Accurate BER Analysis of Orthogonal Space-Time Block Codes with MMSE Channel Estimation

Lennert Jacobs and Marc Moeneclaey
Ghent University, TELIN Department, DIGCOM Research Group
Sint-Pietersnieuwstraat 41, B-9000 Gent, Belgium
E-mail: {Lennert.Jacobs,Marc.Moeneclaey}@telin.ugent.be

Abstract— **In this paper, we investigate the effect of imperfect channel estimation on the bit error rate (BER) performance of orthogonal space-time block codes (OSTBCs). The multiple-input multiple-output (MIMO) propagation channel is assumed to be affected by uncorrelated flat Rayleigh block fading. We consider a mismatched maximum-likelihood (ML) receiver, which estimates the channel matrix from known pilot symbols according to a minimum mean-square error (MMSE) criterion, and uses the channel estimate in the symbol-by-symbol detection algorithm as if it were the true channel matrix. For both PAM and QAM constellations and OSTBCs satisfying a proposed criterion, we present simple exact closed-form expressions for the BER of the mismatched receiver and the corresponding BER degradation as compared to a receiver that knows the true channel matrix. For OSTBCs that do not satisfy this criterion, a simple approximation of the BER and the associated BER degradation is derived, assuming PAM and QAM constellations. By means of computer simulations, this approximation is shown to be very accurate.**

I. Introduction

In wireless applications, spatial diversity is achieved by using multiple antennas at the transmitter and/or receiver side. These so-called multiple-input multiple-output (MIMO) communication systems can achieve a maximum diversity order of $N_\mathrm{t}N_\mathrm{r}$ (with N_t and N_r denoting the number of transmit and receive antennas, respectively), provided that proper space-time coding is used. In 1998, Alamouti invented a simple coding scheme for data transmission using two transmit antennas [1], with the remarkable benefit that the maximum-likelihood decoding algorithm reduces to symbol-by-symbol detection, based only on linear processing at the receiver. Tarokh *et al*. generalized Alamouti's scheme to an arbitrary number of transmit antennas by introducing the concept of orthogonal space-time block coding [2], [3]. As the orthogonal space-time block codes (OSTBCs) achieve full spatial diversity, and require only linear processing at the receiver, these codes are a very attractive transmit diversity technique.

Tarokh *et al*. [4] demonstrated, by means of simulations, that significant gains can be achieved by increasing the number of transmit antennas at the expense of almost no extra decoding complexity. Kim *et al*. [5] presented exact closed-form bit error rate (BER) expressions of OSTBCs for phase-shift keying (PSK) constellations. In [6], exact BER expressions of OSTBCs with square quadrature amplitude modulation (QAM) constellations in slow Rayleigh fading channels were derived. Exact closed-form BER equations for pulse amplitude modulation (PAM), QAM en PSK constellations in correlated Rayleigh MIMO channels, were derived in [7].

However, it is important to note that the above BER results were derived under the assumption that the channel state information (CSI) of the propagation channel is known at the receiver. In practical wireless applications, the receiver has to estimate the channel response, which inevitably results in a performance penalty. In [8], the effect of channel estimation errors on the BER was demonstrated by means of simulations. In [9], expressions for the exact decoding error probability (DEP) were obtained for the case of square OSTBCs and PSK constellations. The symbol error rate (SER) of OSTBCs in presence of channel estimation was studied in [10] and [11]. In [12], analytical BER expressions as well as the tight Chernoff bound were given for orthogonal space-time block coded systems employing M-PSK modulation. High-SNR expressions for the pairwise error probability (PEP) were derived under quite general conditions in [13], using an eigenvalue approach. In [14], an exact closed-form expression for the PEP of both orthogonal and non-orthogonal space-time codes in the case of least-squares channel estimation was obtained by means of characteristic functions. However, from the PEP one can compute only an upper bound on the BER, which in a fading environment does not converge to the true BER at high SNR. To avoid this drawback, we consider in this paper the true BER of OSTBCs, and present a simple closed-form expression for the BER in case of PAM and QAM signal constellations and minimum mean-square error (MMSE) channel estimation. We investigate under which condition on the OSTBCs this BER expression is exact, and point out that for OSTBCs that do not satisfy this condition the BER expression provides a very accurate approximation.

II. Signal model

Consider an orthogonal space-time block coded communication system with N_t transmit antennas and N_r receive antennas, and a flat Rayleigh fading propagation channel. Transmission is organized in frames: in one frame, each transmit antenna sends K_p known pilot symbols and K coded data symbols; the pilot symbols enable channel estimation at the receiver. Within one frame of $N_\mathrm{fr} = K + K_\mathrm{p}$ symbols, the channel is assumed to be constant (block fading). The $N_\mathrm{r} \times N_\mathrm{fr}$ received signal matrix $\mathbf{R}_\mathrm{tot}$ is given by

$$\mathbf{R}_\mathrm{tot} = [\mathbf{R}_\mathrm{p}, \mathbf{R}] = \mathbf{H}\,[\mathbf{A}_\mathrm{p}, \mathbf{A}] + \mathbf{W}, \tag{1}$$

978-1-4244-2204-3/08/$25.00 ©2008 IEEE

where the $N_\mathrm{t} \times K_\mathrm{p}$ pilot matrix $\mathbf{A}_\mathrm{p}$ and the $N_\mathrm{t} \times K$ data matrix $\mathbf{A}$ consist of the pilot symbols and the coded data symbols, respectively, transmitted at each transmit antenna. The propagation channel is represented by the $N_\mathrm{r} \times N_\mathrm{t}$ complex random matrix $\mathbf{H}$, whose elements are independent identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance (i.e., each channel coefficient has independent real and imaginary parts with zero mean and variance 1/2). The $N_\mathrm{r} \times N_\mathrm{fr}$ matrix $\mathbf{W}$ represents additive spatially and temporally white noise and consists of i.i.d. ZMCSCG random variables with variance N_0.

An orthogonal space-time block code transforms a block $\mathbf{s}(k)$ of N_s information symbols $s_i(k)$, $1 \leq i \leq N_\mathrm{s}$, into a $N_\mathrm{t} \times K_\mathrm{c}$ coded symbol matrix $\mathbf{C}(k)$, with k denoting the block index. Hence, assuming that K is a multiple of K_c, the transmitted data symbol matrix is given by $\mathbf{A} = \sqrt{E_\mathrm{s}}\,[\mathbf{C}(1), \cdots, \mathbf{C}(K/K_\mathrm{c})]$. Restricting our attention to OSTBCs from complex linear processing orthogonal designs [2], the entries of the symbol matrices are linear combinations of the information symbols and their complex conjugate, such that $\mathbf{C}$ can be written as (we drop the index k for notational convenience)

$$\mathbf{C} = \sum_{i=1}^{N_\mathrm{s}} \left(\mathbf{C}_i s_i + \mathbf{C}_i^{'} s_i^*\right). \tag{2}$$

The $N_\mathrm{t} \times K_\mathrm{c}$ matrices $\mathbf{C}_i$ and $\mathbf{C}_i^{'}$ comprise the coefficients of s_i and s_i^* in the matrix $\mathbf{C}$, respectively. Without loss of generality, we may assume that the matrices $\mathbf{C}$ are scaled in such way that they satisfy the following orthogonality condition:

$$\mathbf{C}\mathbf{C}^H = \left(\lambda \sum_{i=1}^{N_\mathrm{s}} |s_i|^2\right) \mathbf{I}_{N_\mathrm{t}}, \tag{3}$$

where $(\cdot)^H$ denotes the Hermitian transpose, $\lambda = K_\mathrm{c}/N_\mathrm{s}$ and $\mathbf{I}_{N_\mathrm{t}}$ is the $N_\mathrm{t} \times N_\mathrm{t}$ identity matrix. In this way, considering a normalized information symbol constellation ($\mathbb{E}[|s_i|^2] = 1$), the above property yields

$$\frac{1}{N_\mathrm{t} K_\mathrm{c}} \mathbb{E}\left[\|\mathbf{C}\|^2\right] = 1, \tag{4}$$

where $\|.\|$ denotes the Frobenius norm. From this, it follows that the average energy of the transmitted coded symbols is given by E_s:

$$\frac{1}{N_\mathrm{t} K} \mathbb{E}\left[\|\mathbf{A}\|^2\right] = E_\mathrm{s}. \tag{5}$$

Similarly, the average energy of the pilot symbols is E_p.

III. Pilot-based channel estimation

The receiver can estimate $\mathbf{H}$ using $\mathbf{R}_\mathrm{p}$ and the known pilot matrix $\mathbf{A}_\mathrm{p}$. Assuming orthogonal training sequences, i.e., the matrix $\mathbf{A}_\mathrm{p}$ has orthogonal rows such that $\mathbf{A}_\mathrm{p}\mathbf{A}_\mathrm{p}^H = K_\mathrm{p}E_\mathrm{p}\mathbf{I}_{N_\mathrm{t}}$, the MMSE channel estimate [15] is given by

$$\hat{\mathbf{H}} = \frac{1}{N_0 + K_\mathrm{p}E_\mathrm{p}} \mathbf{R}_\mathrm{p}\mathbf{A}_\mathrm{p}^H. \tag{6}$$

Defining the channel estimation error as $\hat{\mathbf{\Delta}} = \mathbf{H} - \hat{\mathbf{H}}$, the following properties can be derived from (6):

- $\hat{\mathbf{H}}$ and $\hat{\mathbf{\Delta}}$ are Gaussian and statistically independent;
- The components of $\hat{\mathbf{H}}$ are i.i.d. ZMCSCG random variables with

$$\sigma^2_{\hat{\mathbf{H}}} = \mathbb{E}\left[\left|\hat{\mathbf{H}}_{m,n}\right|^2\right] = \frac{K_\mathrm{p}E_\mathrm{p}}{N_0 + K_\mathrm{p}E_\mathrm{p}}; \tag{7}$$

- The components of $\hat{\mathbf{\Delta}}$ are i.i.d. ZMCSCG random variables with

$$\sigma^2_{\hat{\mathbf{\Delta}}} = \mathbb{E}\left[\left|\hat{\mathbf{\Delta}}_{m,n}\right|^2\right] = \frac{N_0}{N_0 + K_\mathrm{p}E_\mathrm{p}}. \tag{8}$$

Allocating a large total energy $K_\mathrm{p}E_\mathrm{p}$ to pilot symbols yields an accurate channel estimate (see (8)), but on the other hand gives rise to a reduction of the symbol energy E_s. Denoting by E_b the energy per information bit and writing $E_\mathrm{p} = \gamma E_\mathrm{s}$, we have

$$E_\mathrm{s} = \frac{K}{K + \gamma K_\mathrm{p}} \rho \log_2(M) E_\mathrm{b}, \tag{9}$$

where $\rho = N_\mathrm{s}/(N_\mathrm{t}K_\mathrm{c})$ and M denote the code rate and the number of constellation points, respectively.

IV. ML detection algorithm

If $\mathbf{H}$ is known by the receiver, maximum-likelihood (ML) detection of the data symbol matrix $\mathbf{C}(k)$ reduces to

$$\hat{\mathbf{C}}(k) = \arg\min_{\widetilde{\mathbf{C}}(k)} \left\|\mathbf{R}'(k) - \sqrt{E_\mathrm{s}}\mathbf{H}\widetilde{\mathbf{C}}(k)\right\|^2, \tag{10}$$

where $\mathbf{R}'(k)$ is the received signal matrix corresponding to the transmitted symbol matrix $\mathbf{C}(k)$, and the minimization is over the valid code matrices satisfying (2). Since only the estimated channel matrix $\hat{\mathbf{H}}$ is known instead of $\mathbf{H}$, we consider a receiver that uses $\hat{\mathbf{H}}$ in the same way an ML receiver would apply $\mathbf{H}$ (we omit the index k for notational convenience):

$$\hat{\mathbf{C}} = \arg\min_{\widetilde{\mathbf{C}}} \left\|\mathbf{R}' - \sqrt{E_\mathrm{s}}\hat{\mathbf{H}}\widetilde{\mathbf{C}}\right\|^2. \tag{11}$$

This type of receiver is often called a mismatched receiver.

Taking (3) into account, the detection algorithm (11) for the information symbols s_i reduces to symbol-by-symbol detection:

$$\hat{s}_i = \arg\min_{\tilde{s}_i} |u_i - \tilde{s}_i|, \; 1 \leq i \leq N_\mathrm{s}. \tag{12}$$

In (12), the decision variable u_i is given by

$$u_i = \frac{\mathrm{tr}\left(\mathbf{C}_i^H\hat{\mathbf{H}}^H\mathbf{R}' + \mathbf{R}'^H\hat{\mathbf{H}}\mathbf{C}_i^{'}\right)}{\lambda\sqrt{E_\mathrm{s}}\left\|\hat{\mathbf{H}}\right\|^2}, \tag{13}$$

where $\mathrm{tr}(\cdot)$ denotes the trace.

V. BIT ERROR RATE ANALYSIS

For a mismatched receiver that assumes $\hat{\mathbf{H}}$ to be the correct channel matrix, the received signal $\mathbf{R}'$ that corresponds to the data matrix $\mathbf{C}$ can be decomposed as

$$\mathbf{R}' = \sqrt{E_\mathrm{s}}\hat{\mathbf{H}}\mathbf{C} + \sqrt{E_\mathrm{s}}\hat{\boldsymbol{\Delta}}\mathbf{C} + \mathbf{W}', \tag{14}$$

where $\sqrt{E_\mathrm{s}}\hat{\mathbf{H}}\mathbf{C}$ is the useful component, $\mathbf{W}'$ is the Gaussian channel noise, and $\sqrt{E_\mathrm{s}}\hat{\boldsymbol{\Delta}}\mathbf{C}$ is additional noise caused by the channel estimation error; the latter noise is Gaussian when conditioned on $\mathbf{C}$. As compared to a receiver with perfect channel knowledge (PCK), the detection performance of the mismatched receiver is degraded: the useful component is reduced (because it follows from (7) that $\sigma^2_{\hat{\mathbf{H}}} \leq 1$) and the total noise variance is increased.

When the mismatched receiver and the PCK receiver achieve some target BER at $E_\mathrm{b}/N_0 = (E_\mathrm{b}/N_0)_\mathrm{MMSE}$ and $E_\mathrm{b}/N_0 = (E_\mathrm{b}/N_0)_\mathrm{PCK}$, respectively, the BER degradation of the mismatched receiver as compared to the PCK receiver is expressed as $10\log_{10}((E_\mathrm{b}/N_0)_\mathrm{MMSE}/(E_\mathrm{b}/N_0)_\mathrm{PCK})$ [dB].

Expanding (13) yields $u_i = s_i + n_i$, $1 \leq i \leq N_\mathrm{s}$, where the disturbance term n_i contains contributions from the channel noise $\mathbf{W}'$ and the channel estimation error $\hat{\boldsymbol{\Delta}}$, and is Gaussian when conditioned on $\mathbf{C}$. When the CSI is known at the receiver (i.e., $\hat{\boldsymbol{\Delta}} = \mathbf{0}$), n_i is a ZMCSCG random variable independent of $\mathbf{C}$ with variance $N_0/(\lambda E_\mathrm{s}\|\mathbf{H}\|^2)$. Owing to the simple symbol-by-symbol detection algorithm of the information symbols, analytical BER expressions for the PCK receiver can be easily obtained (e.g., [16], [17], [6]). However, in this paper we investigate the effect of channel estimation errors on the BER of a mismatched receiver.

A. M-PAM constellation

In a fading environment, the BER is obtained by averaging the conditional BER (conditioned on the channel matrix) over the channel statistics. Here, we take the average over the statistics of the estimated channel:

$$\mathrm{BER}_{M-\mathrm{PAM}} = \mathbb{E}_{\hat{\mathbf{H}}}\left[\mathrm{BER}_{M-\mathrm{PAM}}(\hat{\mathbf{H}})\right]. \tag{15}$$

Since the code matrix $\mathbf{C}$ of an OSTBC comprises N_s information symbols s_i, represented by the transmitted information symbol vector $\mathbf{s} = (s_1, \cdots, s_{N_\mathrm{s}})$, the BER conditioned on the estimated channel matrix is given by

$$\begin{aligned}\mathrm{BER}_{M-\mathrm{PAM}}\left(\hat{\mathbf{H}}\right) &= \mathbb{E}_i\left[\mathrm{BER}_{M-\mathrm{PAM},i}(\hat{\mathbf{H}})\right] \quad (16)\\ &= \frac{1}{N_\mathrm{s}}\sum_{i=1}^{N_\mathrm{s}}\mathrm{BER}_{M-\mathrm{PAM},i}(\hat{\mathbf{H}}), \quad (17)\end{aligned}$$

where $\mathrm{BER}_{M-\mathrm{PAM},i}(\hat{\mathbf{H}})$ is the BER corresponding to the information symbol s_i, conditioned on $\hat{\mathbf{H}}$. A decision error occurs when the decision variable u_i is inside the decision area of a symbol b which is different from the transmitted symbol s_i. Hence, $\mathrm{BER}_{M-\mathrm{PAM},i}(\hat{\mathbf{H}})$ is given by

$$\begin{aligned}\mathrm{BER}_{M-\mathrm{PAM},i}\left(\hat{\mathbf{H}}\right) &= \mathbb{E}_\mathbf{s}\left[\sum_{b\in\Psi}\frac{N(s_i,b)}{\log_2 M}P_i(\mathbf{s},b,\hat{\mathbf{H}})\right]\\ &= \frac{1}{M^{N_\mathrm{s}}}\sum_{\mathbf{s}\in\Psi^{N_\mathrm{s}}}\sum_{b\in\Psi}\frac{N(s_i,b)}{\log_2 M}P_i(\mathbf{s},b,\hat{\mathbf{H}}), \quad (18)\end{aligned}$$

where Ψ denotes the normalized M-PAM constellation and $N(s_i, b)$ represents the Hamming distance between the binary representations of the transmitted symbol s_i and the decoded symbol b. $P_i(\mathbf{s}, b, \hat{\mathbf{H}})$ is the probability that u_i is located in the decision area of the constellation point b (when the transmitted symbol vector and the channel estimate are known). With $d_1(s_i, b)$ and $d_2(s_i, b)$ denoting the distances between the transmitted constellation point s_i and the boundaries of the decision area of b, with $d_1(s_i, b) < d_2(s_i, b)$ (we set $d_2(s_i, b) = \infty$ if b is an outer constellation point), $P_i(\mathbf{s}, b, \hat{\mathbf{H}})$ reduces to

$$P_i(\mathbf{s},b,\hat{\mathbf{H}}) = Q\left(\frac{d_1(s_i,b)}{\sigma_{\mathrm{R}_i}(\mathbf{s},\hat{\mathbf{H}})}\right) - Q\left(\frac{d_2(s_i,b)}{\sigma_{\mathrm{R}_i}(\mathbf{s},\hat{\mathbf{H}})}\right), \tag{19}$$

where $\sigma_{\mathrm{R}_i}(\mathbf{s}, \mathbf{H})$ denotes the standard deviation of the real part of u_i and $Q(.)$ is the Gaussian Q-function, defined as

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{x^2}{2}\right)\mathrm{d}x. \tag{20}$$

Let the $N_\mathrm{t} \times N_\mathrm{t}$ matrix $\mathbf{C}_{\mathrm{R}_i}(\mathbf{s})$, $1 \leq i \leq N_\mathrm{s}$, be defined as

$$\mathbf{C}_{\mathrm{R}_i}(\mathbf{s}) \triangleq \mathbf{C}\left(\mathbf{C}_i + \mathbf{C}_i^{'}\right)^H, \tag{21}$$

which is a function of the information symbol vector $\mathbf{s}$ through the code matrix $\mathbf{C}$ (see (2)). It can be shown that

$$\sigma^2_{\mathrm{R}_i}(\mathbf{s},\hat{\mathbf{H}}) = \frac{N_0}{2\lambda E_\mathrm{s}\|\hat{\mathbf{H}}\|^2}\left(1 + \frac{\sigma^2_{\hat{\boldsymbol{\Delta}}}}{N_0}\frac{E_\mathrm{s}\|\hat{\mathbf{H}}\mathbf{C}_{\mathrm{R}_i}(\mathbf{s})^H\|^2}{\lambda\|\hat{\mathbf{H}}\|^2}\right). \tag{22}$$

The expectation of (19) over $\hat{\mathbf{H}}$ is difficult to compute, because the expression between parentheses in (22) depends on $\hat{\mathbf{H}}$. However, this dependence is removed if the considered OSTBC satisfies the following criterion for $1 \leq i \leq N_\mathrm{s}$:

$$\mathbf{C}_{\mathrm{R}_i}(\mathbf{s})^H\mathbf{C}_{\mathrm{R}_i}(\mathbf{s}) = \beta_{\mathrm{R}_i}(\mathbf{s})\,\mathbf{I}_{N_\mathrm{t}}, \tag{23}$$

where $\beta_{\mathrm{R}_i}(\mathbf{s}) = \|\mathbf{C}_{\mathrm{R}_i}(\mathbf{s})\|^2/N_\mathrm{t}$. When (23) holds, the variance (22) simplifies to

$$\sigma^2_{\mathrm{R}_i}(\mathbf{s},\hat{\mathbf{H}}) = \frac{N_0}{2\lambda E_\mathrm{s}\|\hat{\mathbf{H}}\|^2}\left(1 + \frac{\sigma^2_{\hat{\boldsymbol{\Delta}}}}{N_0}\frac{E_\mathrm{s}\beta_{\mathrm{R}_i}(\mathbf{s})}{\lambda}\right), \tag{24}$$

and the averaged Q-functions in (19) reduce to [16]

$$\mathbb{E}_{\hat{\mathbf{H}}}\left[Q\left(\frac{d_j(s_i,b)}{\sigma_{\mathrm{R}_i}(\mathbf{a},\hat{\mathbf{H}})}\right)\right] = \Omega\left(\frac{2\lambda d_j^2(s_i,b)}{1 + \frac{\beta_{\mathrm{R}_i}(\mathbf{s})}{\lambda\gamma K_\mathrm{p}} + \frac{1}{\gamma K_\mathrm{p}}\frac{N_0}{E_\mathrm{s}}}\frac{E_\mathrm{s}}{N_0}\right), \tag{25}$$

for $j \in \{1, 2\}$. In (25), the function $\Omega(\theta)$ is defined as

$$\Omega(\theta) \triangleq \left[\frac{1-\mu}{2}\right]^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left[\frac{1+\mu}{2}\right]^k, \qquad (26)$$

with $L = N_\mathrm{r} N_\mathrm{t}$ denoting the spatial diversity achieved by the OSTBC, and $\mu = \sqrt{\theta/(2+\theta)}$.

Taking (9) into account, the ratio of the BER of the mismatched receiver to the BER with PCK ($K_\mathrm{p} = 0$) is easily derived (at high E_b/N_0):

$$\frac{\mathrm{BER}_{M-\mathrm{PAM,MMSE}}}{\mathrm{BER}_{M-\mathrm{PAM,PCK}}} = \left(1 + \frac{\gamma K_\mathrm{p}}{K}\right)^L \times \frac{\mathbb{E}_{i,\mathbf{s}}\left[\left(1 + \frac{\beta_{\mathrm{R}_i}(\mathbf{s})}{\lambda\gamma K_\mathrm{p}}\right)^L \xi(s_i)\right]}{\mathbb{E}_s\left[\xi(s)\right]}, \qquad (27)$$

where the function $\xi(s)$ is defined as

$$\xi(s) \triangleq \sum_{b\in\Psi} N(s,b) \left(\frac{1}{d_1^{2L}(s,b)} - \frac{1}{d_2^{2L}(s,b)}\right). \qquad (28)$$

Given that the ratio (27) does not depend on E_b/N_0, it follows that the PCK receiver and the mismatched receiver give rise to the same diversity order (which equals $N_\mathrm{r} N_\mathrm{t}$ for OSTBCs), as observed in [15]. As stated before, the mismatched receiver must have a larger E_b/N_0 ratio than the PCK receiver, in order that both receivers have the same BER. Taking into account that a diversity order of $L = N_\mathrm{r} N_\mathrm{t}$ is achieved (BER $\propto (E_\mathrm{b}/N_0)^{-L}$), the amount (in dB) by which the E_b/N_0 ratio of the mismatched receiver should be increased to obtain the same BER as the PCK receiver is given by

$$\frac{10}{L}\log_{10}\left(\frac{\mathrm{BER}_{M-\mathrm{PAM,MMSE}}}{\mathrm{BER}_{M-\mathrm{PAM,PCK}}}\right). \qquad (29)$$

For OSTBCs that do not satisfy criterion (23), replacing the variance (22) by its average over the entries of $\hat{\mathbf{H}}$, when conditioned on $\|\hat{\mathbf{H}}\|^2$, also results in (24). In this way, (25) and (27) provide simple approximations of the BER and the BER degradation of the mismatched receiver.

B. M-QAM constellation

Now we consider square M-QAM transmission with Gray mapping, which is equivalent to $\sqrt{M}$-PAM transmission for both the in-phase and quadrature information bits. An in-phase (quadrature) decision error occurs when the real (imaginary) part of the decision variable u_i, corresponding to the transmitted information symbol s_i, is located outside the projection of the decision area of s_i on the real (imaginary) axis. Since the BER can be computed as the average of the BERs for the in-phase and quadrature information bits, the computation follows the same lines as for PAM.

Defining the matrix $\mathbf{C}_{\mathrm{I}_i}(\mathbf{s})$ as

$$\mathbf{C}_{\mathrm{I}_i}(\mathbf{s}) \triangleq \mathbf{C}\left(\mathbf{C}_i - \mathbf{C}_i^{'}\right)^H, \qquad (30)$$

the exact BER for both the in-phase and quadrature information bits can be computed analytically if the OSTBC satisfies the following criterion for $1 \le i \le N_\mathrm{s}$:

$$\begin{cases} \mathbf{C}_{\mathrm{R}_i}(\mathbf{s})^H \mathbf{C}_{\mathrm{R}_i}(\mathbf{s}) = \frac{\|\mathbf{C}_{\mathrm{R}_i}(\mathbf{s})\|^2}{N_\mathrm{t}} \mathbf{I}_{N_\mathrm{t}} \\ \mathbf{C}_{\mathrm{I}_i}(\mathbf{s})^H \mathbf{C}_{\mathrm{I}_i}(\mathbf{s}) = \frac{\|\mathbf{C}_{\mathrm{I}_i}(\mathbf{s})\|^2}{N_\mathrm{t}} \mathbf{I}_{N_\mathrm{t}} \end{cases}, \qquad (31)$$

where $\mathbf{C}_{\mathrm{R}_i}(\mathbf{s})$ is given by (21). For OSTBCs satisfying (31), the high-SNR ratio of the BER of the mismatched receiver to the BER with PCK ($K_\mathrm{p} = 0$), using a normalized QAM constellation Φ with Gray mapping, is given by

$$\frac{\mathrm{BER}_{M-\mathrm{QAM,MMSE}}}{\mathrm{BER}_{M-\mathrm{QAM,PCK}}} = \left(1 + \frac{\gamma K_\mathrm{p}}{K}\right)^L \times \frac{\mathbb{E}_{i,\mathbf{s}}\left[\left(1 + \frac{\beta_{\mathrm{R}_i}(\mathbf{s})}{\lambda\gamma K_\mathrm{p}}\right)^L \xi_\mathrm{R}(s_i) + \left(1 + \frac{\beta_{\mathrm{I}_i}(\mathbf{s})}{\lambda\gamma K_\mathrm{p}}\right)^L \xi_\mathrm{I}(s_i)\right]}{\mathbb{E}_s\left[\xi_\mathrm{R}(s) + \xi_\mathrm{I}(s)\right]}, \qquad (32)$$

where the functions $\xi_\mathrm{R}(s)$ and $\xi_\mathrm{I}(s)$ are defined as

$$\begin{cases} \xi_\mathrm{R}(s) \triangleq \sum_{b\in\Phi_\mathrm{R}} N_\mathrm{R}(s,b) \left(\frac{1}{d_{1,\mathrm{R}}^{2L}(s,b)} - \frac{1}{d_{2,\mathrm{R}}^{2L}(s,b)}\right) \\ \xi_\mathrm{I}(s) \triangleq \sum_{b\in\Phi_\mathrm{I}} N_\mathrm{I}(s,b) \left(\frac{1}{d_{1,\mathrm{I}}^{2L}(s,b)} - \frac{1}{d_{2,\mathrm{I}}^{2L}(s,b)}\right) \end{cases}, \qquad (33)$$

where Φ_R and Φ_I denote the projection of Φ on the real axis and the imaginary axis, respectively. The quantities $N_\mathrm{p}(s,b)$, $d_{1,\mathrm{p}}(s,b)$ and $d_{2,\mathrm{p}}(s,b)$ are defined in a similar way as $N(s,b)$, $d_1(s,b)$ and $d_2(s,b)$ for PAM, with $\mathrm{p} = \mathrm{R}$ and $\mathrm{p} = \mathrm{I}$ referring to the in-phase and quadrature signals, respectively. Examples of OSTBCs that satisfy (31) are given in [1], [2, eq. (38),(40)] and [3, eq. (62)].

For OSTBCs not satisfying (31), the resulting expressions yield simple approximations of the actual BER and the BER degradation of the mismatched receiver.

VI. Numerical results

The code matrix of Alamouti's code [1] ($N_\mathrm{t} = K_\mathrm{c} = N_\mathrm{s} = 2$), the simplest and best-known OSTBC, is given by

$$\mathcal{H} = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix}. \qquad (34)$$

Given that Alamouti's code satisfies (31), with $\|\mathbf{C}_{\mathrm{R}_i}(\mathbf{s})\|^2 = \|\mathbf{C}_{\mathrm{I}_i}(\mathbf{s})\|^2 = N_\mathrm{t}\left(|s_1|^2 + |s_2|^2\right)$ for $i \in \{1, 2\}$, the exact BER curves for PAM and QAM constellations can be derived analytically. Fig. 1 shows the exact BER curves for QPSK transmission employing Alamouti's code, for both the mismatched receiver and the PCK receiver (we assume $E_\mathrm{p} = E_\mathrm{s}$); also shown are computer simulation results for the mismatched receiver that confirm the analytical result. The corresponding BER degradation amounts to $1.15\,\mathrm{dB}$, irrespective of N_r.

The OSTBC ($N_\mathrm{t} = 3, K_\mathrm{c} = 4, N_\mathrm{s} = 3$) given by [2, eq. (39)]

$$\mathcal{G} = \begin{pmatrix} s_1 & -s_2^* & \frac{s_3^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} \\ s_2 & s_1^* & \frac{s_3^*}{\sqrt{2}} & \frac{-s_3^*}{\sqrt{2}} \\ \frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & \frac{-s_1 - s_1^* + s_2 - s_2^*}{2} & \frac{s_2 + s_2^* + s_1 - s_1^*}{2} \end{pmatrix}, \qquad (35)$$

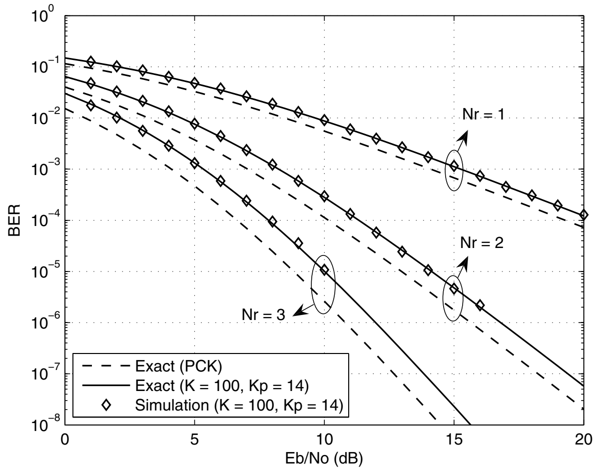

Fig. 1. BER of Alamouti's code, QPSK

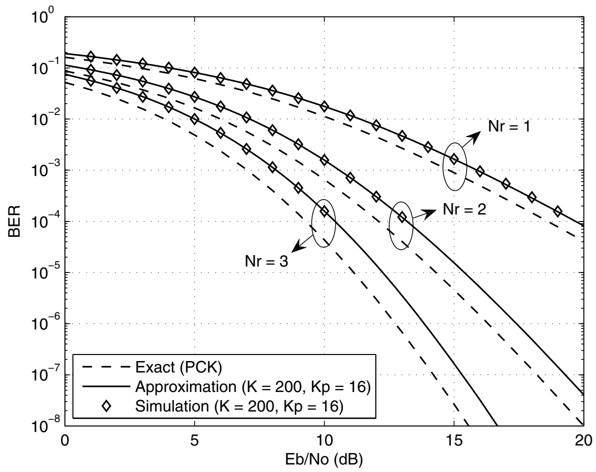

Fig. 2. BER of OSTBC given by [2, eq. (39)], 16-QAM

does not satisfy criterion (31). Fig. 2 illustrates the BER for a 16-QAM constellation, resulting from the PCK receiver (exact result) and the mismatched receiver (analytical approximation and simulation result). The simulations indicate that the approximation for the mismatched receiver is very accurate. The BER degradation amounts to $1.04\,\mathrm{dB}$, $1.06\,\mathrm{dB}$ or $1.09\,\mathrm{dB}$ when N_r equals 1, 2 or 3, respectively.

VII. Conclusions

In this work, we investigated the effect of MMSE channel estimation on the BER performance of orthogonal space-time block codes. We considered a mismatched receiver and a MIMO propagation channel that is affected by uncorrelated flat Rayleigh block fading. MMSE channel estimation is carried out, based on known pilot symbols sent among the data. The mismatched receiver utilizes in the symbol-by-symbol detection algorithm the estimated channel matrix as if it were the true channel.

For a class of OSTBCs, specified by a proposed criterion, we derived simple closed-form expressions of the exact BER and the associated BER degradation, which depend on the number of pilot symbols, data symbols, transmit antennas and receive antennas, on the considered constellation and on some specific properties of the OSTBC.

For OSTBCs that do not belong to this specific class, we derived a simple approximation of the BER and the BER degradation caused by channel estimation errors for both PAM and QAM symbol constellations. Simulations show that these approximations are very accurate.

Acknowledgments

This work was supported by the European Commission in the framework of the FP7 Network of Excellence in Wireless COMmunications NEWCOM++ (contract no. 216715). The first author also gratefully acknowledges the support from the Fund for Scientific Research in Flanders (FWO-Vlaanderen).

References

[1] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1459–1478, Oct. 1998.

[2] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, Jul. 1999.

[3] X.-B. Liang, "Orthogonal designs with maximal rates," *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2468–2503, Oct. 2003.

[4] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 451–460, Mar. 1999.

[5] I.-M. Kim and V. Tarokh, "Variable-rate space-time block codes in M-ary PSK systems," *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 362–373, Apr. 2003.

[6] S.-H. Kim, I.-S. Kang, and J.-S. No, "Exact bit error probability of orthogonal space-time block codes with QAM," in *Proc. IEEE Int. Symp. on Information Theory (ISIT)*, Yokohama, Japan, 29 Jun.-4 Jul. 2003, p. 63.

[7] I.-M. Kim, "Exact BER analysis of OSTBCs in spatially correlated MIMO channels," *IEEE Trans. Commun.*, vol. 54, no. 8, pp. 1365–1373, Aug. 2006.

[8] D. Mavares and R. Torres, "Channel estimation error effects on the performance of STB codes in flat frequency rayleigh channels," in *Proc. IEEE Vehicular Technology Conf. (VTC-Fall)*, Orlando, USA, 6-9 Oct. 2003, pp. 647–651 Vol.1.

[9] P. Garg, R. Mallik, and H. Gupta, "Exact error performance of square orthogonal space-time block coding with channel estimation," *IEEE Trans. Commun.*, vol. 54, no. 3, pp. 430–437, Mar. 2006.

[10] E. Ko, C. Kang, and D. Hong, "Effect of imperfect channel information on M-QAM SER performance of orthogonal space-time block codes," in *Proc. IEEE Vehicular Technology Conf. (VTC-Spring)*, Jeju Island, Korea, 22-25 Apr. 2003, pp. 722–726 vol.1.

[11] L. Tao, L. Jianfeng, H. Jianjun, and Y. Guangxin, "Performance analysis for orthogonal space-time block codes in the absence of perfect channel state information," in *Proc. IEEE Int. Symp. Personal, Indoor and Mobile Radio Communications (PIMRC)*, Beijing, China, 7-10 Sep. 2003, pp. 1012–1016 vol.2.

[12] C. Shan, P. Kam, and A. Nallanathan, "Theoretical performance of space-time block coded system with channel estimation," in *Proc. IEEE Global Telecommunications Conf. (Globecom)*, Dallas, USA, 29 Nov.-3 Dec. 2004, pp. 3666–3670 Vol.6.

[13] M. Brehler and M. Varanasi, "Asymptotic error probability analysis of quadratic receivers in rayleigh-fading channels with applications to a unified analysis of coherent and noncoherent space-time receivers," *IEEE Trans. Inform. Theory*, vol. 47, no. 6, pp. 2383–2399, Sep. 2001.

[14] P. Garg, R. Mallik, and H. Gupta, "Performance analysis of space-time coding with imperfect channel estimation," *IEEE Trans. Wireless Commun.*, vol. 4, no. 1, pp. 257–265, Jan. 2005.

[15] G. Taricco and E. Biglieri, "Space-time decoding with imperfect channel estimation," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1874–1888, Jul. 2005.
[16] J. Proakis, *Digital Communications*, 4th ed. McGraw-Hill, 2001.
[17] S. Chennakeshu and J. Anderson, "Error rates for rayleigh fading multichannel reception of MPSK signals," *IEEE Trans. Commun.*, vol. 43, no. 234, pp. 338–346, Feb./Mar./Apr. 1995.