Synchronization Sensitivity of Multicarrier Systems

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Abstract

In this paper, we give an overview of the sensitivity of a number of multicarrier systems to carrier and clock synchronization errors. A comparison is made between orthogonal frequency division multiplexing/multiple access (OFDM(A)) and two combinations of the orthogonal multicarrier technique and the code-division multiple access (CDMA) technique, i.e., multicarrier CDMA (MC-CDMA), where the spreading is accomplished in the frequency domain, and multicarrier direct-sequence CDMA (MC-DS-CDMA), where the spreading is done in the time domain. We evaluate the effect of small synchronization errors on the BER performance by deriving simple analytical expressions for the BER degradation that are based upon truncated Taylor series expansions. To allow a fair comparison, all considered systems are able to accommodate the same number of users, and each user operates at the same data rate. Under these conditions, we show that all multicarrier systems exhibit the same sensitivity to carrier phase jitter and timing jitter. Further, when the number of carriers is equal to the maximum number of users, the different multicarrier systems also are equally affected by a carrier frequency offset or a clock frequency offset.

I. INTRODUCTION

During the last decade, we have witnessed a widespread deployment of digital communication services requiring an exchange of digital information at constantly increasing data rates (e.g., audio and video conferencing, internet applications, digital television,...). To satisfy this increasing demand for higher data rates, the data rates over the existing transmission media must be enhanced. Particularly multicarrier (MC) systems have received considerable attention in the context of high data rate communications, as they combine a high spectral efficiency with an immunity to channel dispersion [1]. One of the MC systems is the well-studied orthogonal frequency-division multiplexing (OFDM) system. The conventional OFDM system has been proposed and/or accepted for various applications such as transmission over twisted pair cables (xDSL) [2], broadcasting of digital audio (DAB) and digital television (DTTB) [3], mobile radio [4], or wireless local area networks (WLAN) [5]-[6]. A technique that is closely related to OFDM is orthogonal frequency-division multiple access (OFDMA). In contrast with OFDM, where all carriers are modulated by the same user, in OFDMA the different carriers are modulated by different users. The OFDMA technique has been proposed for the return path of the CATV (cable area TV) network [7].

Recently, different combinations of the MC technique and the code-division multiple access technique (CDMA) have been proposed [8]. Two of these combinations that make use of carriers satisfying the orthogonality constraint with minimum frequency separation are multicarrier CDMA (MC-CDMA) and multicarrier direct-sequence CDMA (MC-DS-CDMA). In the MC-CDMA technique [9]-[11], the original data stream is first multiplied with the spreading sequence and then modulated on the orthogonal carriers. As the chips belonging to the same symbol are modulated on different carriers, the spreading is done in the frequency domain. In the MC-DS-CDMA technique [12]-[13], on the other hand, the serial-to-parallel converted data stream is multiplied with the spreading sequence, and then the chips belonging to the same symbol modulate the same carrier: the spreading is accomplished in the time domain. Both MC-CDMA and MC-DS-CDMA have been considered for mobile radio communications [9]-[13].

The transmitter of a digital communication system contains a clock that indicates the timing instants at which the data symbols must be transmitted. Furthermore, the transmitter contains a carrier oscillator, necessary for the upconversion of the data-carrying baseband signal, yielding the bandpass signal to be transmitted. At the receiver, the received bandpass signal is downconverted using a local carrier oscillator. The resulting baseband signal is sampled at timing instants determined by the receiver clock. Based on the resulting samples, a decision is taken about the transmitted data symbols. As the reliability of this decision is maximum when the frequencies and phases of the carrier oscillator and clock at the transmitter are related to those at the receiver, the receiver must estimate the frequencies and phases of the carrier oscillator and the clock used at the transmitter. Because of interference, noise and other disturbances, these estimates are not perfect, resulting in carrier and clock synchronization errors.

In the literature, it has been reported that multicarrier systems are very sensitive to some types of carrier and clock synchronization errors, especially when a large number of carriers is used. The effect of different types of carrier and clock synchronization errors has been investigated in [14]-[25] for OFDM(A), MC-CDMA and MC-DS-CDMA, respectively. However, in these papers, the effect of the different types of synchronization errors on the different multicarrier systems is studied through simulations or complicated analytical expressions. Hence, the influence of the different system parameters is not easily understood.

In this paper, we derive simple analytical expressions for the performance degradation of the considered multicarrier systems, caused by synchronization errors. Based on these expressions, we compare the sensitivity of the different multicarrier systems to synchronization errors. To allow a fair comparison between the different multicarrier systems, we assume that all considered systems are able to accommodate up to N users, each user operating at a data rate R_s. In section 2, we describe the different multicarrier systems. The sensitivity to carrier and clock synchronization errors is considered in sections 3 and 4, respectively. The conclusions are drawn in section 5. We restrict our attention to the case of downlink transmission. In this case, the basestation synchronizes the different user signals and upconverts the sum of the different user signals with the same carrier oscillator, such that all users and all carriers exhibit the same carrier and clock synchronization errors.

II. Multicarrier Systems

By means of the orthogonal multicarrier system from Fig. 1, we transmit a sequence of vectors $\{\mathbf{x}_i\}$. The vector $\mathbf{x}_i$ consists of N_F components; the k^{th} component of $\mathbf{x}_i$ is denoted $x_{i,k}$. The transmitter computes the N_F-point inverse fast Fourier transform (IFFT) of $\mathbf{x}_i$, and then cyclically extends the resulting block of N_F samples with a prefix of N_p samples. This yields FFT blocks of $N_F + N_p$ samples, that are applied sequentially at a rate $1/T$ to a square-root raised cosine transmit filter $P(f)$ with rolloff α and unit-energy impulse response $p(t)$. The m^{th} sample $s_{i,m}$ of the i^{th} IFFT block that is applied to the transmit filter is given by

$$s_{i,m} = \frac{1}{\sqrt{N_F + N_p}} \sum_{k=0}^{N_F-1} x_{i,k} e^{j2\pi \frac{km}{N_F}}, \qquad m = -N_p, -N_p+1, \ldots, 0, 1, \ldots, N_F - 1. \tag{1}$$

Hence, the k^{th} IFFT output gives rise to a baseband signal with carrier frequency $k/(N_F T)$. In order to avoid aliasing when sampling the received signal at a rate $1/T$, the carriers in the rolloff area of the transmit pulse are not modulated, i.e., they have zero amplitude. Hence, of the N_F available carriers, only $N_c \leq (1-\alpha)N_F$ carriers are actually modulated. Assuming N_c to be odd, the set of carriers actually modulated is given by $I_c = \{0, \ldots, (N_c-1)/2\} \cup \{N_F - (N_c-1)/2, \ldots, N_F - 1\}$. Hence, $x_{i,k}$ is nonzero only for $k \in I_c$, and the summation interval in (1) can be restricted to $k \in I_c$.

The transmitted signal $s(t)$ reaches the receiver of the reference user through a multipath fading channel with transfer function $H_{ch}(f)$. The channel output is affected by a carrier phase error $\phi(t)$. Further, the received signal is disturbed by additive white Gaussian noise (AWGN) $w(t)$, with uncorrelated real and imaginary parts, each having a noise power spectral density of $N_0/2$.

The received signal $r(t)$ is applied to the receiver filter, which is matched to the transmit filter, and sampled. The resulting samples are affected by a clock phase error $\epsilon_{i,m}T$, which is the deviation from the timing instant $t_{i,m} = (i(N_F + N_p) + m)T$. We assume that the length of the cyclic prefix is longer than the duration T_{ch} of the impulse response of the composite channel with transfer function $H(f) = H_{ch}(f)|P(f)|^2$, such that in each received IFFT block at least N_F consecutive samples are not affected by interference from adjacent blocks. The receiver removes the samples in the cyclic prefix and keeps only the samples with indices $m = 0, \ldots, N_F - 1$. The receiver adjusts its sampling clock phase by means of coarse synchronization, such that the sample corresponding to $m = 0$ is located between the earliest and latest receiver timing indicated in Fig. 2: the N_F samples to be processed are free from interference from other transmitted blocks. This is possible as the length of the cyclic prefix is assumed to be larger than T_{ch}. The N_F selected samples are demodulated using an N_F-point FFT. Each FFT output is applied to a one-tap equalizer with equalizer coefficient $g_{i,k}$ that scales and rotates the k^{th} FFT output, resulting in $y_{i,k}$, which is a scaled version of $x_{i,k}$, affected by interference and noise.

$$y_{i,k} = \sqrt{\frac{N_F}{N_F + N_p}} \sum_{k' \in I_c} x_{i,k'} A_{i,k,k'} + W_{i,k} \tag{2}$$

where

$$A_{i,k,k'} = g_{i,k} \frac{1}{N_F} \sum_{m=0}^{N_F-1} e^{-j2\pi \frac{m(k-k')}{N_F}} \tilde{H}_{k'}(\phi_{i,m}, \epsilon_{i,m}) \tag{3}$$

$$\tilde{H}_k(\phi_{i,m}, \epsilon_{i,m}) = H_k e^{j\phi_{i,m}} e^{j2\pi \frac{mod(k;N_F)}{N_F} \epsilon_{i,m}} \tag{4}$$

$$H_k = \frac{1}{T} H\left(\frac{mod(k; N_F)}{N_F T}\right) \tag{5}$$

$\phi_{i,m} = \phi(t_{i,m})$, and $W_{i,k}$ is the Gaussian noise contribution with

$$E[W_{i,k} W^*_{i',k'}] = N_0 \delta_{i-i'} \delta_{k-k'} |g_{i,k}|^2. \tag{6}$$

In (4) and (5), $mod(k; N_F)$ is the modulo-N_F reduction of k, yielding a result in the interval $[-N_F/2, N_F/2]$. In the absence of synchronization errors, i.e., $\phi_{i,m} = \epsilon_{i,m} = 0$, (2) reduces to

$$y_{i,k} = \sqrt{\frac{N_F}{N_F + N_p}} x_{i,k} g_{i,k} H_k + W_{i,k} \tag{7}$$

which indicates that there is no interference between different carriers when synchronization is perfect. Comparing (7) to (2), we observe that synchronization errors affect the useful component ($A_{i,k,k} \neq g_{i,k} H_k$) and give rise to intercarrier interference ($A_{i,k,k'} \neq 0$ for $k' \neq k$).

In each of the multicarrier systems to be described below, the data symbols to be transmitted are related in a specific way to the sequence $\{\mathbf{x}_i\}$, and the equalizer output sequence $\{\mathbf{y}_i\}$ is processed in a particular way to obtain the decision variables, that are applied to the decision device that makes a decision regarding the transmitted data symbols. The performance of the MC systems is measured by the signal-to-noise ratio (SNR), which is defined as the ratio of the power of the average useful component (P_U) to the sum of the powers of the total interference (P_I) and the noise (P_N) at the input of the decision device. Note that for OFDM(A) and MC-DS-CDMA these quantities will depend on the index of the considered carrier, whereas for MC-CDMA, these quantities are independent of the carrier index.

A. OFDM

The conceptual block diagram of the traditional OFDM system is shown in Fig. 3a. The complex-valued data symbols to be transmitted at rate R_s are organized into blocks of N_c data symbols; $a_{i,k}$ denotes the k^{th} data symbol transmitted within the i^{th} block. The index k belongs to a set I_c of N_c carrier indices. The data symbols $a_{i,k}$ are applied to the IFFT inputs from Fig. 1, i.e., $x_{i,k} = a_{i,k}$.

The traditional OFDM technique is not a multiple access technique as all carriers are modulated with data symbols from the same user. To support multiple users, we combine the OFDM technique with the time-division multiple access (TDMA) scheme. In this case, the time axis is partitioned into a number of non-overlapping time slots, as shown in Fig. 3b. The time slots are grouped into frames of N time slots. During each frame, each user is assigned a time slot. Each time slot consists of a burst of N_B FFT blocks, during which N_c data symbols per FFT block can be transmitted in parallel. We denote by $a_{i,k,\ell}$ the k^{th} data symbol in the i^{th} block to be transmitted to user ℓ. Note that in OFDM, the number N_c of carriers can be chosen independently of the number N of users. In OFDM, the sampling rate equals $1/T = (N_F + N_p)NR_s/N_c$, and the corresponding carrier spacing Δf and system bandwidth B are shown in table I.

In the following, we focus on the detection of the data symbols transmitted to the reference user ($\ell = 0$). The detection of the data symbol $a_{i,k,0}$ is based upon the decision variable $z_{i,k}$ which equals the equalizer output corresponding to the k^{th} carrier.

B. OFDMA

The OFDMA system (Fig. 4) is closely related to traditional OFDM. However, in OFDMA, the data streams that are transmitted on the different carriers belong to different users. Denote by $a_{i,\ell}$ the i^{th} data symbol transmitted at a rate R_s to the ℓ^{th} user. The data symbols $a_{i,\ell}$ belonging to the different users are transmitted in parallel on the N_c carriers. During one OFDMA block, one data symbol per user is transmitted (see Fig. 4b). This indicates that, as each user is assigned a different carrier, the number N_c of carriers equals the maximum number N of users. At maximum load, $x_{i,\ell} = a_{i,\ell}, \ell \in I_c$. The sampling rate equals $1/T = (N_F + N_p)R_s$. The corresponding carrier spacing Δf and system bandwidth B are shown in table I.

The detection of the data symbol $a_{i,0}$, transmitted to the reference user ($\ell = 0$), is based upon $z_{i,0}$, i.e. the equalizer output corresponding to the carrier on which the data of the reference user was transmitted.

C. MC-CDMA

The conceptual block diagram of the MC-CDMA system is shown in Fig. 5a. The data symbols $a_{i,\ell}$ are generated at a rate R_s, where $a_{i,\ell}$ denotes the i^{th} symbol transmitted to user ℓ. Each data symbol is multiplied with a spreading sequence $\{c_{i,n,\ell}|n = 0, \ldots, N_s - 1\}$ with a spreading factor N_s, where $c_{i,n,\ell}$ is the n^{th} chip of the sequence that spreads the data symbols for user ℓ during the i^{th} symbol interval. The N_s components corresponding to the same symbol index i are located in the same time slot on different carriers, i.e. the spreading is done in the frequency domain (Fig. 5b). Each component has a duration of $1/R_s$. The n^{th} chip is mapped on the n^{th} carrier, with n belonging to a set I_c of $N_c = N_s$ carrier indices: $x_{i,n} = a_{i,\ell} c_{i,n,\ell}$. The sampling rate equals $1/T = (N_F + N_p)R_s$ and the corresponding carrier spacing Δf and system bandwidth B are shown in table I.

In a multiuser scenario, the basestation broadcasts to all users the sum of the different user signals. In order that the mobile receiver of the reference user ($\ell = 0$) can distinguish between the different user signals, each user is assigned a unique spreading sequence. In this paper, we assume orthogonal spreading sequences, which consist of user-dependent Walsh-Hadamard (WH) sequences of length N_s, multiplied with a complex-valued random scrambling sequence that is common to all users. Hence, the maximum number of users that can be accommodated equals N_s, i.e. the number of WH sequences of length N_s. This indicates that the maximum number N of users equals the spreading factor N_s, which in turn equals the number N_c of modulated carriers.

In MC-CDMA, the equalizer outputs are multiplied with the complex conjugate of the corresponding chip of the reference user spreading sequence, and summed over the $N_c = N_s$ carriers to obtain the sample z_i, from which a decision is made about the data symbol $a_{i,0}$.

D. MC-DS-CDMA

The conceptual block diagram of the MC-DS-CDMA system is shown in Fig. 6a. In MC-DS-CDMA, the complex data symbols to be transmitted at a rate R_s to user ℓ are split into N_c symbol sequences, each having a rate R_s/N_c, and each modulating a different carrier of the multicarrier system. We denote by

$a_{i,k,\ell}$ the i^{th} symbol sent on carrier k to user ℓ. The data symbol is multiplied with a spreading sequence $\{c_{i,n,\ell}|n = 0, \ldots, N_s - 1\}$ with spreading factor N_s, where $c_{i,n,\ell}$ denotes the n^{th} chip of the sequence that spreads the data symbols transmitted to user ℓ during the i^{th} symbol interval. The N_s components corresponding to the same symbol index i are located in successive time slots on the same carrier (see Fig. 6b); each component $a_{i,k,\ell}c_{i,n,\ell}$ has a duration of $(N_c/N_s)R_s$. The inputs of the multicarrier system of Fig. 1 are $x_{iN_c+n,k} = a_{i,k,\ell}c_{i,n,\ell}$. Note that the spreading sequence does not depend on the carrier index k: all N_c data symbols from user ℓ that are transmitted during the same symbol interval of duration N_c/R_s are spread with the same spreading sequence. The sampling rate equals $1/T = (N_F + N_p)(N_s/N_c)R_s$. The corresponding carrier spacing Δf and system bandwidth B are shown in table I.

In a multiuser scenario, each user is assigned a different spreading sequence. For MC-DS-CDMA, we consider the same set of orthogonal spreading sequences as for MC-CDMA. Note that in MC-DS-CDMA, the number N_c of carriers can be selected independently of the spreading factor N_s, which in turn equals the maximum number N of users.

Each equalizer output $y_{iN_c+n,k}$ is multiplied with the complex conjugate of the corresponding chip $c_{i,n,0}$ of the spreading sequence of the reference user, and summed over N_s consecutive samples to obtain the samples $z_{i,k}$ at the input of the decision device.

E. Comparison of System Parameters

Considering that $N_F \gg N_p$, we observe in table I that, when the load is maximum, the bandwidths occupied by the different MC techniques are the same and equal NR_s Hz. Furthermore, it is observed that, under the same condition, the carrier spacing is the same for OFDM and MC-DS-CDMA (for given ratio of the number N_c of carriers to the number N of users), and for OFDMA and MC-CDMA.

III. CARRIER SYNCHRONIZATION ERRORS

In this section, we compare the sensitivity of the different MC systems to carrier synchronization errors in the absence of clock synchronization errors. To clearly isolate the effect of the carrier synchronization errors, we consider the case of an ideal channel. The case of a dispersive channel will be discussed in section 5. Further, we assume the maximum load (number of active users equals N), and the energy per symbol is equal for all users and all carriers, and is given by E_s. In [15]-[17], it is shown that a constant mismatch between the phases of the carrier oscillator at the transmitter and the receiver can be corrected by the one-tap equalizers without loss of performance, for any of the multicarrier systems. However, in the presence of time-varying carrier phase errors, the system performance will be degraded. In the following, we separately consider the case of a carrier frequency offset and carrier phase jitter.

A. *Carrier Frequency Offset*

A carrier frequency offset ΔF gives rise to $\phi(t) = 2\pi\Delta Ft + \phi(0)$. The effect of a carrier frequency offset on the MC systems is twofold [14]-[18]. First, a carrier frequency offset introduces a frequency shift of the downconverted received signal. This results in signal distortion and power loss at the receiver filter output as a part of the received signal falls outside the bandwidth of the receiver filter. This effect results in an attenuation of the useful component and the introduction of interference at the receiver filter output. Secondly, the carrier frequency offset introduces a rotation at a constant speed of $2\pi\Delta F$ rad/s of the samples at the input of the FFT. This rotation of the FFT input samples gives rise to an additional reduction of the useful component and additional interference at the FFT outputs. Further, the FFT outputs are rotating at a constant speed of $2\pi(N_F + N_p)\Delta FT$ rad/block. The one-tap equalizers are able to compensate for the systematic rotation of the FFT outputs without loss of performance. However, the equalizer is not able to correct for the reduction of the useful component without enhancing the noise power level, nor to eliminate the interference. Hence, the MC systems are degraded in the presence of a carrier frequency offset. The degradation at the receiver filter output (caused by the frequency shift of the downconverted signal) is negligibly small as compared to the degradation at the FFT outputs (caused by the rotation of the receiver filter output samples).

When the carrier frequency offset is larger than the carrier spacing ($\Delta F > \Delta f$) of the MC system, the resulting degradation is very large, as the data transmitted on carrier k are strongly attenuated at the k^{th} FFT output. In order to keep the degradation within reasonable bounds, we restrict our attention to carrier frequency offsets smaller than the carrier spacing ($\Delta F < \Delta f$). For both OFDM(A) and MC-DS-CDMA, it can be verified [14]-[15], [17] that the powers of the average useful component, the interference and the noise (under the assumption of the maximum load and all carriers having the same energy per symbol) are given by

$$P_{U_k} = \frac{N_F}{N_F + N_p} E_s \left| D_{N_F}(\Delta FT) \right|^2 \tag{8}$$

$$P_{I_k} = \frac{N_F}{N_F + N_p} E_s \sum_{k' \in I_c; k' \neq k} \left| D_{N_F}\left(\frac{k' - k}{N_F} + \Delta FT \right) \right|^2 \tag{9}$$

$$P_{N_k} = N_0, \tag{10}$$

where

$$D_{N_F}(x) = \frac{1}{N_F} \sum_{m=0}^{N_F - 1} e^{j2\pi mx} = e^{j\pi(N_F - 1)x} \frac{\sin(\pi N_F x)}{N_F \sin(\pi x)}. \tag{11}$$

The total interference power (9) depends on the number N_c of modulated carriers, as the summation over k' ranges over the set I_c of N_c modulated carriers. A simple upper bound on the interference power is obtained by extending the summation over all N_F available carriers, i.e. $k' = 0, \ldots, N_F - 1$. This yields

$$P_{I_k} \leq \frac{N_F}{N_F + N_p} E_s \left(1 - |D_{N_F}(\Delta FT)|^2\right). \quad (12)$$

Note that the powers of the useful component (8), the noise (10) and the upper bound on the interference power (12) are independent of the carrier index k. In the following, we drop the index k.

We approximate the powers of the average useful component and the interference by a truncated Taylor series, keeping up to quadratic terms, around $\Delta FT = 0$. For $|N_F \Delta FT| \ll 1$, this approximation yields

$$P_U \approx \frac{N_F}{N_F + N_p} E_s \quad (13)$$

$$P_I \approx \frac{N_F}{N_F + N_p} E_s \frac{1}{3} \left(\pi N_F \Delta FT\right)^2. \quad (14)$$

From (13) and (14) it follows that the main effect of a carrier frequency offset is the interference, of which the power quadratically increases with the frequency offset. Further, we observe that the attenuation of the useful component is negligibly small, when $|N_F \Delta FT| \ll 1$. The corresponding degradation of the signal-to-noise ratio, caused by the presence of the carrier frequency offset is given by

$$Deg \approx 10 \log \left(1 + SNR(0) \frac{1}{3} \left(\pi N_F \Delta FT\right)^2\right), \quad (15)$$

where $SNR(0) = (N_F/(N_F + N_p))(E_s/N_0)$ is the signal-to-noise ratio in the absence of synchronization errors.

It can be verified [16] that for MC-CDMA, the powers of the average useful component, the interference and the noise are obtained by arithmetically averaging of the corresponding powers for OFDM(A) and MC-DS-CDMA over all modulated carriers $k \in I_c$. As (8), (10) and (12) are independent of the carrier index, the powers of the average useful component, the interference and the noise in MC-CDMA are the same as in OFDM(A) and MC-DS-CDMA. Hence the degradation for MC-CDMA can also be approximated by (15).

The degradation (15) is independent of the carrier index, and depends only on $SNR(0)$ [1] and on the ratio of the carrier frequency offset to the carrier spacing, i.e. $N_F \Delta FT = \Delta F/\Delta f = N_c \Delta F/B = (N_c/N)\Delta F/R_s$. Hence, for given $\Delta F/R_s$, the degradation strongly increases with the ratio N_c/N. When $N_c = N$, the degradation for all MC systems is the same. Hence, the ratio N_c/N determines which of the MC systems is more sensitive to a carrier frequency offset: when this ratio is larger (smaller) than 1, OFDM and MC-DS-CDMA are more (less) sensitive than OFDMA and MC-CDMA (for which $N_c = N$). Fig. 7 shows the degradation (15) for the different multicarrier systems as function of $(N_c/N)\Delta F/R_s$, along with the actual degradation. As can be observed, the approximation (15) corresponds well to the actual degradation when $|N_F \Delta FT| \ll 1$. To obtain small degradations, the carrier frequency offset must

[1] The value of the BER corresponding to $SNR(0)$ depends on the considered (normalized) constellation.

be kept small as compared to the carrier spacing, i.e. $|\Delta F/\Delta f| \ll 1$. In this case, the degradation is proportional to $(N_F \Delta F T)^2 = (\Delta F/\Delta f)^2 = (N_c \Delta F/B)^2 = ((N_c/N)\Delta F/R_s)^2$.

B. Carrier Phase Jitter

When the degradation caused by the carrier frequency offset can not be tolerated, the carrier frequency offset must be corrected in front of the FFT by means of a synchronization algorithm. In this case, the MC system is affected only by the phase jitter resulting from the synchronizer. The carrier phase jitter $\phi(t)$ is modelled as a zero-mean stationary random process with jitter spectrum $S_\phi(f)$ and jitter variance σ_ϕ^2 [15]-[16], [19]-[21]. For small jitter variances, i.e. $\sigma_\phi^2 \ll 1$, the phase rotation $\exp\{j\phi(t)\}$ at the FFT outputs can be approximated by the Taylor series: $\exp\{j\phi(t)\} \approx 1 + j\phi(t)$. When the load is maximum and all users and all carriers have the same energy per symbol, it can be verified [15], [19], [21] that OFDM(A) and MC-DS-CDMA yield the same expression for the power of the average useful component, the interference and the noise:

$$P_{U_k} = \frac{N_F}{N_F + N_p} E_s \tag{16}$$

$$P_{I_k} = \frac{N_F}{N_F + N_p} E_s \sum_{k' \in I_c} \int_{-\infty}^{+\infty} S_\phi(f) \left| D_{N_F}\left(\frac{k'-k}{N_F} + fT\right)\right|^2 df \tag{17}$$

$$P_{N_k} = N_0. \tag{18}$$

As the summation over k' in (17) ranges over the set I_c, the interference power depends on the number N_c of modulated carriers. By extending the summation interval over all N_F available carriers ($k' = 0, \ldots, N_F - 1$), a simple upper bound on the interference power is found:

$$P_{I_k} \leq \frac{N_F}{N_F + N_p} E_s \sigma_\phi^2, \tag{19}$$

where the jitter variance is given by

$$\sigma_\phi^2 = \int_{-\infty}^{+\infty} S_\phi(f) df. \tag{20}$$

Note that the powers of the average useful component (16), the noise (18) and the upper bound on the power of the interference (19) are independent of the carrier index. It can be verified [16], [20] that for MC-CDMA, the powers of the average useful component, the interference and the noise are obtained by arithmetically averaging the corresponding powers in OFDM(A) or MC-DS-CDMA over all modulated carriers $k \in I_c$. As these powers in OFDM(A) and MC-DS-CDMA turn out to be independent of the carrier index, it follows that the sensitivity to carrier phase jitter is the same for all multicarrier systems.

The corresponding degradation is independent of the carrier index, the spectral contents of the jitter, the spreading factor N_s, the number of carriers N_c and the maximum number N of users, but only depends on the jitter variance:

$$Deg \approx 10\log\left(1 + SNR(0)\sigma_\phi^2\right), \tag{21}$$

The performance degradation caused by the carrier phase jitter is shown in Fig. 8. Further, the exact degradation, assuming the carrier phase jitter is Gaussian distributed, is shown. As we observe, the degradation (21) yields a good approximation for the actual degradation. For small jitter variances, the degradation (21) is proportional to σ_ϕ^2.

IV. Clock Synchronization Errors

In this section, we compare the effect of clock synchronization errors on the different multicarrier systems in the absence of carrier synchronization errors. To clearly isolate the effect of the clock synchronization errors, we consider the case of an ideal channel. The case of a dispersive channel will be discussed in section 5. Further, the load is taken maximum (number of active users equal to N), and the energy per symbol is equal to E_s for all carriers and all users. In [15]-[16], it is shown that a constant mismatch between the phases of the clocks at the transmitter and the receiver does not introduce a performance degradation, provided that the carriers inside the rolloff area are not modulated, and the cyclic prefix is sufficiently long. However, in the presence of time-varying timing errors, the performance will degrade. In the following, we separately consider the case of a clock frequency offset and timing jitter.

A. Clock Frequency Offset

When the receiver of the reference user has a free-running clock with a relative clock frequency offset $\Delta T/T$ ($\Delta T/T \ll 1$) as compared to the frequency $1/T$ of the basestation clock, the timing deviation linearly increases with time: $\epsilon_{i,m} = \epsilon_0 + (m + i(N_F + N_p))\Delta T/T$ [15]-[16], [22]-[23]. Hence, an increasing misalignment between the time-domain samples at the transmitter and the receiver is introduced. The receiver performs a coarse synchronization by selecting the sample $m = 0$ between the earliest and latest possible timing, indicated in Fig. 2, such that the N_F successive samples kept for further processing remain in the region where interference from adjacent blocks is absent. After coarse synchronization, the timing deviation is given by $\epsilon_{i,m} = \epsilon_i + m\Delta T/T$, where ϵ_i is the timing deviation of the first of the N_F samples of the considered block that are processed by the receiver.

The clock frequency offset gives rise to a reduction of the useful component and to interference at the FFT outputs. Hence, the MC systems are degraded by a clock frequency offset. When all users and all carriers exhibit the same energy per symbol, and the load is maximum, it can be verified [15], [22]-[23] that for OFDM(A) and MC-DS-CDMA the powers of the average useful component, the interference and the noise are given by

$$P_{U_k} = \frac{N_F}{N_F+N_p} E_s \left| D_{N_F}\left(\frac{mod(k;N_F)}{N_F}\frac{\Delta T}{T}\right)\right|^2 \quad (22)$$

$$P_{I_k} = \frac{N_F}{N_F+N_p} E_s \sum_{k'\in I_c;k'\neq k} \left| D_{N_F}\left(\frac{k'-k}{N_F}+\frac{mod(k';N_F)}{N_F}\frac{\Delta T}{T}\right)\right|^2 \quad (23)$$

$$P_{N_k} = N_0. \quad (24)$$

A simple but accurate approximation for the powers of the average useful component (22) and the interference (23) can be found when $|N_F \Delta T/T| \ll 1$ by using truncated Taylor series around $\Delta T/T = 0$, where only terms up to quadratic are kept:

$$P_{U_k} \approx \frac{N_F}{N_F+N_p} E_s \quad (25)$$

$$P_{I_k} \approx \frac{N_F}{N_F+N_p} E_s \frac{1}{3}\left(\pi mod(k;N_F)\frac{\Delta T}{T}\right)^2. \quad (26)$$

From (25) and (26), we observe that the clock frequency offset mainly causes interference; the interference power turns out to be proportional to the square of the clock frequency offset when $|N_F \Delta T/T| \ll 1$. Further, when $|N_F \Delta T/T| \ll 1$, the useful component is essentially not attenuated. The corresponding degradation yields

$$Deg_k \approx 10\log\left(1+SNR(0)\frac{1}{3}\left(\pi mod(k;N_F)\frac{\Delta T}{T}\right)^2\right). \quad (27)$$

The degradation (27) depends on the carrier index and becomes maximum for carriers close to the rolloff area. We define the average degradation by replacing, in the expression of the SNR, the interference power by its arithmetical average over the modulated carriers. The corresponding degradation yields [23]

$$Deg_{Av} \approx 10\log\left(1+SNR(0)\left(\frac{\pi}{6}N_F\frac{\Delta T}{T}\right)^2\right). \quad (28)$$

In MC-CDMA, the powers of the average useful component, the interference and the noise are obtained by arithmetically averaging the corresponding powers of OFDM(A) and MC-DS-CDMA, over all modulated carriers $k \in I_c$ [16]. The resulting powers can be approximated by truncated Taylor series around $\Delta T/T = 0$, similar as in OFDM(A) and MC-DS-CDMA. The corresponding degradation equals the average degradation (28) of OFDM(A) and MC-DS-CDMA.

For all multicarrier systems, the degradation strongly increases with $N_F \Delta T/T$. Hence, for given $\Delta T/T$, the degradation rapidly increases when the number N_c ($\lesssim N_F$) of modulated carriers increases. The (average) degradation, caused by a clock frequency offset is shown in Fig. 9, along with the actual (average) degradation. As can be observed, the approximation (28) corresponds well to the actual degradation when $|N_F \Delta T/T| \ll 1$. For $N_c = N$, all MC systems exhibit the same degradation. For the systems where the number of carriers can be chosen independently of the number of users (i.e. OFDM and MC-DS-CDMA), the average degradation is larger or smaller than for OFDMA and MC-CDMA, depending on whether $N_c > N$ or $N_c < N$, i.e. when the number of carriers is larger or smaller than the number of users. To obtain small degradations, it is required that $|N_F \Delta T/T| \ll 1$, in which case the degradation is proportional to $(N_F \Delta T/T)^2$.

B. Timing Jitter

To avoid the degradation associated with a clock frequency offset, we can perform synchronized sampling at the output of the receiver filter. When a timing synchronization algorithm is used to adjust the sampling clock, the MC systems are affected only by the timing jitter resulting from the synchronizer. The timing jitter $\epsilon_{i,m}T$ is modelled as a zero-mean stationary random process with jitter spectrum $S_\epsilon(e^{j2\pi fT})$ and jitter variance σ_ϵ^2 [15]-[16], [24]-[25]. When the jitter variance is small, i.e. $\sigma_\epsilon^2 \ll 1$, the phase rotation $\exp(j2\pi mod(k;N_F)/N_F\epsilon_{i,m})$ at the FFT outputs can be approximated by a truncated Taylor series: $\exp(j2\pi mod(k;N_F)/N_F\epsilon_{i,m}) \approx 1 + j2\pi mod(k;N_F)/N_F\epsilon_{i,m}$.

In OFDM(A) and MC-DS-CDMA ([15], [25]), the powers of the average useful component, the interference and the noise turn out to be the same, when the load is maximum and all users and all carriers have the same energy per symbol:

$$P_{U_k} = \frac{N_F}{N_F + N_p} E_s \tag{29}$$

$$P_{I_k} = \frac{N_F}{N_F + N_p} E_s \sum_{k' \in I_c} \left(\frac{2\pi mod(k';N_F)}{N_F} \right)^2 \cdot \int_{-\frac{1}{2T}}^{+\frac{1}{2T}} S_\phi(e^{j2\pi fT}) \left| D_{N_F}\left(\frac{k'-k}{N_F} + fT \right) \right|^2 df \tag{30}$$

$$P_{N_k} = N_0. \tag{31}$$

The total interference power depends on the carrier index k and becomes maximum for carriers close to the rolloff area. We define the average degradation, by replacing in the expression of the SNR, the powers of the average useful component, the interference and the noise by their arithmetical average over all modulated carriers. The average degradation still depends on the number of modulated carriers, as the summation over the carrier indices k and k' ranges over the set I_c of N_c modulated carriers. A simple upper bound on the average degradation is found by extending the summations over all N_F available carriers, i.e. $k, k' = 0, \ldots, N_F - 1$. The corresponding degradation yields

$$Deg_{Av} \leq 10 \log \left(1 + SNR(0) \frac{\pi^2}{3} \sigma_\epsilon^2 \right), \tag{32}$$

where the jitter variance is given by

$$\sigma_\epsilon^2 = \int_{-\frac{1}{2T}}^{+\frac{1}{2T}} S_\epsilon(e^{j2\pi fT})df. \tag{33}$$

This average degradation is independent of the spectral contents of the jitter, the spreading factor, the number of carriers and the number of users, but only depends on the jitter variance.

In MC-CDMA, the powers of the average useful component, interference and noise are obtained by arithmetically averaging the corresponding powers of OFDM(A) and MC-DS-CDMA over all modulated carriers $k \in I_c$ [16]. Hence, the degradation of MC-CDMA is the same as the average degradation of OFDM(A) and MC-DS-CDMA, and can be upper bounded by (32).

The (average) degradation (32) caused by timing jitter is shown in Fig. 10. Further, the exact average degradation, assuming the timing jitter is Gaussian distributed, is shown. As we observe, the degradation (32) yields a good approximation for the actual degradation. For small jitter variances, the degradation is proportional to σ_ϵ^2.

V. Conclusions and Remarks

In this paper, we have presented simple but accurate analytical expressions for the degradations caused by different types of synchronization errors, and compared the sensitivity of several multicarrier systems to these synchronization errors. For the maximum load, an ideal channel and the energy per symbol equal for all carriers and all users, the results can be summarized as follows.

- The degradation caused by a carrier frequency offset is proportional to $((N_c/N)\Delta F/R_s)^2$, with $\Delta F/R_s$ denoting the ratio of the carrier frequency offset to the symbol rate. The degradation is independent of the carrier index. For given $\Delta F/R_s$, the degradation depends on the ratio N_c/N; when the ratio N_c/N is larger (smaller) than 1, OFDM and MC-DS-CDMA yield a larger (smaller) degradation than OFDMA and MC-CDMA (where $N_c = N$).
- The MC systems are very sensitive to a clock frequency offset. The degradations of OFDM(A) and MC-DS-CDMA depend on the carrier index, and are proportional to $(N_F\Delta T/T)^2$. When $N_c/N = 1$, the average degradation of OFDM(A) and MC-DS-CDMA is the same as the degradation of MC-CDMA. This degradation strongly increases with $N_F\Delta T/T$, and does not depend on the spreading factor. For given $\Delta T/T$, the sensitivity depends on the ratio N_c/N. For N_c/N larger (smaller) than 1, OFDM and MC-DS-CDMA are more (less) sensitive than OFDMA and MC-CDMA. To obtain a small degradation, it is required that $|N_F\Delta T/T| \ll 1$.
- All MC systems exhibit the same sensitivity to carrier phase jitter. The degradation is independent of the carrier index, the spreading factor, the number of carriers, the number of users and the spectral contents of the jitter. The degradation only depends on the jitter variance.

- Timing jitter causes a degradation that depends on the carrier index for OFDM(A) and MC-DS-CDMA. The average (over all FFT outputs) degradation of OFDM(A) and MC-DS-CDMA is equal to the degradation for MC-CDMA. This degradation is independent of the spreading factor, the number of carriers, the number of users and the spectral contents of the jitter, but only depends on the jitter variance.

In this paper, we separately considered the effect of the different synchronization errors. For small synchronization errors, i.e. for which the approximations obtained in this paper are valid, it can be verified that the degradation, caused by two or more types of synchronization errors, can be approximated by the sum of the degradations caused by the different synchronization errors separately.

In this paper, we have restricted our attention to the effect of synchronization errors in the downlink, assuming an ideal channel. This analysis can be extended for the uplink and/or slowly varying multipath fading channels (wide-sense stationary uncorrelated scattering (WSSUS) multipath fading channel).

- In the considered OFDM system, the signals of the different users do not interfere as they are physically separated in time. As a result, the analysis for the uplink system reduces, similar as in the downlink, to the analysis of a single user system. Hence, assuming an ideal channel, the effect of the synchronization errors on uplink OFDM is the same as in the downlink [26].
- When all users exhibit the same jitter spectra, it can be verified [19], [21], [25]-[26] that the degradation caused by carrier phase jitter or timing jitter for OFDMA and MC-DS-CDMA, for an ideal channel, is the same in the uplink and the downlink.
- In uplink OFDMA and MC-DS-CDMA, the contributions of all users are affected by a different carrier or clock frequency offset. As a result, there is a larger amount of interference from the other users as compared to the downlink, where the carrier or clock frequency offset are the same for all users. Hence, assuming an ideal channel, the degradation caused by carrier or clock frequency offset in uplink OFDMA and MC-DS-CDMA is larger than in the downlink [17], [23], [26].
- It can be shown that for both uplink and downlink OFDM(A) and MC-DS-CDMA, assuming perfect power control, the degradations in the presence of a slowly varying multipath fading channel, caused by synchronization errors, are the same as in the case of an ideal channel [17], [23], [26].
- In downlink MC-CDMA, similar expressions for the degradation caused by the different types of synchronization errors can be obtained in the presence of a slowly varying multipath fading channel [26].
- In uplink MC-CDMA, the different user signals are not aligned in time, and are sent over different multipath fading channels. This causes significant interference between the user signals that cannot be compensated by one-tap equalizers, even in the absence of synchronization errors. To combat the

interference in uplink MC-CDMA, a much more complicated receiver structure is needed. Hence, the simple analytical expressions obtained in this paper cannot easily be extended to the case of uplink MC-CDMA.

REFERENCES

[1] J.A.C. Bingham. 1990. Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come. *IEEE Comm. Mag.* 1990; **28**(5):5-14.

[2] P.S. Chow, J.C. Tu, J.M. Cioffi. 1991. Performance Evaluation of a Multichannel Transceiver System for ADSL and VHDSL. *IEEE J. on Select. Areas in Comm.* 1991; **9**(8):909-919.

[3] H. Sari, G. Karam, I. Jeanclaude. 1995. Transmission Techniques for Digital Terrestrial TV Broadcasting. *IEEE Comm. Mag.* 1995; **33**(2):100-109.

[4] G. Santella. 1995. Bit Error Rate Performances of M-QAM Orthogonal Multicarrier Modulation in Presence of Time-Selective Multipath Fading. *in Proceedings ICC'95 Seattle, WA*:1683-1688.

[5] IEEE 802.11. 1997. *IEEE standard for Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*.

[6] ETSI. 1996. Radio Equipment and Systems, HIgh PErformance Radio Local Area Network (HIPERLAN) Type 1. *European Telecommunications Standard, ETS 300-652*

[7] H. Sari, Y. Lévy and G. Karam. 1996. Orthogonal Frequency-Division Multiple Access for the Return Channel on CATV Networks. *in Proceedings International Conference on Telecommunications ICT'96, Istanbul*:602-607.

[8] S. Hara, R. Prasad. 1997. Overview of Multicarrier CDMA. *IEEE Communications Magazine* 1997; **35**(12):126-133.

[9] K. Fazel, L. Papke. 1993. On the Performance of Convolutionally Sequenced CDMA/OFDM for Mobile Communication System. *in Proceedings IEEE PIMRC'93, Yokohama, Japan*:468-472.

[10] N. Yee, J-P. Linnartz, G. Fettweis. 1993. Multicarrier CDMA in Wireless Radio Networks, *in Proceedings IEEE PIMRC'93, Yokohama, Japan*:109-113.

[11] A. Chouly, A. Brajal, S. Jourdan. 1993. Orthogonal Multicarrier Techniques Applied to Direct Sequence Spread Spectrum CDMA techniques. *in Proceedings IEEE Globecom'93, Houston, USA*:1723-1728.

[12] V.M. DaSilva, E.S. Sousa. 1993. Performance of Orthogonal CDMA Sequences for Quasi-Synchronous Communication Systems. *in Proceedings IEEE ICUPC'93, Ottawa, Canada*:995-999.

[13] S. Kondo, L.B. Milstein. 1996. Performance of Multicarrier DS-CDMA Systems. *IEEE Transactions on Communications* 1996; **44**(2):238-246.

[14] T. Pollet, M. Moeneclaey. 1996. The Effect of Carrier Frequency Offset on the Performance of Band-Limited Single Carrier and OFDM Signals. *in Proceedings Globecom 96, London*:719-723.

[15] H. Steendam, M. Moeneclaey. 2000. Sensitivity of Orthogonal Frequency-Division Multiplexed Systems to Carrier and Clock Synchronisation Errors. *Signal Processing* 2000; **80**(7):1217-1229.

[16] H. Steendam, M. Moeneclaey. 1999. The Sensitivity of MC-CDMA to Synchronisation Errors. *European Trans. on Telecomm. ETT special issue on MC-SS*; **10**(4):429-436.

[17] H. Steendam, M. Moeneclaey. 2001. The Effect of Carrier Frequency Offsets on Downlink and Uplink MC-DS-CDMA. *IEEE Journal on Sel. Areas in Communications* 2001; **19**(12):2528-2536.

[18] L. Tomba and W.A. Krzymien. 1996. Effect of Carrier Phase Noise and Frequency Offset on the Performance of Multicarrier CDMA Systems. *in Proceedings ICC 1996, Dallas TX*:1513-1517.

[19] H. Steendam, M. Moeneclaey, H. Sari. 1998. The Effect of Carrier Phase Jitter on the Performance of Orthogonal Frequency-Division Multiple Access Systems. *IEEE Trans. on Comm.* 1998; **46**(4):456-459.

[20] H. Steendam, M. Moeneclaey. 1999. The Effect of Carrier Phase Jitter on MC-CDMA. *IEEE Trans. on Comm.* 1999; **47**(2):195-198.

[21] H. Steendam, M. Moeneclaey. 2001. The Effect of Carrier Phase Jitter on MC-DS-CDMA. *in Proceedings ICC'01, Helsinki, Finland*:1881-1884.

[22] T. Pollet, M. Moeneclaey. 1995. Synchronizability of OFDM Signals. *in Proceedings Globecom 95, Singapore*:2054-2058.

[23] H. Steendam, M. Moeneclaey. 2004. A Comparison between Uplink and Downlink MC-DS-CDMA Sensitivity to Static Clock Phase and Frequency Offsets. *submitted to IEEE Transactions on Wireless Communications*.

[24] L. Tomba and W.A. Krzymien. 1998. A Model for the Analysis of Timing Jitter in OFDM Systems. *in Proceedings ICC 1998, Atlanta GA*:1227-1231.

[25] H. Steendam, M. Moeneclaey. 2004. The Effect of Timing Jitter on MC-DS-CDMA. *to appear in IEEE Transactions on Communications*; **52**(3).

[26] H. Steendam. *The Effect of Synchronization Systems on Multicarrier Systems*. PhD thesis, 2000. http://telin.UGent.be/ hs/full/PHD.zip.

TABLE I

OVERVIEW OF SYSTEM PARAMETERS ($\beta = (N_F + N_p)/N_F \approx 1$ WHEN $N_F \gg N_p$)

	Δf	B
OFDM	$\beta\frac{NR_s}{N_c}$	βNR_s
OFDMA	βR_s	βNR_s
MC-CDMA	βR_s	βNR_s
MC-DS-CDMA	$\beta\frac{N_sR_s}{N_c}$	βN_sR_s

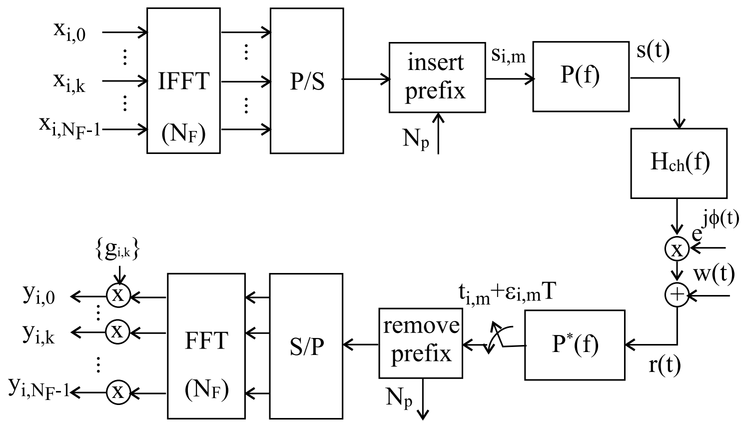

Fig. 1. Multicarrier system

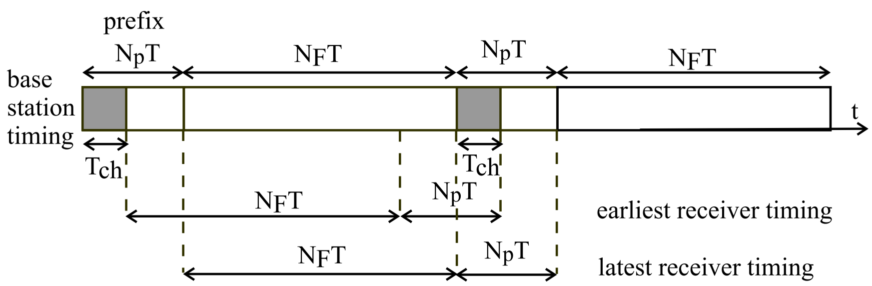

Fig. 2. Earliest and latest possible timing to avoid interference between FFT blocks

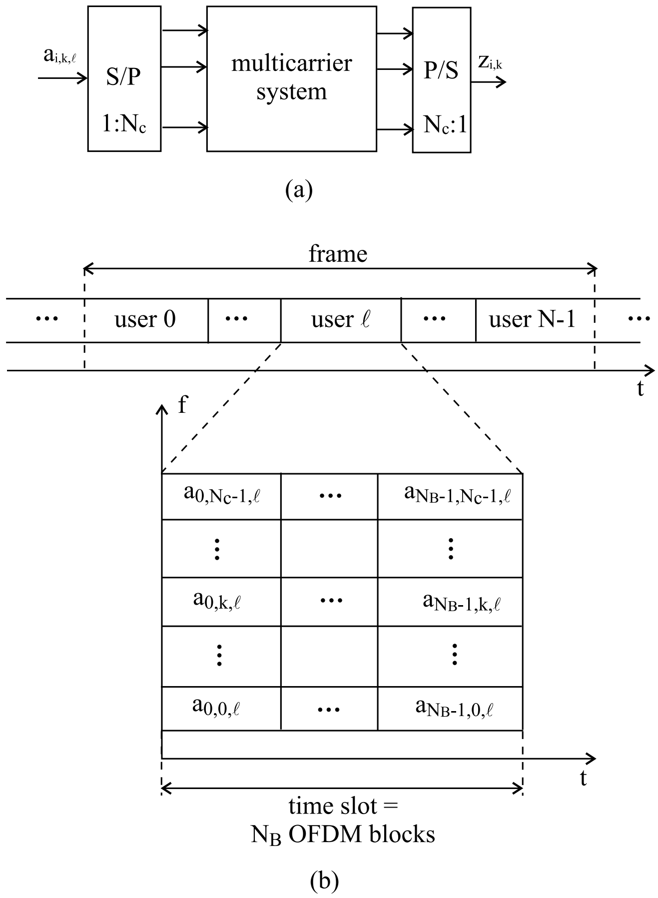

Fig. 3. The OFDM system (a) transceiver (b) transmitted data

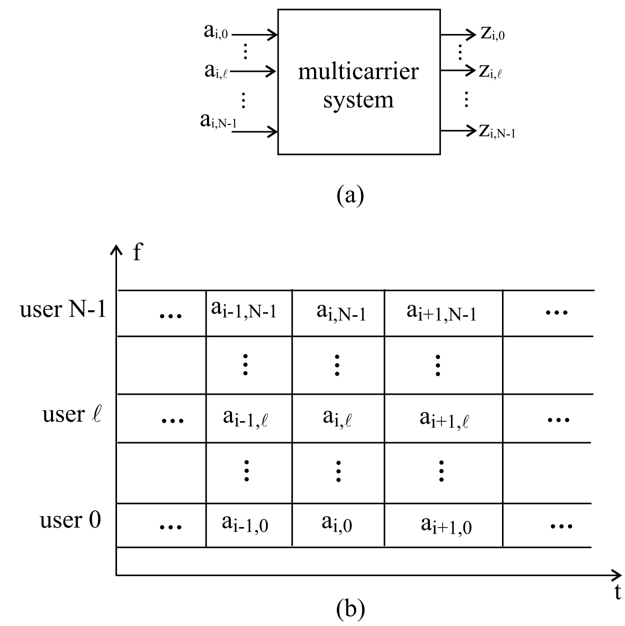

Fig. 4. The OFDMA system (a) transceiver (b) transmitted data ($N_c = N$)

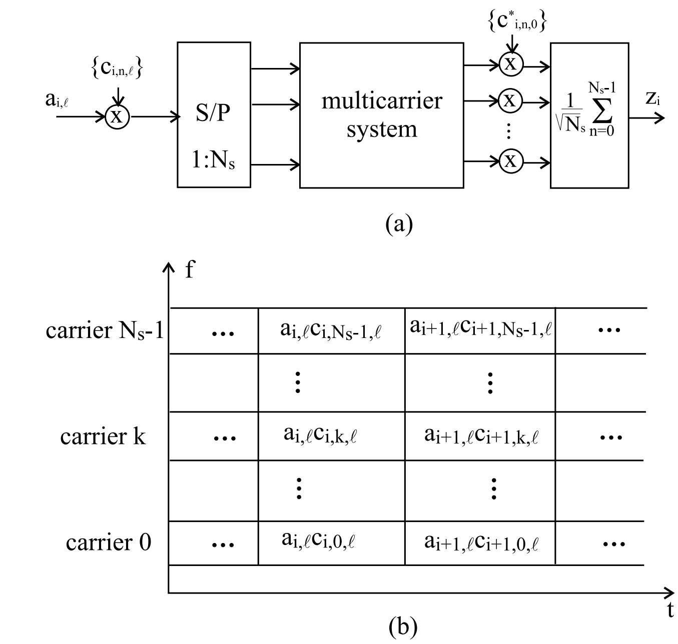

Fig. 5. The MC-CDMA system (a) transceiver (b) transmitted data ($N_s = N_c = N$)

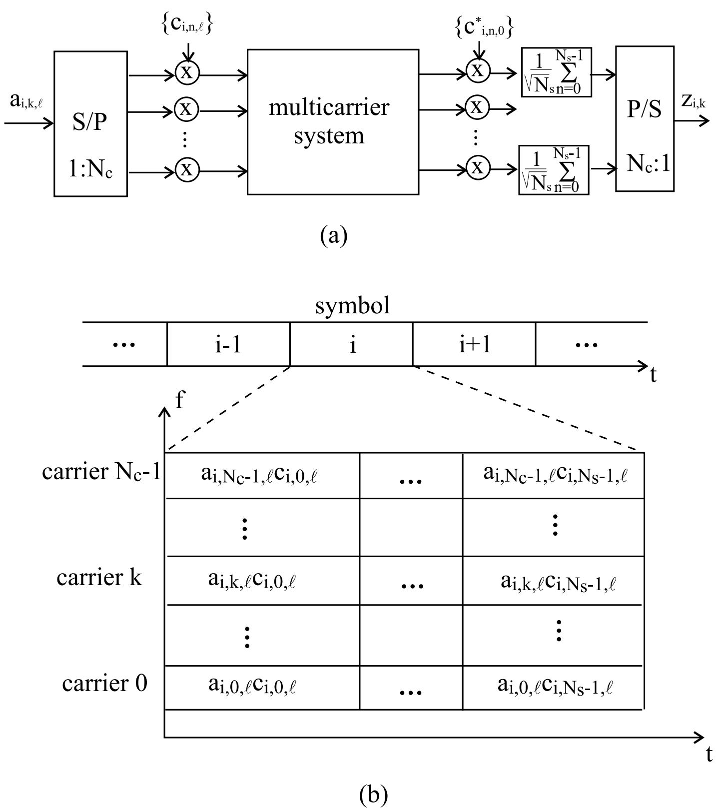

Fig. 6. The MC-DS-CDMA system (a) transceiver (b) transmitted data ($N_s = N$)

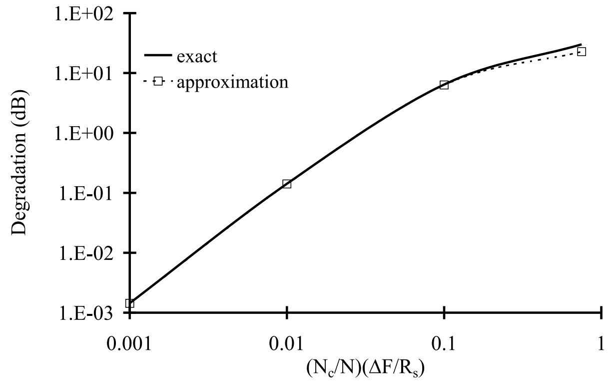

Fig. 7. Degradation caused by carrier frequency offset ($SNR(0) = 20dB$)

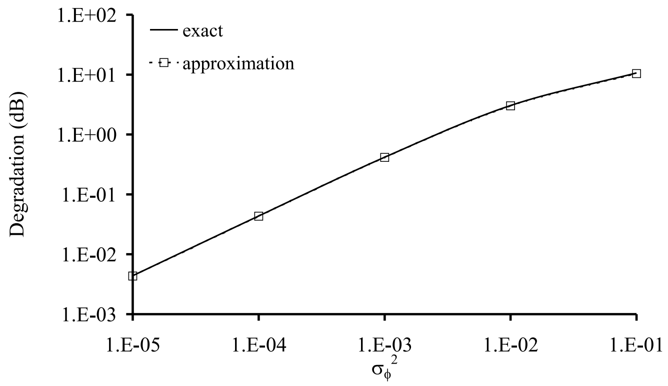

Fig. 8. Degradation caused by carrier phase jitter ($SNR(0) = 20dB$)

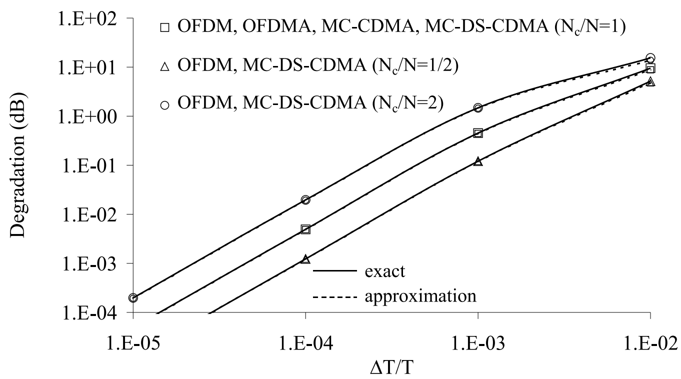

Fig. 9. Degradation caused by clock frequency offset ($N = 64,\ SNR(0) = 20dB, N_F = N_c$)

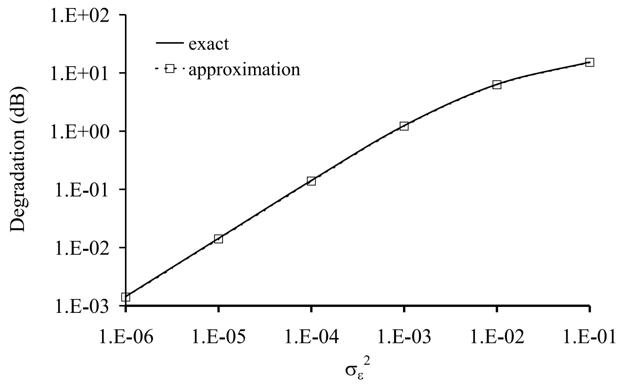

Fig. 10. Degradation caused by timing jitter ($SNR(0) = 20dB$)