BER Analysis of Square OSTBCs with LMMSE Channel Estimation in Arbitrarily Correlated Rayleigh Fading Channels

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***Abstract*—In this paper, we examine the bit error rate (BER) performance of square orthogonal space-time block codes (OSTBCs) under arbitrarily correlated Rayleigh fading channels. We consider a mismatched maximum-likelihood receiver that obtains the channel state information through pilot-based linear minimum mean-square error channel estimation. For PAM and QAM constellations, a closed-form approximation of the BER is presented, which yields very accurate BER results over a wide range of signal-to-noise ratios.**

***Index Terms*—Space-time block coding, imperfect channel estimation, error analysis, correlated Rayleigh fading.**

I. Introduction

ORTHOGONAL space-time block codes (OSTBCs) [1], [2] have become a popular transmit diversity technique, since they combine the ability to achieve full spatial diversity with a remarkably simple symbol-by-symbol decoding algorithm, based on linear processing at the receiver. For pulse amplitude modulation (PAM), quadrature amplitude modulation (QAM), and phase-shift keying (PSK) constellations, the exact bit error rate (BER) of OSTBCs was derived in [3] under the assumption of correlated Rayleigh fading channels with perfect channel state information (PCSI). In practice, however, PCSI is not available and the receiver has to estimate the channel response. For independent and identically distributed (i.i.d.) Rayleigh fading channels with linear minimum mean-square error (LMMSE) channel estimation, an exact closed-form BER expression was presented in [4] for PAM and QAM constellations. A closed-form expression for the pairwise error probability (PEP) of space-time block codes under arbitrarily correlated Ricean fading channels with imperfect channel estimation (ICE) was given in [5]. In this contribution, we extend the result from [4] to arbitrarily correlated Rayleigh fading channels. Introducing a high signal-to-noise ratio (SNR) approximation of the channel error covariance matrix, we obtain a closed-form BER expression for OSTBCs with LMMSE channel estimation, which we verify to be very accurate in the range from low to high SNR. We denote by $\mathrm{vec}(\mathbf{X})$ the vector that is obtained by stacking the columns of the matrix $\mathbf{X}$, and by $\mathbf{A} \otimes \mathbf{B}$ the Kronecker product of the matrices $\mathbf{A}$ and $\mathbf{B}$.

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II. Signal Model

We consider a multiple-input multiple-output (MIMO) OSTBC system with L_t transmit and L_r receive antennas. Each square OSTBC from a complex orthogonal design [1], [2] is defined by an $L_\mathrm{t} \times L_\mathrm{t}$ coded symbol matrix $\mathbf{C}$, the entries of which are linear combinations of N_s information symbols $s_i = s_{i,\mathrm{R}} + j s_{i,\mathrm{I}}$, $1 \leq i \leq N_\mathrm{s}$, with $s_{i,\mathrm{R}}$ and $s_{i,\mathrm{I}}$ denoting the real and imaginary parts of s_i, respectively, and their complex conjugate s_i^*, such that

$$\mathbf{C} = \sum_{i=1}^{N_\mathrm{s}} \left(\mathbf{C}_i \, s_i + \mathbf{C}_i^{'} \, s_i^* \right) \tag{1}$$

where the $L_\mathrm{t} \times L_\mathrm{t}$ matrices $\mathbf{C}_i$ and $\mathbf{C}_i^{'}$ comprise the coefficients of s_i and s_i^*, respectively. Without loss of generalization, we scale $\mathbf{C}$ in such way that it satisfies the following orthogonality condition

$$\mathbf{C}^H \mathbf{C} = \mathbf{C} \, \mathbf{C}^H = \lambda \, \|\mathbf{s}\|^2 \, \mathbf{I}_{L_\mathrm{t}} \tag{2}$$

where $\lambda \triangleq L_\mathrm{t}/N_\mathrm{s}$, $\mathbf{s} = [s_1, s_2, \ldots, s_{N_\mathrm{s}}]^T$ is the data symbol vector, and $\mathbf{I}_{L_\mathrm{t}}$ denotes the $L_\mathrm{t} \times L_\mathrm{t}$ identity matrix.

We assume that each transmitted data frame consists of K_p known pilot symbols and K coded data symbols per transmit antenna, with K being a multiple of L_t. In this way, K/L_t coded data matrices $\mathbf{C}(k)$, with $1 \leq k \leq K/L_\mathrm{t}$, are sent within one frame. Furthermore, we use orthogonal pilot sequences, i.e., the $L_\mathrm{t} \times K_\mathrm{p}$ pilot matrix $\mathbf{C}_\mathrm{p}$ has orthogonal rows such that

$$\mathbf{C}_\mathrm{p} \mathbf{C}_\mathrm{p}^H = K_\mathrm{p} \mathbf{I}_{L_\mathrm{t}}. \tag{3}$$

We represent the $L \triangleq L_\mathrm{t} \, L_\mathrm{r}$ MIMO channel coefficients by the $L_\mathrm{r} \times L_\mathrm{t}$ complex-valued random matrix $\mathbf{H}$, which is assumed to remain constant during the length of one frame of $K + K_\mathrm{p}$ symbols, such that the receiver separately observes the $L_\mathrm{r} \times L_\mathrm{t}$ matrices

$$\mathbf{R}(k) = \sqrt{E_\mathrm{s}} \, \mathbf{H} \, \mathbf{C}(k) + \mathbf{W}(k) \tag{4a}$$

with $1 \leq k \leq K/L_\mathrm{t}$, and the $L_\mathrm{r} \times K_\mathrm{p}$ matrix

$$\mathbf{R}_\mathrm{p} = \sqrt{E_\mathrm{p}} \, \mathbf{H} \, \mathbf{C}_\mathrm{p} + \mathbf{W}_\mathrm{p} \tag{4b}$$

where the channel noise matrices $\mathbf{W}(k)$ and $\mathbf{W}_\mathrm{p}$ consist of i.i.d. zero-mean (ZM) circularly symmetric complex Gaussian (CSCG) random variables (RVs) with variance N_0. Taking (2) and (3) into account and using a normalized symbol constellation, i.e., $\mathbb{E}\left[\|s_i\|^2\right] = 1$, E_s and E_p in (4a) and (4b) denote the average data and pilot symbol energy, respectively. Defining $\mathbf{h} \triangleq \mathrm{vec}(\mathbf{H})$, the elements of $\mathbf{h}$ are arbitrarily correlated ZM CSCG RVs with a positive definite covariance matrix $\boldsymbol{\mathcal{R}}_{\mathbf{hh}} \triangleq \mathbb{E}[\mathbf{h} \, \mathbf{h}^H]$.

Using the L-dimensional column vectors $\mathbf{r} \triangleq \mathrm{vec}(\mathbf{R})$ and $\mathbf{w} \triangleq \mathrm{vec}(\mathbf{W})$, and $\mathbf{r}_\mathrm{p} \triangleq \mathrm{vec}(\mathbf{R}_\mathrm{p})$ and $\mathbf{w}_\mathrm{p} \triangleq \mathrm{vec}(\mathbf{W}_\mathrm{p})$, respectively, and omitting the block index k in (4a) for notational convenience, (4a) and (4b) are equivalent to [5]

$$\mathbf{r} = \sqrt{E_\mathrm{s}}\,\mathbf{B}\,\mathbf{h} + \mathbf{w} \tag{5a}$$

$$\mathbf{r}_\mathrm{p} = \sqrt{E_\mathrm{p}}\,\mathbf{B}_\mathrm{p}\,\mathbf{h} + \mathbf{w}_\mathrm{p} \tag{5b}$$

where $\mathbf{B} \triangleq \mathbf{C}^T \otimes \mathbf{I}_{L_\mathrm{r}}$ and $\mathbf{B}_\mathrm{p} \triangleq \mathbf{C}_\mathrm{p}^T \otimes \mathbf{I}_{L_\mathrm{r}}$. Note that $\mathbf{B}^H\mathbf{B} = \mathbf{B}\,\mathbf{B}^H = \lambda\,\|\mathbf{s}\|^2\,\mathbf{I}_L$ and $\mathbf{B}_\mathrm{p}^H\mathbf{B}_\mathrm{p} = K_\mathrm{p}\mathbf{I}_L$.

III. Mismatched Receiver

Using the vector channel model (5b), the LMMSE channel estimate is given by [5]

$$\hat{\mathbf{h}} = \frac{\sqrt{E_\mathrm{p}}}{N_0}\left(\mathbf{I}_L + \frac{K_\mathrm{p}E_\mathrm{p}}{N_0}\boldsymbol{\mathcal{R}}_{\mathbf{hh}}\right)^{-1}\boldsymbol{\mathcal{R}}_{\mathbf{hh}}\,\mathbf{B}_\mathrm{p}^H\,\mathbf{r}_\mathrm{p} \tag{6}$$

Defining the channel estimation error as $\boldsymbol{\varepsilon} = \mathbf{h} - \hat{\mathbf{h}}$, the following properties can be derived from (6):

- $\hat{\mathbf{h}}$ and $\boldsymbol{\varepsilon}$ are Gaussian and statistically independent.
- The components of $\hat{\mathbf{h}}$ are ZM CSCG RVs, the covariance matrix of which is given by

$$\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}\hat{\mathbf{h}}} \triangleq \mathbb{E}\left[\hat{\mathbf{h}}\,\hat{\mathbf{h}}^H\right] = \frac{K_\mathrm{p}E_\mathrm{p}}{N_0}\boldsymbol{\mathcal{R}}_{\mathbf{hh}}\boldsymbol{\mathcal{R}}_{\mathbf{hh}}\left(\mathbf{I}_L + \frac{K_\mathrm{p}E_\mathrm{p}}{N_0}\boldsymbol{\mathcal{R}}_{\mathbf{hh}}\right)^{-1}. \tag{7}$$

- The components of $\boldsymbol{\varepsilon}$ are ZM CSCG RVs, the covariance matrix of which is given by

$$\boldsymbol{\mathcal{R}}_{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}} \triangleq \mathbb{E}[\boldsymbol{\varepsilon}\,\boldsymbol{\varepsilon}^H] = \left(\mathbf{I}_L + \frac{K_\mathrm{p}E_\mathrm{p}}{N_0}\boldsymbol{\mathcal{R}}_{\mathbf{hh}}\right)^{-1}\boldsymbol{\mathcal{R}}_{\mathbf{hh}}. \tag{8}$$

  Note that for high SNR, the elements of the channel noise vector $\boldsymbol{\varepsilon}$ can be considered as i.i.d. ZM CSCG RVs with variance $N_0/(K_\mathrm{p}E_\mathrm{p})$, since $\boldsymbol{\mathcal{R}}_{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}}$ becomes

$$\boldsymbol{\mathcal{R}}_{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}} \approx \frac{N_0}{K_\mathrm{p}E_\mathrm{p}}\mathbf{I}_L, \qquad \frac{K_\mathrm{p}E_\mathrm{p}}{N_0} \gg 1. \tag{9}$$

Accurate channel estimation can be obtained by allocating a large total energy $K_\mathrm{p}E_\mathrm{p}$ to pilot symbols. However, this leads inevitably to a reduction of the symbol energy E_s available for data transmission. With E_b, $\gamma \triangleq E_\mathrm{p}/E_\mathrm{s}$, M, and $\rho \triangleq N_\mathrm{s}/L_\mathrm{t}^2$ denoting the energy per information bit, the ratio of E_p to E_s, the constellation size, and the code rate, respectively, it can be shown that

$$E_\mathrm{s} = \frac{K}{K + \gamma K_\mathrm{p}}\rho\log_2(M)E_\mathrm{b}. \tag{10}$$

We consider a mismatched maximum-likelihood (ML) receiver that uses the estimated channel (6) instead of the true channel. Taking (2) into account, it is readily verified that the detection algorithm for the information symbols s_i reduces to symbol-by-symbol detection

$$\hat{s}_i = \arg\min_{\tilde{s}}|u_i - \tilde{s}|, \qquad 1 \le i \le N_\mathrm{s} \tag{11}$$

where the decision variable u_i is given by

$$u_i = \frac{\hat{\mathbf{h}}^H\left(\mathbf{C}_i^* \otimes \mathbf{I}_{L_\mathrm{r}}\right)\mathbf{r} + \mathbf{r}^H\left(\mathbf{C}_i^{'T} \otimes \mathbf{I}_{L_\mathrm{r}}\right)\hat{\mathbf{h}}}{\lambda\sqrt{E_\mathrm{s}}\|\hat{\mathbf{h}}\|^2}. \tag{12}$$

IV. Bit Error Rate Analysis

For a mismatched receiver, the received signal (5a) can be decomposed as

$$\mathbf{r} = \sqrt{E_\mathrm{s}}\,\mathbf{B}\,\hat{\mathbf{h}} + \sqrt{E_\mathrm{s}}\,\mathbf{B}\,\boldsymbol{\varepsilon} + \mathbf{w} \tag{13}$$

where $\sqrt{E_\mathrm{s}}\,\mathbf{B}\,\hat{\mathbf{h}}$ is the useful component, $\mathbf{w}$ is the Gaussian channel noise, and $\sqrt{E_\mathrm{s}}\,\mathbf{B}\,\boldsymbol{\varepsilon}$ is additional noise caused by the channel estimation error; the latter noise is Gaussian when conditioned on the data symbol vector $\mathbf{s}$. Using (13), expanding (12) yields $u_i = s_i + n_i$, $1 \le i \le N_\mathrm{s}$, where the disturbance term n_i contains contributions from the channel noise $\mathbf{w}$ and the channel estimation error $\boldsymbol{\varepsilon}$, and is Gaussian when conditioned on $\mathbf{s}$. When PCSI is available at the receiver, i.e., $\boldsymbol{\varepsilon} = \mathbf{0}$, n_i is a ZM CSCG RV independent of $\mathbf{s}$ with variance $N_0/(\lambda E_\mathrm{s}\,\|\mathbf{h}\|^2)$. In this section, we derive the BER of a mismatched ML receiver using LMMSE channel estimation. We consider square M-QAM transmission with Gray mapping, which reduces to $\sqrt{M}$-PAM transmission for both the in-phase and quadrature-phase information bits. Therefore, the BER for M-PAM with Gray mapping can be obtained in a similar manner.

For square M-QAM and equally likely symbol vectors $\mathbf{s}$, the BERs related to the in-phase and quadrature-phase bits of s_i, $1 \le i \le N_\mathrm{s}$, can be shown to be equal, and independent of the index i. Hence, the average BER equals the BER related to the in-phase information bits of s_i, irrespective of i, and a decision error will occur when the real part of (12) is located inside the decision area of a QAM symbol $b \neq s_i$. The projection on the real axis of the decision area of the QAM symbol $b = b_\mathrm{R} + jb_\mathrm{I}$, with b_R and b_I denoting the real and imaginary parts of b, respectively, is denoted as the decision region of b_R. We write the BER as

$$\mathrm{BER} = \frac{1}{M^{N_\mathrm{s}}}\sum_{\mathbf{s}\in\Psi^{N_\mathrm{s}}}\sum_{b_\mathrm{R}\in\Psi_\mathrm{R}}\frac{d_\mathrm{H}(s_{i,\mathrm{R}}, b_\mathrm{R})}{\frac{1}{2}\log_2 M}\,\mathbb{E}_{\hat{\mathbf{h}}}\left[P_{i,\mathrm{R}}(\mathbf{s}, b_\mathrm{R}, \hat{\mathbf{h}})\right] \tag{14}$$

where Ψ and Ψ_R denote the sets of the M-QAM constellation points and their real parts, respectively, $d_\mathrm{H}(s_{i,\mathrm{R}}, b_\mathrm{R})$ is the Hamming distance between the in-phase bits allocated to s_i and b, and $P_{i,\mathrm{R}}(\mathbf{s}, b_\mathrm{R}, \hat{\mathbf{h}})$ is the probability that the real part of (12) is located inside the decision area of b_R, when $\mathbf{s}$ and $\hat{\mathbf{h}}$ are known. With $d_1(s_{i,\mathrm{R}}, b_\mathrm{R})$ and $d_2(s_{i,\mathrm{R}}, b_\mathrm{R})$ denoting the distances between $s_{i,\mathrm{R}}$ and the boundaries of the decision area of b_R, with $d_1(s_{i,\mathrm{R}}, b_\mathrm{R}) < d_2(s_{i,\mathrm{R}}, b_\mathrm{R})$ (we set $d_2(s_{i,\mathrm{R}}, b_\mathrm{R}) = \infty$ if b is an outer constellation point), $P_{i,\mathrm{R}}(\mathbf{s}, b_\mathrm{R}, \hat{\mathbf{h}})$ reduces to

$$P_{i,\mathrm{R}}(\mathbf{s}, b_\mathrm{R}, \hat{\mathbf{h}}) = Q\left(\frac{d_1(s_{i,\mathrm{R}}, b_\mathrm{R})}{\sigma_{i,\mathrm{R}}(\mathbf{s}, \hat{\mathbf{h}})}\right) - Q\left(\frac{d_2(s_{i,\mathrm{R}}, b_\mathrm{R})}{\sigma_{i,\mathrm{R}}(\mathbf{s}, \hat{\mathbf{h}})}\right) \tag{15}$$

where $\sigma_{i,\mathrm{R}}(\mathbf{s}, \hat{\mathbf{h}})$ denotes the standard deviation of the real part of n_i and $Q(.)$ is the Gaussian Q-function. Assuming that the high-SNR approximation (9) is valid, it can be shown that $\sigma_{i,\mathrm{R}}^2(\mathbf{s}, \hat{\mathbf{h}})$ is given by

$$\sigma_{i,\mathrm{R}}^2(\mathbf{s}, \hat{\mathbf{h}}) = \frac{N_0}{2\lambda E_\mathrm{s}\|\hat{\mathbf{h}}\|^2}\left(1 + \frac{\lambda\|\mathbf{s}\|^2}{\gamma K_\mathrm{p}}\right). \tag{16}$$

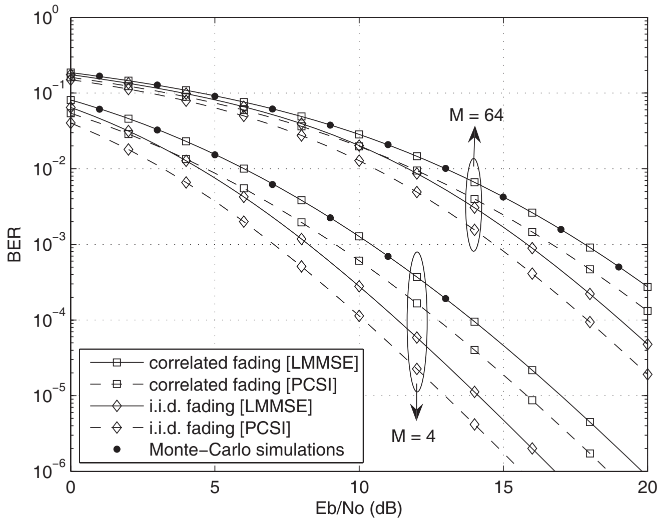

Fig. 1. BER of Alamouti's code with M-QAM transmission, for correlated and i.i.d. Rayleigh fading, and for LMMSE channel estimation and PCSI.

Since $\sigma^2_{i,\mathrm{R}}(\mathbf{s},\hat{\mathbf{h}})$ depends on $\hat{\mathbf{h}}$ through $\|\hat{\mathbf{h}}\|^2$ only, a closed-form expression for (14) can be obtained by averaging the Q-functions in (15) over the statistics of $\|\hat{\mathbf{h}}\|^2$. Using a moment generating function (MGF) approach, it is readily verified that the PDF of $\|\hat{\mathbf{h}}\|^2$ is given by [6]

$$p_{\|\hat{\mathbf{h}}\|^2}(x) = \sum_{m=1}^{\kappa}\sum_{n=1}^{c_m} D_{m,n}\,\frac{x^{n-1}\exp\left(-\frac{x}{\lambda_m}\right)}{(n-1)!\,(\lambda_m)^n} \tag{17}$$

where $x \geq 0$ and λ_m's, $m = 1, 2, \ldots, \kappa$, are the distinct eigenvalues of $\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}\hat{\mathbf{h}}}$ given by (7), with corresponding algebraic multiplicities c_m. In (17), the parameters $D_{m,n}$ are given by

$$D_{m,n} = \frac{(\lambda_m)^{n-c_m}}{(c_m-n)!}\left[\frac{\mathrm{d}^{c_m-n}}{\mathrm{d}s^{c_m-n}}\Psi_m(s)\right]\Big|_{s=-\frac{1}{\lambda_m}} \tag{18}$$

where

$$\Psi_m(s) = \prod_{\substack{l=1\\ l\neq m}}^{\kappa}(1+\lambda_l\, s)^{-c_l}\,. \tag{19}$$

Using (17) and the result from [7, Eq. (14.4-15)], it is easily shown that averaging the Q-functions in (15) over the statistics of $\|\hat{\mathbf{h}}\|^2$ yields

$$\mathbb{E}_{\hat{\mathbf{h}}}\left[Q\left(\frac{d_q(s_{i,\mathrm{R}},b_\mathrm{R})}{\sigma_{i,\mathrm{R}}(\mathbf{s},\hat{\mathbf{h}})}\right)\right] = \sum_{m=1}^{\kappa}\sum_{n=1}^{c_m} D_{m,n} \times\left[\frac{1-\mu_m}{2}\right]^n\sum_{k=0}^{n-1}\binom{n-1+k}{k}\left[\frac{1+\mu_m}{2}\right]^k \tag{20}$$

where $q = \{1, 2\}$, and μ_m is given by

$$\mu_m \triangleq \left[\frac{\lambda\, d_q^2(s_{i,\mathrm{R}},b_\mathrm{R})\frac{E_\mathrm{s}}{N_0}\left(1+\frac{\lambda\|\mathbf{s}\|^2}{\gamma K_\mathrm{p}}\right)^{-1}\lambda_m}{1+\lambda\, d_q^2(s_{i,\mathrm{R}},b_\mathrm{R})\frac{E_\mathrm{s}}{N_0}\left(1+\frac{\lambda\|\mathbf{s}\|^2}{\gamma K_\mathrm{p}}\right)^{-1}\lambda_m}\right]^{\frac{1}{2}}. \tag{21}$$

V. Numerical Results

To obtain our numerical results, we consider Alamouti's code [1] ($L_\mathrm{t} = N_\mathrm{s} = 2$), which is given by

$$\mathcal{C} = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \tag{22}$$

and we assume $E_\mathrm{p} = E_\mathrm{s}$ and a mismatched dual-antenna receiver ($L_\mathrm{r} = 2$) performing LMMSE channel estimation. For 4-QAM and 64-QAM, Fig. 1 shows the BER curves for Alamouti's code under correlated and uncorrelated Rayleigh fading, for $K = 100$ and $K_\mathrm{p} = 14$. Also shown are the BER results for PCSI, with $K = 100$ and $K_\mathrm{p} = 0$. For correlated fading, the covariance matrix $\boldsymbol{\mathcal{R}}_{\mathbf{hh}}$ is assumed to be given by $\boldsymbol{\mathcal{R}}_{\mathbf{hh}} = \boldsymbol{\mathcal{R}}_\mathrm{t}\otimes\boldsymbol{\mathcal{R}}_\mathrm{r}$, where $\boldsymbol{\mathcal{R}}_\mathrm{t}$ and $\boldsymbol{\mathcal{R}}_\mathrm{r}$ are given by

$$\boldsymbol{\mathcal{R}}_\mathrm{t} = \begin{pmatrix} 1 & 0.2+j0.3 \\ 0.2-j0.3 & 1 \end{pmatrix} \tag{23a}$$

$$\boldsymbol{\mathcal{R}}_\mathrm{r} = \begin{pmatrix} 1 & 0.5-j0.7 \\ 0.5+j0.7 & 1 \end{pmatrix}. \tag{23b}$$

Monte-Carlo simulations indicate that the presented BER expression is not only asymptotically exact but also yields very accurate BER results for low to moderate SNR. We observe from the figure that antenna correlation and LMMSE channel estimation both give rise to a horizontal shift of the BER curve at high SNR, with respect to the case of i.i.d. fading and PCSI. Moreover, the amount of degradation caused by LMMSE channel estimation is essentially independent of the amount of correlation.

VI. Conclusions

In this work, we investigated the effect of LMMSE channel estimation on the BER performance of square orthogonal space-time block codes under arbitrarily correlated Rayleigh fading channels. We presented a closed-form BER expression which is asymptotically exact and yields very accurate BER results also in the low to moderate SNR region.

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