,k',\ell}$ and $Y_{k,k',\ell}$ are independent of the user index ℓ.

To clearly isolate the effect of the timing jitter, we consider the case where the channel is ideal ($|H_k| = 1, k \in I_c$), the load is maximum $(N_s = N_u)$, and the energy per symbol is the same for all subcarriers and all users ($E_{s,k,\ell} = E_s$ for $k \in I_c, \ell = 0, \ldots, N_s - 1$). In this case, the degradation is given by (15). As in uplink transmission, an upper bound on the degradation is obtained by extending the summation over k' in (15) over all N_F available subcarriers, yielding (16). Further, similar to the uplink, we can define the average SNR by replacing in (10) the powers of the average useful component, the total interference, and the noise by their arithmetical average over all subcarriers. The degradation of this average SNR, for $N_F \gg 1$, is given by (17).

Hence, the degradation in the downlink is the same as in the uplink, assuming that the jitter spectra in the uplink are the same for all users. This degradation is independent of the number of subcarriers, of the spreading factor, and of the spectral contents of the jitter, but only depends on the jitter variance.

IV. Conclusions and Remarks

In this paper, we have determined the effect of timing jitter on the performance of MC-DS-CDMA, for both uplink and downlink transmission, assuming orthogonal spreading sequences. We have determined analytical expressions for the SNR at the input of the decision device, and the degradation of the SNR caused by timing jitter. We have compared the resulting degradation for both uplink and downlink transmission, under the assumption of the full load $(N_u = N_c)$. When the jitter spectra of the different users are the same, the degradation turns out to be the same for the uplink and the downlink. Furthermore, we have shown that the degradation of the MC-DS-CDMA system is independent of the number of subcarriers, the spreading factor, and the spectral contents of the jitter, but only depends on the jitter variance.

In order to check the validity of the linearization $\exp(jx) \approx 1 + jx$ in (6), we have simulated the MC-DS-CDMA decision variables $z_{i,k}$ from (4) in the absence of noise but in the presence of timing jitter, without using the linearization. We have verified that the interference power computed from this simulation is close to the theoretical value $\sigma_\epsilon^2 \pi^2/3$ resulting from the linearization, provided that the jitter variance σ_ϵ^2 is less than 10^{-2}, or, equivalently, the root mean square timing jitter is less than 10% of the sampling interval T. For jitter variances exceeding 10^{-2}, the linearization is no longer accurate, but this is not a suitable operating condition, as the resulting degradation becomes unacceptably large.

Although the derivation of the performance degradation caused by the timing jitter has been carried out under the assumption of a static channel, (10)–(12) are also valid for slowly varying multipath fading channels that are essentially constant over N_s FFT blocks (which corresponds to a duration N_c/R_s). In this case, the SNR (10) denotes the instantaneous SNR, related to a particular realization of the channel. The relevant performance measure for the slowly varying multipath fading channel is the (degradation of) the *average* SNR, which is obtained by replacing in (11) the quantities $|H_{k,0}|^2$ and $|\tilde{H}_{k',\ell}|^2$ by $E[|H_{k,0}|^2]$ and $E[|\tilde{H}_{k',\ell}|^2]$, respectively. The degradation of this average SNR (in decibels) corresponds to the increase in E_s/N_0 (or E_b/N_0) that is required to keep the

BER in the presence of timing errors essentially the same as the BER for perfect timing. When $E[|H_{k,0}|^2] = 1$ for $k \in I_c$ and $\ell = 0, 1, \ldots, N_u - 1$, the degradation of the average SNR for the slowly fading channel is the same as the degradation (15) of the SNR for an AWGN channel.

It can be verified from [10] and [11] that the average degradation (17) for MC-DS-CDMA is exactly the same as the corresponding degradations for OFDM and MC-CDMA, assuming that the three multicarrier systems have the same subcarrier spacing.

REFERENCES

[1] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*. Norwood, MA: Artech House, 2000.

[2] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications," *IEEE Signal Processing Mag.*, vol. 17, pp. 29–48, May 2000.

[3] N. Morinaga, M. Nakagawa, and R. Kohno, "New concepts and technologies for achieving highly reliable and high capacity multimedia wireless communication systems," *IEEE Commun. Mag.*, vol. 38, pp. 34–40, Jan. 1997.

[4] S. Hara and R. Prasad, "Overview of Multicarrier CDMA," *IEEE Commun. Mag.*, vol. 35, pp. 126–133, Dec. 1997.

[5] G. Santella, "Bit-error rate performances of *M*-QAM orthogonal multicarrier modulation in presence of time-selective multipath fading," in *Proc. Int. Conf. Communications*, Seattle, WA, June 1995, pp. 1683–1688.

[6] V. M. DaSilva and E. S. Sousa, "Performance of orthogonal CDMA sequences for quasi-synchronous communication systems," in *Proc. IEEE ICUPC*, Ottawa, ON, Canada, Oct. 1993, pp. 995–999.

[7] S. Kondo and L. B. Milstein, "Performance of multicarrier DS-CDMA systems," *IEEE Trans. Commun.*, vol. 44, pp. 238–246, Feb. 1996.

[8] E. A. Sourour and M. Nakagawa, "Performance of orthogonal multicarrier CDMA in a multipath fading channel," *IEEE Trans. Commun.*, vol. 44, pp. 356–367, Mar. 1996.

[9] H. Steendam and M. Moeneclaey, "Sensitivity of orthogonal frequency-division multiplexed systems to carrier and clock synchronization errors," *Signal Processing*, vol. 80, pp. 1217–1229, July 2000.

[10] ——, "The effect of synchronization errors on MC-CDMA performance," in *Proc. Int. Conf. Communications*, Vancouver, BC, Canada, June 1999, pp. 1510–1514.

[11] L. Tomba and W. A. Krzymien, "A model for the analysis of timing jitter in OFDM systems," in *Proc. Int. Conf. Communications*, Atlanta, GA, June 1998, pp. 1227–1231.

[12] H. Steendam and M. Moeneclaey, "The effect of carrier phase jitter on MC-CDMA performance," *IEEE Trans. Commun.*, vol. 47, pp. 195–198, Feb. 1999.

[13] H. Steendam, "The Effect of Synchronization Errors on Multicarrier Systems," Ph.D. dissertation, Ghent Univ., Gent, Belgium, Oct. 2000.

Heidi Steendam (M'00) received the Diploma and the Ph.D. degree, both in electrical engineering, from Ghent University, Gent, Belgium, in 1995 and 2000, respectively.

She is currently a Professor in the Department of Telecommunications and Information Processing, Ghent University. Her main research interests are in statistical communication theory, carrier and symbol synchronization, bandwidth-efficient modulation and coding, spread spectrum (multicarrier spread spectrum), satellite and mobile communication. She is the author of more than 50 scientific papers in international journals and conference proceedings.

Marc Moeneclaey (M'93–SM'99–F'02) received the Diploma and the Ph.D. degree, both in electrical engineering, from Ghent University, Gent, Belgium, in 1978 and 1983, respectively.

He is currently a Professor in the Department of Telecommunications and Information Processing, Ghent University. His main research interests are in statistical communication theory, carrier and symbol synchronization, bandwidth-efficient modulation and coding, spread spectrum, satellite and mobile communication. He is the author of more than 200 scientific papers in international journals and conference proceedings. Together with H. Meyr (RWTH Aachen) and S. Fechtel (Siemens AG), he is a co-author of the book *Digital Communication Receivers—Synchronization, Channel Estimation, and Signal Processing* (New York: Wiley, 1998).

Between 1992–1994, Dr. Moeneclaey served as Editor for Synchronization for the IEEE TRANSACTIONS ON COMMUNICATIONS.