Accurate Closed-Form Approximation of BER for OSTBCs with Estimated CSI on Spatially Correlated Rayleigh Fading MIMO Channels

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***Abstract*—In this contribution, we present a novel accurate closed-form approximation for the bit error rate (BER) of orthogonal space-time block codes (OSTBCs) employing quadrature amplitude modulation with Gray-mapping on spatially correlated Rayleigh fading MIMO channels, in the presence of channel estimation errors. The presented BER expression allows to accurately examine the impact of spatial correlation and channel estimation errors on the BER for any OSTBC or signal-to-noise ratio, even in case of highly correlated channels.**

***Index Terms*—Error analysis, OSTBC, channel estimation, correlated MIMO channels, Rayleigh fading.**

I. Introduction

ORTHOGONAL space-time block codes (OSTBCs) [1] are considered a very appealing transmit diversity technique, since they achieve full diversity with a remarkably simple symbol-by-symbol optimal detection algorithm. Accordingly, their error performance has been studied extensively in the literature. Under the assumption of correlated Rayleigh fading channels with perfect channel state information (PCSI), exact bit error rate (BER) expressions for pulse amplitude modulation (PAM), quadrature amplitude modulation (QAM), and phase-shift keying (PSK) constellations can be found in [2] and [3].

In practical wireless systems, however, the receiver has to estimate the channel, in which case the assumption of PCSI is no longer valid. In general, the resulting BER cannot be obtained in closed form, and, therefore, closed-form BER *approximations* for various cases have been presented in literature (e.g., [4]–[7] and references therein). The closed-form BER expressions from [4], for PAM and QAM constellations with linear minimum mean-square error (LMMSE) channel estimation, yield exact results for square OSTBCs and approximate results for non-square OSTBCs, but are limited to independent and identically distributed (i.i.d.) Rayleigh fading channels. Recently, under identical channel conditions, the loss of signal-to-noise ratio (SNR) due to estimation errors was quantified in [5] for PAM/PSK/QAM constellations, based on high-SNR approximations of the symbol error rate (SER). Assuming spatially correlated Rayleigh fading, an asymptotically exact BER approximation has been obtained in closed form in [6] for square OSTBCs using LMMSE channel estimation and PAM or QAM constellations. In [7], a simple closed-form expression is given for the asymptotic BER degradation due to channel estimation, which is exact only in case of Rayleigh fading, square OSTBCs, and PSK constellations, but is also used to approximate the BER when the latter conditions are not met. Note that for high SNR, the results from [6] and [7] converge in case of BPSK or QPSK.

In this contribution, we present a novel generalized closed-form BER approximation that is shown to be very accurate at any SNR, for square and non-square OSTBCs employing M-QAM with Gray mapping on spatially correlated Rayleigh fading channels with LMMSE channel estimation. The presented expression comprises the analytical BER results on i.i.d. Rayleigh channels from [4] as special cases and converges to the asymptotically exact BER result from [6] for high SNR in case of square OSTBCs. Moreover, it is shown that the presented BER expression yields more accurate results for low-to-moderate SNR than the high-SNR approximations from [6] and [7]. Hence, in contrast with the analytical BER results available from the literature, the presented closed-form expression allows to very accurately examine the impact of spatial correlation and channel estimation errors on the BER performance of OSTBCs for any SNR value, even in case of highly correlated channels.

II. Signal Model

We consider a wireless MIMO communication system utilizing L_t transmit and L_r receive antennas. Assuming OSTBCs from complex orthogonal designs [1], each OSTBC transforms a symbol vector $\mathbf{s} = [s_1, s_2, \ldots, s_{N_\mathrm{s}}]^T$ of N_s information symbols into an $L_\mathrm{t} \times K_\mathrm{c}$ coded symbol matrix $\mathbf{C}$

$$\mathbf{C} = \sum_{i=1}^{N_\mathrm{s}} \left(\mathbf{C}_i \, s_i + \mathbf{C}_i^{'} \, s_i^* \right), \tag{1}$$

where the $L_\mathrm{t} \times K_\mathrm{c}$ matrices $\mathbf{C}_i$ and $\mathbf{C}_i^{'}$ consist of the weights of s_i and s_i^*, respectively. We assume that $\mathbf{C}$ is scaled such that

$$\mathbf{C}\mathbf{C}^H = \lambda \|\mathbf{s}\|^2 \mathbf{I}_{L_\mathrm{t}}, \tag{2}$$

where $\lambda \triangleq K_\mathrm{c}/N_\mathrm{s}$ and $\mathbf{I}_{L_\mathrm{t}}$ denotes the $L_\mathrm{t} \times L_\mathrm{t}$ identity matrix.

Data transmission is organized in frames dedicating K_p time slots to pilot symbols and K time slots to coded data symbols. Assuming that K is a multiple of K_c, K/K_c code matrices $\mathbf{C}(k)$, $1 \le k \le K/K_\mathrm{c}$, are sent within one frame. To recover the channel state information we use orthogonal pilot sequences, i.e., the $L_\mathrm{t} \times K_\mathrm{p}$ pilot matrix $\mathbf{C}_\mathrm{p}$ has orthogonal rows such that

$$\mathbf{C}_\mathrm{p}\mathbf{C}_\mathrm{p}^H = K_\mathrm{p} \mathbf{I}_{L_\mathrm{t}}. \tag{3}$$

The $L \triangleq L_\mathrm{t} L_\mathrm{r}$ MIMO channel coefficients are represented by the $L_\mathrm{r} \times L_\mathrm{t}$ complex-valued random matrix $\mathbf{H}$, which remains constant during the length of one frame of $K + K_\mathrm{p}$

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symbols. Stacking the elements of $\mathbf{H}$ into the column vector $\mathbf{h} \triangleq \text{vec}(\mathbf{H})$, the elements of which are assumed to be arbitrarily correlated zero-mean (ZM) circularly symmetric complex Gaussian (CSCG) random variables (RVs) with a positive definite covariance matrix $\boldsymbol{\mathcal{R}} \triangleq \mathbb{E}[\mathbf{h}\mathbf{h}^H]$. Adopting the vector model from [6], the received signals corresponding to the OSTBC matrices $\mathbf{C}(k)$ and the pilot matrix $\mathbf{C}_\text{p}$, respectively, are given by

$$\mathbf{r}(k) = \sqrt{E_\text{s}}\,\mathbf{B}(k)\,\mathbf{h} + \mathbf{w}(k), \tag{4a}$$

$$\mathbf{r}_\text{p} = \sqrt{E_\text{p}}\,\mathbf{B}_\text{p}\,\mathbf{h} + \mathbf{w}_\text{p}, \tag{4b}$$

where $\mathbf{B}(k) \triangleq \mathbf{C}(k)^T \otimes \mathbf{I}_{L_\text{r}}$ and $\mathbf{B}_\text{p} \triangleq \mathbf{C}_\text{p}^T \otimes \mathbf{I}_{L_\text{r}}$, with $\otimes$ denoting the Kronecker product. In the remainder of the paper, we will omit the block index k in (4a) for notational convenience. Note that (2) and (3) imply that $\mathbf{B}^H\mathbf{B} = \lambda\|\mathbf{s}\|^2\mathbf{I}_L$ and $\mathbf{B}_\text{p}^H\mathbf{B}_\text{p} = K_\text{p}\mathbf{I}_L$, such that E_s and E_p denote the average transmitted energy per symbol interval per antenna during the transmission of data and pilot symbols, respectively. The additive channel noise vectors $\mathbf{w}$ and $\mathbf{w}_\text{p}$ consist of i.i.d. ZM CSCG RVs with variance N_0. To guarantee a fair comparison in terms of energy, regardless of the number of pilot symbols, we obtain the system performance for a given E_b/N_0, where E_b is defined as the energy required to transmit one bit. In this way, we have

$$E_\text{s} = \frac{K}{K + \gamma K_\text{p}}\rho\log_2(M)E_\text{b}, \tag{5}$$

where $\gamma \triangleq E_\text{p}/E_\text{s}$, $\rho \triangleq N_\text{s}/(L_\text{t}K_\text{c})$, and M is the number of constellation points.

We assume that the channel is estimated by means of pilot-based LMMSE channel estimation, yielding [8]

$$\hat{\mathbf{h}} = \frac{\sqrt{E_\text{p}}}{N_0}\left(\mathbf{I}_L + \frac{K_\text{p}E_\text{p}}{N_0}\boldsymbol{\mathcal{R}}\right)^{-1}\boldsymbol{\mathcal{R}}\,\mathbf{B}_\text{p}^H\,\mathbf{r}_\text{p}. \tag{6}$$

With $\boldsymbol{\varepsilon} \triangleq \mathbf{h} - \hat{\mathbf{h}}$ being the MMSE channel estimation error, it is readily verified that $\mathbf{h}$ can be written as the sum of two uncorrelated terms

$$\mathbf{h} = \hat{\mathbf{h}} + \boldsymbol{\varepsilon}. \tag{7}$$

It follows from (4b), (6), (7) and the ZM CSCG nature of $\mathbf{h}$ (i.e., Rayleigh fading) that the components of $\hat{\mathbf{h}}$ and $\boldsymbol{\varepsilon}$ are *independent* ZM CSCG RVs; the corresponding covariance matrices are given by

$$\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}} \triangleq \mathbb{E}\left[\hat{\mathbf{h}}\hat{\mathbf{h}}^H\right] = \frac{K_\text{p}E_\text{p}}{N_0}\boldsymbol{\mathcal{R}}\left(\mathbf{I}_L + \frac{K_\text{p}E_\text{p}}{N_0}\boldsymbol{\mathcal{R}}\right)^{-1}\boldsymbol{\mathcal{R}}, \tag{8}$$

$$\boldsymbol{\mathcal{R}}_{\boldsymbol{\varepsilon}} \triangleq \mathbb{E}[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^H] = \left(\mathbf{I}_L + \frac{K_\text{p}E_\text{p}}{N_0}\boldsymbol{\mathcal{R}}\right)^{-1}\boldsymbol{\mathcal{R}}. \tag{9}$$

Considering a mismatched maximum likelihood (ML) receiver that uses the estimated channel instead of the true channel, it is readily verified that the detection algorithm for the different information symbols s_i comprised in the matrix $\mathbf{B}$ reduces to symbol-by-symbol detection

$$\hat{s}_i = \arg\min_{\tilde{s}}|u_i - \tilde{s}|, \qquad 1 \le i \le N_\text{s}, \tag{10}$$

where the minimization is over all symbols $\tilde{s}$ belonging to the considered symbol constellation and the decision variable u_i is given by

$$u_i = \frac{\hat{\mathbf{h}}^H\left(\mathbf{C}_i^* \otimes \mathbf{I}_{L_\text{r}}\right)\mathbf{r} + \mathbf{r}^H\left(\mathbf{C}_i^{'T} \otimes \mathbf{I}_{L_\text{r}}\right)\hat{\mathbf{h}}}{\lambda\sqrt{E_\text{s}}\|\hat{\mathbf{h}}\|^2}. \tag{11}$$

In this contribution, we assume that the transmitted data symbols s_i result from the Gray mapping of information bits to a square M-QAM constellation $\Psi = \{b_\text{R} + jb_\text{I} \mid b_\text{R} \in \Psi',\, b_\text{I} \in \Psi'\}$, where the set Ψ' contains the $\sqrt{M}$ real (and imaginary) parts of the M-QAM constellation points. We denote the real and imaginary parts of the symbols s_i, of the detected symbols $\hat{s}_i$, and of the decision variables u_i by ($s_{i,\text{R}}$,$s_{i,\text{I}}$), ($\hat{s}_{i,\text{R}}$,$\hat{s}_{i,\text{I}}$) and ($u_{i,\text{R}}$,$u_{i,\text{I}}$), respectively. The corresponding symbol-by-symbol detection reduces to $\hat{s}_{i,\text{q}} = \arg\min_{\tilde{s}}|u_{i,\text{q}} - \tilde{s}|$, with $\text{q} \in \{\text{R},\text{I}\}$, where $\tilde{s}$ takes values from Ψ'.

III. Bit error rate analysis

Using the particular channel decomposition (7), the received signal (4a) can be written as

$$\mathbf{r} = \sqrt{E_\text{s}}\,\mathbf{B}\hat{\mathbf{h}} + \sqrt{E_\text{s}}\,\mathbf{B}\boldsymbol{\varepsilon} + \mathbf{w}, \tag{12}$$

where $\sqrt{E_\text{s}}\,\mathbf{B}\hat{\mathbf{h}}$ is the useful component, $\mathbf{w}$ denotes the Gaussian channel noise, and $\sqrt{E_\text{s}}\,\mathbf{B}\boldsymbol{\varepsilon}$ can be treated as an additional noise term caused by the channel estimation error; note that $\hat{\mathbf{h}}$ and $\boldsymbol{\varepsilon}$ are statistically independent. Since the error vector $\boldsymbol{\varepsilon}$ consists of ZM CSCG RVs with covariance matrix (9), the additional noise vector $\sqrt{E_\text{s}}\,\mathbf{B}\boldsymbol{\varepsilon}$ is Gaussian when conditioned on the data symbol vector $\mathbf{s}$. Substituting (12) into (11), the decision variable u_i reduces to

$$u_i = s_i + n_i, \qquad 1 \le i \le N_\text{s}, \tag{13}$$

where the disturbance term n_i contains contributions from the channel noise $\mathbf{w}$ and the channel estimation error $\boldsymbol{\varepsilon}$. When conditioned on $\mathbf{s}$ and $\hat{\mathbf{h}}$, n_i is zero-mean Gaussian; its real and imaginary parts have variances $\sigma_{i,\text{R}}^2(\hat{\mathbf{h}},\mathbf{s})$ and $\sigma_{i,\text{I}}^2(\hat{\mathbf{h}},\mathbf{s})$, respectively, that are given by

$$\sigma_{i,\text{q}}^2(\hat{\mathbf{h}},\mathbf{s}) = \frac{N_0}{2\lambda E_\text{s}\|\hat{\mathbf{h}}\|^2}\left(1 + \frac{x_{i,\text{q}}(\hat{\mathbf{h}},\mathbf{s})E_\text{s}}{\lambda N_0\|\hat{\mathbf{h}}\|^2}\right), \tag{14}$$

with $\text{q} \in \{\text{R},\text{I}\}$, where

$$x_{i,\text{q}}(\hat{\mathbf{h}},\mathbf{s}) \triangleq \hat{\mathbf{h}}^H\left(\mathbf{C}_{i,\text{q}}^T(\mathbf{s}) \otimes \mathbf{I}_{L_\text{r}}\right)\boldsymbol{\mathcal{R}}_{\boldsymbol{\varepsilon}}\left(\mathbf{C}_{i,\text{q}}^*(\mathbf{s}) \otimes \mathbf{I}_{L_\text{r}}\right)\hat{\mathbf{h}}, \tag{15}$$

and we have introduced $\mathbf{C}_{i,\text{R}}(\mathbf{s}) \triangleq \mathbf{C}(\mathbf{C}_i + \mathbf{C}_i^{'})^H$ and $\mathbf{C}_{i,\text{I}}(\mathbf{s}) \triangleq \mathbf{C}(\mathbf{C}_i - \mathbf{C}_i^{'})^H$. In case of PCSI, we have $\boldsymbol{\mathcal{R}}_{\boldsymbol{\varepsilon}} = \mathbf{0}$, yielding $x_{i,\text{q}}(\hat{\mathbf{h}},\mathbf{s}) = 0$, so that $\sigma_{i,\text{R}}^2(\hat{\mathbf{h}},\mathbf{s}) = \sigma_{i,\text{I}}^2(\hat{\mathbf{h}},\mathbf{s}) = N_0/(2\lambda E_\text{s}\|\mathbf{h}\|^2)$.

The BER of the mismatched receiver is obtained by carrying out the following steps, for $\text{q} \in \{\text{R},\text{I}\}$:

(i) Compute, for all $b_\text{q} \in \Psi'$ and all $\mathbf{s}$, $\Pr[\hat{s}_{i,\text{q}} = b_\text{q}|\mathbf{s}]$ by averaging $\Pr[\hat{s}_{i,\text{q}} = b_\text{q}|\hat{\mathbf{h}},\mathbf{s}]$ over $\hat{\mathbf{h}}$. The probability $\Pr[\hat{s}_{i,\text{q}} = b_\text{q}|\hat{\mathbf{h}},\mathbf{s}]$ is easily obtained from (13) and (14) as

$$\Pr[\hat{s}_{i,\text{q}} = b_\text{q}|\hat{\mathbf{h}},\mathbf{s}] = Q\left(\frac{D_1(s_{i,\text{q}},b_\text{q})}{\sigma_{i,\text{q}}(\hat{\mathbf{h}},\mathbf{s})}\right) - Q\left(\frac{D_2(s_{i,\text{q}},b_\text{q})}{\sigma_{i,\text{q}}(\hat{\mathbf{h}},\mathbf{s})}\right), \tag{16}$$

where $Q(.)$ is the Gaussian Q-function [9, Eq. (4.1)], and $D_1(s_{i,\mathrm{q}}, b_\mathrm{q})$ and $D_2(s_{i,\mathrm{q}}, b_\mathrm{q})$ denote the distances between $s_{i,\mathrm{q}}$ and the boundaries of the decision area of b_q. Note that we assume that $D_1(s_{i,\mathrm{q}}, b_\mathrm{q}) < D_2(s_{i,\mathrm{q}}, b_\mathrm{q})$ and that $D_2(s_{i,\mathrm{q}}, b_\mathrm{q}) \to \infty$ if b_q corresponds to an outer constellation point of Ψ_q.

(ii) Compute, for all $b_\mathrm{q} \in \Psi'$ and all $s_{i,\mathrm{q}} \in \Psi'$, the Hamming distance $d_\mathrm{H}(s_{i,\mathrm{q}}, b_\mathrm{q})$ between the binary representations of b_q and $s_{i,\mathrm{q}}$;

(iii) Compute $N_{e,\mathrm{q}}$ by taking the arithmetical average of $d_\mathrm{H}(s_{i,\mathrm{q}}, b_\mathrm{q})\Pr[\hat{s}_{i,\mathrm{q}} = b_\mathrm{q}|\mathbf{s}]$ over all $\mathbf{s}$ and over $i = 1, \cdots, N_\mathrm{s}$, and by summing the result over all $b_\mathrm{q} \in \Psi'$. Finally, the BER is computed as $(N_{e,\mathrm{R}} + N_{e,\mathrm{I}})/\log_2(M)$. Note that $N_{e,\mathrm{R}}$ and $N_{e,\mathrm{I}}$ are the average number of erroneous in-phase bits and quadrature bits per information symbol.

All averaging operations needed to compute the BER from $\Pr[\hat{s}_{i,\mathrm{q}} = b_\mathrm{q}|\hat{\mathbf{h}}, \mathbf{s}]$ are finite sums, except for the averaging over the joint distribution of the components of the channel estimate $\hat{\mathbf{h}}$ in (i). Considering the complicated way $\sigma_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s})$ depends on $\hat{\mathbf{h}}$ (see (14)), the average of $\Pr[\hat{s}_{i,\mathrm{q}} = b_\mathrm{q}|\hat{\mathbf{h}}, \mathbf{s}]$ over $\hat{\mathbf{h}}$ cannot be expressed in closed form. This problem is often circumvented by obtaining (an estimate of) the BER through Monte Carlo simulation, where the receiver operation is simulated for a large number of transmitted frames, or through Monte Carlo integration, where the expectation over $\hat{\mathbf{h}}$ is replaced by a finite sum, involving many realizations of $\hat{\mathbf{h}}$. However, as these Monte Carlo methods tend to be computationally intensive, we will search for a closed-form approximation of the BER allowing a fast computation.

In [4], [6], and [7], closed-form approximations of the BER have been obtained by substituting for $\sigma^2_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s})$ a function that is inversely proportional to $\|\hat{\mathbf{h}}\|^2$, allowing analytical averaging of $\Pr[\hat{s}_{i,\mathrm{q}} = b_\mathrm{q}|\hat{\mathbf{h}}, \mathbf{s}]$ over the channel estimate. In [4], $\sigma^2_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s})$ is replaced by its average over $\hat{\mathbf{h}}$, conditioned on $\|\hat{\mathbf{h}}\|^2$, whereas in [6], $\sigma^2_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s})$ is computed with the error covariance matrix $\boldsymbol{\mathcal{R}}_{\boldsymbol{\varepsilon}}$ being replaced by its high-SNR limit. However, the approaches from [4] and [6] are restricted to *i.i.d.* Rayleigh fading channels and to *square* OSTBCs, respectively, and cannot be extended to non-square OSTBCs on correlated fading channels. The simple closed-form expression for the asymptotic BER degradation due to channel estimation presented in [7] is exact only in case of square OSTBCs and PSK constellations; the extension to non-square OSTBCs has been obtained by substituting for $E[\mathbf{C}^H\mathbf{C}]$ a diagonal matrix with identical diagonal elements that are the arithmetical average of the diagonal elements of $E[\mathbf{C}^H\mathbf{C}]$. However, the latter approximation is not very well motivated and the resulting BER approximation is accurate at high SNR only. Therefore we now look for a novel closed-form approximation for both square and non-square OSTBCs under spatially correlated Rayleigh fading that is accurate from low to high SNR.

The key to obtaining the closed-form BER approximation is the observation that analytical averaging over $\hat{\mathbf{h}}$ in case of Rayleigh fading is possible when $\sigma^2_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s})$ is inversely proportional to $\|\hat{\mathbf{h}}\|^2$. Therefore, we approximate $x_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s})$ from (15) as

$$x_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s}) \approx \alpha_{i,\mathrm{q}}(\mathbf{s})\, \|\hat{\mathbf{h}}\|^2, \qquad (17)$$

so that the variance (14) of the real and imaginary parts of u_i is (approximately) inversely proportional to $\|\hat{\mathbf{h}}\|^2$:

$$\sigma^2_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s}) \approx \frac{N_0}{2\lambda E_\mathrm{s}\|\hat{\mathbf{h}}\|^2}\left(1 + \frac{\alpha_{i,\mathrm{q}}(\mathbf{s})E_\mathrm{s}}{\lambda N_0}\right), \qquad (18)$$

A proper proportionality factor $\alpha_{i,\mathrm{q}}(\mathbf{s})$ in (17) can be obtained using the MMSE criterion

$$\begin{aligned}\alpha_{i,\mathrm{q}}(\mathbf{s}) &= \arg\min_{\tilde{\alpha}} \mathbb{E}\left[\left|x_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s}) - \tilde{\alpha}\,\|\hat{\mathbf{h}}\|^2\right|^2\right], \qquad (19)\\ &= \mathbb{E}\left[x_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s})\|\hat{\mathbf{h}}\|^2\right]\left(\mathbb{E}\left[\|\hat{\mathbf{h}}\|^4\right]\right)^{-1}\end{aligned}$$

which in the case of Rayleigh fading yields

$$\alpha_{i,\mathrm{q}}(\mathbf{s}) = \frac{\mathrm{tr}(\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}})\mathrm{tr}\left(\mathbf{A}_{i,\mathrm{q}}(\mathbf{s})\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}}\right) + \mathrm{tr}\left(\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}}\mathbf{A}_{i,\mathrm{q}}(\mathbf{s})\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}}\right)}{\mathrm{tr}(\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}})\mathrm{tr}(\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}}) + \mathrm{tr}(\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}}\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}})}, \qquad (20)$$

where $\mathrm{tr}(\cdot)$ denotes the trace and $\mathbf{A}_{i,\mathrm{q}}(\mathbf{s})$ is defined as

$$\mathbf{A}_{i,\mathrm{q}}(\mathbf{s}) \triangleq \left(\mathbf{C}^T_{i,\mathrm{q}}(\mathbf{s}) \otimes \mathbf{I}_{L_\mathrm{r}}\right)\boldsymbol{\mathcal{R}}_{\boldsymbol{\varepsilon}}\left(\mathbf{C}^*_{i,\mathrm{q}}(\mathbf{s}) \otimes \mathbf{I}_{L_\mathrm{r}}\right). \qquad (21)$$

A closed-form BER expression for arbitrarily correlated Rayleigh fading can now be obtained in a similar way as in [6], since (18) is inversely proportional to $\|\hat{\mathbf{h}}\|^2$ and the probability density function of $\|\hat{\mathbf{h}}\|^2$ reduces to a weighted sum of χ^2-distributions. Using the approximation (17), it follows from (18) and [10, 14.4-15] that averaging the Q-functions in (16) over $\|\hat{\mathbf{h}}\|^2$ results in the following closed-form expression

$$\mathbb{E}_{\hat{\mathbf{h}}}\left[Q\left(\frac{D_\nu(s_{i,\mathrm{q}}, b_\mathrm{q})}{\sigma_{i,\mathrm{q}}(\hat{\mathbf{h}}, \mathbf{s})}\right)\right] = \sum_{m=1}^{\kappa}\sum_{n=1}^{c_m} D_{m,n} \times \left[\frac{1-\mu_m}{2}\right]^n \sum_{k=0}^{n-1}\binom{n-1+k}{k}\left[\frac{1+\mu_m}{2}\right]^k, \qquad (22)$$

where $\nu \in \{1, 2\}$, $D_{m,n}$ is the coefficient of $(1 + \lambda_m s)^{-n}$ in the partial fraction expansion of $\prod_{l=1}^{\kappa}(1 + \lambda_l s)^{-c_l}$, λ_m denotes the m-th distinct eigenvalue of $\boldsymbol{\mathcal{R}}_{\hat{\mathbf{h}}}$ with corresponding algebraic multiplicity c_m, and μ_m is given by

$$\mu_m \triangleq \left[\frac{\lambda D^2_\nu(s_{i,\mathrm{q}}, b_\mathrm{q})\frac{E_\mathrm{s}}{N_0}\left(1 + \frac{\alpha_{i,\mathrm{q}}(\mathbf{s})}{\lambda}\frac{E_\mathrm{s}}{N_0}\right)^{-1}\lambda_m}{1 + \lambda D^2_\nu(s_{i,\mathrm{q}}, b_\mathrm{q})\frac{E_\mathrm{s}}{N_0}\left(1 + \frac{\alpha_{i,\mathrm{q}}(\mathbf{s})}{\lambda}\frac{E_\mathrm{s}}{N_0}\right)^{-1}\lambda_m}\right]^{\frac{1}{2}}. \qquad (23)$$

Considering (22) and (16), a closed-form BER expression for OSTBCs employing M-QAM on arbitrarily correlated Rayleigh fading channels with LMMSE channel estimation, is readily obtained. In case of i.i.d. Rayleigh fading, the resulting BER expression can be easily shown to reduce to the exact BER result for square OSTBCs and to the approximate BER result for non-square OSTBCs in [4]. Furthermore, by applying a high-SNR approximation of the covariance matrix (9) to (21), the presented BER expression converges to the asymptotically exact BER result from [6] for square OSTBCs.

IV. Numerical Results

With $E_\mathrm{p} = E_\mathrm{s}$, we verify in this section the accuracy of the presented closed-form BER approximation by means of straightforward Monte-Carlo simulations.

Assuming QPSK transmission and a single-antenna receiver ($L_\mathrm{r} = 1$) employing LMMSE channel estimation with $K = 20$

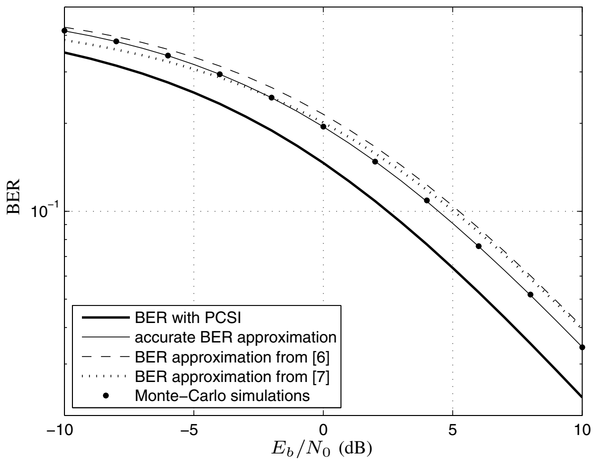

Fig. 1. BER curves for Alamouti's code from the presented accurate BER approximation and from the high-SNR approximations in [6] and [7].

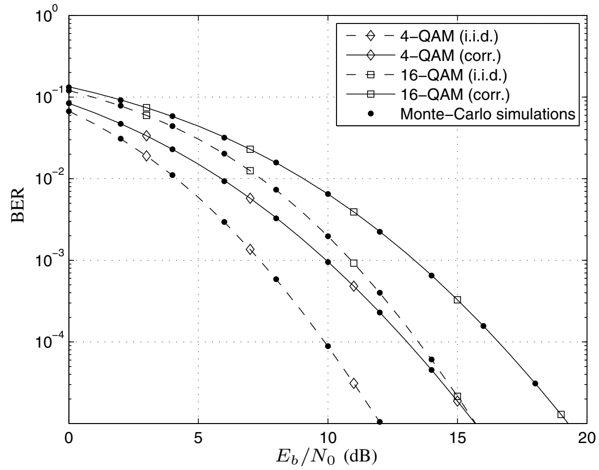

Fig. 2. BER for the 3×4 OSTBC given by [1, Eq. (39)] on i.i.d. and correlated Rayleigh fading channels.

and $K_\mathrm{p} = 4$, Fig. 1 shows the BER for Alamouti's code, which is defined in [1, Eq. (32)] as a square 2×2 OSTBC. In order to clearly illustrate the accuracy of the presented BER approximation, even under extreme channel conditions, we consider two highly correlated Rayleigh channels with covariance matrix

$$\boldsymbol{\mathcal{R}} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \quad (24)$$

where $\rho = 0.999$. Besides the BER results for PCSI, we show in the figure both the curves from the accurate closed-form BER approximation derived in section III, from the high-SNR BER approximation for square OSTBCs in [6], and from the asymptotic BER result from [7]. Since no high-SNR approximations are used in the analytical derivation in section III, it is not surprising to see that the closed-form BER expression presented in this paper is more accurate than the BER approximations from [6] and [7], especially for low-to-moderate SNR and highly correlated channels. Since we consider a square OSTBC employing QPSK transmission, the three BER curves will converge for high SNR.

Fig. 2 displays the BER for the non-square 3×4 OSTBC given by [1, Eq. (39)], scaled such that (2) is satisfied. Assuming a dual-antenna receiver ($L_\mathrm{r} = 2$) using LMMSE channel estimation with $K = 100$ and $K_\mathrm{p} = 16$, we show the curves corresponding to the presented BER approximation for 4-QAM and 16-QAM transmission on i.i.d. and spatially correlated Rayleigh MIMO channels. Note that for i.i.d. channels, the presented BER approximation reduces to the BER approximation from [4]. In case of a correlated channel, the covariance matrix is given by $\boldsymbol{\mathcal{R}} = \boldsymbol{\mathcal{R}}_\mathrm{t} \otimes \boldsymbol{\mathcal{R}}_\mathrm{r}$, where $\otimes$ denotes the Kronecker product, $\boldsymbol{\mathcal{R}}_\mathrm{t}$ is given by

$$\boldsymbol{\mathcal{R}}_\mathrm{t} = \begin{bmatrix} 1 & 0.7 + j0.3 & 0.4 - j0.2 \\ 0.7 - j0.3 & 1 & 0.5 + j0.1 \\ 0.4 + j0.2 & 0.5 - j0.1 & 1 \end{bmatrix}, \quad (25)$$

and $\boldsymbol{\mathcal{R}}_\mathrm{r}$ is given by (24) with $\rho = 0.8$. The simulation results indicate that the presented BER approximation yields very accurate results from low to high SNR.

V. Conclusions

In this contribution, we presented a novel closed-form approximation for the BER of OSTBCs employing QAM on arbitrarily correlated Rayleigh fading channels with LMMSE channel estimation. The resulting BER expression is shown to be very accurate in the range from low to high SNR, even in case of highly correlated channels.

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