BER PERFORMANCE AND OPTIMUM TRAINING STRATEGY FOR UNCODED SIMO AND ALAMOUTI SPACE-TIME BLOCK CODES WITH MMSE CHANNEL ESTIMATION

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ABSTRACT

In multiple antenna systems, channel estimation is of critical importance. In this paper, we investigate the effect of imperfect channel estimation on the performance of both uncoded single-input multiple-output (SIMO) systems and multiple-input multiple-output (MIMO) systems using Alamouti space-time block codes. To this end we consider a mismatched receiver and a propagation channel that is affected by flat Rayleigh block fading. The receiver estimates the channel from known pilot symbols which are transmitted among the data. An analytical expression for the bit error rate (BER) with imperfect channel estimation is derived. Given a number of data symbols we find the optimal number of pilot symbols that minimizes the BER degradation, for both uncoded SIMO communication and Alamouti space-time block coded MIMO communication. For best performance the number of data symbols in one transmission block is to be taken as high as possible, provided that the duration of a block does not exceed the channel coherence time. Analytical results are confirmed by computer simulations.

1. INTRODUCTION

The performance of wireless communication systems is strongly limited by channel fading. The use of multiple transmit and/or receive antennas serves as a convenient means to improve the reliability and throughput on fading channels. These so-called MIMO communication systems can exploit the spatial dimension to combine high data rates with low bit error rates [1]. However, in order to realize these beneficial prospects, the channel state information (CSI) needs to be known at the receiver.

In realistic wireless applications this CSI is not a priori known and the receiver has to estimate the channel response. Channel estimation, however, is never perfect in practice, which results in a performance penalty. In the current paper, we investigate to what extent this imperfect channel estimation deteriorates the performance of both uncoded single-input multiple-output (SIMO) systems and space-time coded MIMO systems with Alamouti space-time block codes (STBCs). A performance analysis of space-time coded systems with channel estimation errors has also been carried out in [3]. However, in contrast to the work from [3], the current paper presents closed-form expressions for the degradation caused by channel estimation errors and for the optimal training sequence length that minimizes this degradation.

2. PILOT-BASED CHANNEL ESTIMATION

We consider a wireless communication system with N_t transmit antennas and N_r receive antennas. The propagation channel is assumed to be a Rayleigh block fading flat MIMO channel that is approximately constant over N_{coh} symbol intervals; N_{coh} denotes the channel coherence time (measured in symbol intervals). Since the CSI is not known by the receiver, we assume that the transmitter sends a sequence of P known pilot symbols at each transmit antenna among the K data symbols of one transmission block, in order to enable channel estimation. Within one block of $N = K + P$ symbols transmitted at each antenna, the general MIMO signal model holds:

$$\mathbf{R}_{tot} = [\mathbf{R}_P\, \mathbf{R}] = \mathbf{H}\, [\mathbf{A}_P\, \mathbf{A}] + \mathbf{W}, \qquad (1)$$

where we assume $N < N_{coh}$. The $N_r \times N$ matrix $\mathbf{R}_{tot}$ comprises the received complex signals at each receive antenna. The $N_t \times N$ matrix $[\mathbf{A}_P\, \mathbf{A}]$ consists of the transmitted symbols at each transmit antenna, and can be decomposed into a $N_t \times P$ pilot matrix $\mathbf{A}_P$ and a $N_t \times K$ data matrix $\mathbf{A}$. The channel is represented by the $N_r \times N_t$ complex random matrix $\mathbf{H}$, whose elements are independent identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with unit variance (i.e., each channel coefficient has independent real and imaginary parts with zero mean and variance 1/2). The $N_r \times N$ matrix $\mathbf{W}$ describes additive spatially and temporally white noise and consists of i.i.d. ZMCSCG random variables with variance N_0.

The average data symbol energy E_s is given by

$$E_s = \frac{1}{N_t K} \mathrm{E}[\|\mathbf{A}\|^2], \qquad (2)$$

where $\|.\|$ denotes the Frobenius norm. The average pilot symbol energy is E_P. The receiver can estimate $\mathbf{H}$ using $\mathbf{R}_P$ and the known pilot matrix $\mathbf{A}_P$. In this contribution we consider the minimum mean-square error (MMSE) estimate which is given by [7]

$$\hat{\mathbf{H}} = \mathbf{R}_P \mathbf{A}_P^H \left(N_0 \mathbf{I}_{N_t} + \mathbf{A}_P \mathbf{A}_P^H \right)^{-1}, \tag{3}$$

where $(.)^H$ denotes the Hermitian transpose. We assume orthogonal training sequences, i.e., the matrix $\mathbf{A}_P$ has orthogonal rows such that $\mathbf{A}_P \mathbf{A}_P^H = PE_P \mathbf{I}_{N_t}$. In this way, the above equation simplifies to

$$\hat{\mathbf{H}} = \frac{1}{N_0 + PE_P} \mathbf{R}_P \mathbf{A}_P^H. \tag{4}$$

Defining the channel estimation error as

$$\hat{\Delta} = \mathbf{H} - \hat{\mathbf{H}}. \tag{5}$$

The following properties can be derived from (4):

- $\hat{\mathbf{H}}$ and $\hat{\Delta}$ are Gaussian and statistically independent;
- The components of $\hat{\mathbf{H}}$ are i.i.d. ZMCSCG random variables with

$$\sigma_{\hat{\mathbf{H}}}^2 = E\left[\left| \hat{H}_{m,n} \right|^2 \right] = \frac{PE_P}{N_0 + PE_P}; \tag{6}$$

- The components of $\hat{\Delta}$ are i.i.d. ZMCSCG random variables with

$$\sigma_{\hat{\Delta}}^2 = E\left[\left| \hat{\Delta}_{m,n} \right|^2 \right] = \frac{N_0}{N_0 + PE_P}. \tag{7}$$

3. ML DETECTION ALGORITHM

If $\mathbf{H}$ is known by the receiver, ML decision of the data symbol matrix $\mathbf{A}$ reduces to

$$\hat{\mathbf{A}} = \arg\min_{\mathbf{A}} \|\mathbf{R} - \mathbf{H}\mathbf{A}\|^2. \tag{8}$$

Since only the estimated channel matrix $\hat{\mathbf{H}}$ is known instead of $\mathbf{H}$, we consider a receiver that uses $\hat{\mathbf{H}}$ in the same way an ML receiver would apply $\mathbf{H}$

$$\hat{\mathbf{A}} = \arg\min_{\mathbf{A}} \left\| \mathbf{R} - \hat{\mathbf{H}}\mathbf{A} \right\|^2. \tag{9}$$

This type of receiver is often called a mismatched receiver. For a mismatched receiver that assumes $\hat{\mathbf{H}}$ to be the correct channel matrix, the received signal $\mathbf{R}$ that corresponds to the data matrix $\mathbf{A}$ can be decomposed as:

$$\mathbf{R} = \hat{\mathbf{H}}\mathbf{A} + \hat{\Delta}\mathbf{A} + \mathbf{W}, \tag{10}$$

where $\hat{\mathbf{H}}\mathbf{A}$ is the useful component, $\mathbf{W}$ is the Gaussian channel noise, and $\hat{\Delta}\mathbf{A}$ is additional Gaussian noise caused by the channel estimation error. As compared to a receiver with perfect CSI, the detection performance of the mismatched receiver is degraded: the useful component is reduced (because it follows from (6) that $\sigma_{\hat{\mathbf{H}}}^2 \leq 1$) and the total noise variance is increased.

3.1. Uncoded SIMO

For SIMO systems, the channel matrix reduces to a $N_r \times 1$ column vector $\mathbf{h}$. When the data symbols are uncoded, (9) simplifies to symbol-by-symbol detection, with the detection of $a(k)$ involving only the k-th column $\mathbf{r}(k)$ of $\mathbf{R}$. Suppressing for notational convenience the time index k, the detection for uncoded SIMO reduces to

$$\hat{a} = \arg\min_{a} |u - a|^2, \tag{11}$$

with

$$u = \frac{\hat{\mathbf{h}}^H \mathbf{r}}{\left\| \hat{\mathbf{h}} \right\|^2}. \tag{12}$$

3.2. Alamouti space-time block code

The Alamouti space-time block code [6], that has been designed for two transmit antennas, transforms two information symbols $c_1(k)$ and $c_2(k)$ into a 2×2 coded symbol matrix $\mathbf{C}(k)$, given by

$$\mathbf{C}(k) = \begin{bmatrix} c_1(k) & -c_2^*(k) \\ c_2(k) & c_1^*(k) \end{bmatrix}. \tag{13}$$

Hence, assuming that an even number K of information symbols is sent, the transmitted data symbol matrix is given by $\mathbf{A} = [\mathbf{C}(1), ..., \mathbf{C}(K/2)]$. Denoting by $[\mathbf{r}_1\, \mathbf{r}_2]$ the $N_r \times 2$ observation matrix corresponding to a coded symbol matrix $\mathbf{C}$ (we omit the time index for notational convenience) and writing $\mathbf{H} = [\mathbf{h}_1\, \mathbf{h}_2]$, the detection algorithm for the information symbols c_1 and c_2 reduces to symbol-by-symbol detection :

$$\hat{c}_i = \arg\min_{c_i} |u_i - c_i|, \; i = 1, 2, \tag{14}$$

with

$$u_1 = \frac{\hat{\mathbf{h}}_1^H \mathbf{r}_1 + \hat{\mathbf{h}}_2^T \mathbf{r}_2^*}{\left\| \hat{\mathbf{h}}_1 \right\|^2 + \left\| \hat{\mathbf{h}}_2 \right\|^2} \tag{15}$$

$$u_2 = \frac{\hat{\mathbf{h}}_2^H \mathbf{r}_1 - \hat{\mathbf{h}}_1^T \mathbf{r}_2^*}{\left\| \hat{\mathbf{h}}_1 \right\|^2 + \left\| \hat{\mathbf{h}}_2 \right\|^2}. \tag{16}$$

4. BIT ERROR RATE COMPUTATION

Taking into account the properties of the MMSE channel estimate and the associated estimation error, we show that the BER resulting from the mismatched receiver in the cases of uncoded SIMO and Alamouti STBC can be easily derived from the BER expressions that hold for the receiver with perfect channel knowledge (PCK). BER expressions for the PCK receiver can be obtained from the literature (e.g., [4, 5, 2])

4.1. Uncoded SIMO

The observation model for a SIMO receiver with PCK is

$$\mathbf{r} = \mathbf{h}a + \mathbf{w}, \tag{17}$$

where $\mathbf{h}$ and $\mathbf{w}$ consist of i.i.d. ZMCSCG random variables with variances $\sigma^2_{\mathbf{H}}$ $(= 1)$ and N_0, respectively. The resulting BER can be expressed as

$$BER_{SIMO-PCK} = f_{SIMO}\left(\frac{E_s\sigma^2_{\mathbf{H}}}{N_0}\right), \tag{18}$$

where $f_{SIMO}(.)$ is a function depending on the symbol constellation type and on the number of receive antennas. The argument of $f_{SIMO}(.)$ in (18) denotes the SNR related to the PCK receiver.
For the mismatched SIMO receiver, the relevant observation model is

$$\mathbf{r} = \hat{\mathbf{h}}a + \hat{\Delta}a + \mathbf{w}. \tag{19}$$

The vectors $\hat{\mathbf{h}}$ and $\hat{\Delta}a$ consist of i.i.d. ZMCSCG random variables with variances $\sigma^2_{\hat{\mathbf{H}}}$ (see (6)) and $E_s\sigma^2_{\hat{\Delta}}$ (see (7)), respectively. Hence, the BER of the mismatched SIMO receiver is given by

$$BER_{SIMO} = f_{SIMO}\left(\frac{E_s\sigma^2_{\hat{\mathbf{H}}}}{N_0 + E_s\sigma^2_{\hat{\Delta}}}\right). \tag{20}$$

The argument of $f_{SIMO}(.)$ in (20) denotes the SNR related to the mismatched receiver.

4.2. Alamouti space-time block code

The observation model for the Alamouti receiver with PCK is

$$[\mathbf{r}_1\,\mathbf{r}_2] = [\mathbf{h}_1\,\mathbf{h}_2]\,\mathbf{C} + [\mathbf{w}_1\,\mathbf{w}_2]\,, \tag{21}$$

with $\mathbf{C}$ given by (13). The 2×2 matrices $[\mathbf{h}_1\,\mathbf{h}_2]$ and $[\mathbf{w}_1\,\mathbf{w}_2]$ consist of i.i.d. ZMCSCG random variables with variances $\sigma^2_{\mathbf{H}}$ $(= 1)$ and N_0, respectively. The resulting BER can be expressed as

$$BER_{Alam-PCK} = f_{Alam}\left(\frac{E_s\sigma^2_{\mathbf{H}}}{N_0}\right), \tag{22}$$

where $f_{Alam}(.)$ is a function depending on the symbol constellation type and on the number of receive antennas. The argument of $f_{Alam}(.)$ in (22) denotes the signal-to-noise ratio (SNR) related to the PCK receiver.
For the mismatched Alamouti receiver, the relevant observation model is

$$[\mathbf{r}_1\,\mathbf{r}_2] = \left[\hat{\mathbf{h}}_1\,\hat{\mathbf{h}}_2\right]\mathbf{C} + \hat{\Delta}\mathbf{C} + [\mathbf{w}_1\,\mathbf{w}_2]\,. \tag{23}$$

The matrix $[\hat{\mathbf{h}}_1\,\hat{\mathbf{h}}_2]$ consist of i.i.d. ZMCSCG random variables with variances $\sigma^2_{\hat{\mathbf{H}}}$ (see (6)). The matrix $\hat{\Delta}\mathbf{C}$ is statistically independent of $[\hat{\mathbf{h}}_1\,\hat{\mathbf{h}}_2]$, and consists of ZMCSCG random variables with the following correlation :

$$\begin{aligned} E&\left[\left(\hat{\Delta}\mathbf{C}\right)_{m,k}\left(\hat{\Delta}\mathbf{C}\right)^*_{m',k'}\right] \\ &= \sum_{n,n'} E\left[\hat{\Delta}_{m,k}\hat{\Delta}^*_{m',k'}\right]\mathbf{C}_{n,k}\mathbf{C}^*_{n',k'} \\ &= \sigma^2_{\hat{\Delta}}\left(\mathbf{C}^H\mathbf{C}\right)_{k',k}\delta_{m-m'}. \end{aligned} \tag{24}$$

From (13) it follows that $\mathbf{C}^H\mathbf{C} = (|c_1|^2 + |c_2|^2)\mathbf{I}_2$, so that the components of $\hat{\Delta}\mathbf{C}$ are i.i.d. with variance $2E_s\sigma^2_{\hat{\Delta}}$. Hence, the BER of the mismatched Alamouti receiver is given by

$$BER_{Alam} = f_{Alam}\left(\frac{E_s\sigma^2_{\hat{\mathbf{H}}}}{N_0 + 2E_s\sigma^2_{\hat{\Delta}}}\right). \tag{25}$$

The argument of $f_{Alam}(.)$ in (25) denotes the SNR related to the mismatched receiver.

5. OPTIMAL TRAINING STRATEGY

In this section we determine the optimal training strategy, such that the BER degradation of the mismatched receiver as compared to the PCK receiver is minimal. Allocating a large total energy PE_P to pilot symbols yields an accurate channel estimate (see (7)), but on the other hand gives rise to a reduction of the symbol energy E_s. Denoting by E_b the energy per information bit, we have

$$KE_s + PE_P = K\rho E_b \log_2(M), \tag{26}$$

where ρ and M denote the code rate and the number of constellation points, respectively ($\rho = 1$ for uncoded transmission, $\rho = 1/2$ for Alamouti STBC). Hence, for given E_b and K, the energy per data symbol E_s decreases with increasing PE_P. Further, denoting by R_s the symbol rate per transmit antenna, the information bitrate is given by

$$R_b = \frac{K}{K+P}N_t \log_2(M)\rho R_s, \tag{27}$$

which indicates that the addition of pilot symbols reduces the bandwidth efficiency.
In the following we take $E_P = \gamma E_s$, in which case (27) yields

$$E_s = \frac{K}{K+\gamma P}\rho \log_2(M) E_b. \quad (28)$$

5.1. Uncoded SIMO

For given K and E_b, we compare the SNRs of the PCK receiver (with $P = 0$) and the mismatched receiver. Considering (28) and taking the ratio of the arguments of the function $f_{SIMO}(.)$ in (18) (with $P = 0$) and (20) yields

$$\frac{\text{SNR}_{SIMO-PCK}}{\text{SNR}_{SIMO}} = \frac{K+\gamma P}{K}\left(1+\frac{1}{\gamma P}+\frac{K+\gamma P}{\gamma PK\log_2 M}\frac{N_o}{E_b}\right) \quad (29)$$

Note that (29) depends on γP, rather than on γ and P separately. For large E_b/N_0 and $E_P = E_s$ ($\gamma = 1$), (29) reduces to

$$\frac{\text{SNR}_{SIMO-PCK}}{\text{SNR}_{SIMO}} = \frac{K+P}{K}\frac{1+P}{P}. \quad (\gamma = 1) \quad (30)$$

Hence, in order that the PCK receiver and the mismatched receiver have the same BER, the latter receiver must have a larger E_b/N_0 ratio than the former receiver, by an amount indicated by (30). This indicates that the PCK receiver and the mismatched receiver give rise to the same diversity order (which equals N_r for SIMO), as observed in [7].
From (30), we find the optimal number P of pilot symbols for a given number K of data symbols

$$P_{opt} = \sqrt{K}. \quad (\gamma = 1) \quad (31)$$

The minimal degradation then becomes

$$\frac{\text{SNR}_{SIMO-PCK}}{\text{SNR}_{SIMO}} = \frac{\left(1+\sqrt{K}\right)^2}{K}, \quad (\gamma = 1) \quad (32)$$

which for large K asymptotically approximates 1 (or $0dB$). For best performance we should take K as high as possible, taking into account that $K + P$ must not exceed the coherence interval N_{coh}. Since P in (29) only occurs in combination with γ, we can obtain the same degradation (32) by taking $P = \sqrt{K}/\gamma$ and $E_P = \gamma E_s$. An advantage of this degree of freedom is that we can reduce the number of pilot symbols $P < \sqrt{K}$ to increase the information bitrate. However, the lower P, the larger E_P: higher peak transmit powers are needed at the transmitter side to maintain optimal performance.

5.2. Alamouti space-time block code

Here we make a similar reasoning as for uncoded SIMO to obtain the degradation in the case of the mismatched receiver for the Alamouti code. From (22) and (25) we obtain

$$\frac{\text{SNR}_{Alam-PCK}}{\text{SNR}_{Alam}} = \frac{K+\gamma P}{K}\left(1+\frac{2}{\gamma P}+\frac{K+\gamma P}{\gamma PK\log_2 M}\frac{N_o}{E_b}\right), \quad (33)$$

which for large E_b/N_0 and $E_P = E_s$ ($\gamma = 1$) reduces to

$$\frac{\text{SNR}_{Alam-PCK}}{\text{SNR}_{Alam}} = \frac{K+P}{K}\frac{2+P}{P}. \quad (\gamma = 1) \quad (34)$$

Hence, the PCK receiver and the mismatched receiver give rise to the same diversity order (which equals $2N_r$ for the Alamouti code), as observed in [7].
From (34), we find the optimal number P of pilot symbols at each transmit antenna for a given number K of data symbols

$$P_{opt} = \sqrt{2K}. \quad (35)$$

The minimal degradation then becomes

$$\frac{\text{SNR}_{SIMO-PCK}}{\text{SNR}_{SIMO}} = \frac{\left(\sqrt{2}+\sqrt{K}\right)^2}{K}, \quad (\gamma = 1) \quad (36)$$

which for large K asymptotically approximates 1 (or $0dB$). Again, we can obtain the degradation (36) with an arbitrary number of pilot symbols by taking $\gamma P = \sqrt{2K}$ and adjusting E_P accordingly.

6. NUMERICAL RESULTS

To obtain our numerical results, we consider QPSK transmission over a MIMO channel affected by independent flat Rayleigh block fading.
Denoting by $BER_{QPSK}(E_s/N_0, L)$ the BER resulting from diversity reception of QPSK symbols over L independent flat Rayleigh fading channels with channel variances 1, the BER for the PCK receiver with N_r receive antennas is given by

$$BER_{QPSK-SIMO-PCK}(E_s/N_0, N_r) = BER_{QPSK}(E_s/N_0, N_r) \quad (37)$$

$$BER_{QPSK-Alam-PCK}(E_s/N_0, N_r) = BER_{QPSK}(E_s/N_0, 2N_r) \quad (38)$$

The expression for $BER_{QPSK}(E_s/N_0, L)$ is available from the literature (e.g., [4]). Using (20) and (25) in (37) and (38), the BER expressions for the mismatched receivers are easily obtained.

Figure 1 shows the minimal degradations (32) and (36) of the SNR, as a function of the number K of data symbols. For high E_b/N_0, this degradation manifests itself in a horizontal shift of the BER curve of the mismatched receiver as compared to the PCK receiver, by an amount indicated in Figure 1. For a given number K of data symbols, the degradation is always larger for Alamouti coded communication than for uncoded SIMO communication. Nevertheless, for large K the degradation is negligible in both cases, provided that the coherence time of the considered channel is sufficiently long to allow large K. Throughout the remainder of this section we assume that the same energy is allocated to pilot and data symbols ($E_P = E_s$) and that $K = 100$. Under these assumptions, we know from (31) that the optimal number of pilot symbols for uncoded SIMO communication is $P = 10$.

Figure 2 illustrates the BER for uncoded SIMO communication with MMSE channel estimation from the optimum number of pilot symbols. The solid lines show the analytical BER result, whereas the markers are obtained through simulation. The dashed lines show the BER in case of perfect channel knowledge. In accordance with (32), the degradation of the SNR amounts to $0.83dB$, irrespective of the number of receive antennas.

Figure 3 illustrates the BER for the Alamouti STBC in case of MMSE channel estimation with $P = 14$. In accordance with (36), the degradation of the SNR amounts to $1.15dB$.

Figure 4 shows the BER versus the number of pilot symbols for the Alamouti code at $Eb/N0 = 10dB$. Again, the solid lines show the analytical result whereas the markers are obtained through simulation. The dashed lines show the BER in case of perfect channel knowledge. It is clear from the figure that selecting the optimal number of pilot symbols is not a very critical issue.

7. CONCLUSIONS

In this work, we investigated the effect of imperfect channel estimation on the performance of both uncoded SIMO systems and MIMO systems with Alamouti space-time block codes. In both cases we considered a mismatched receiver and a propagation channel that is affected by flat Rayleigh block fading. Channel estimation was done from known pilot symbols sent among the data. We found that the channel estimation errors generate extra noise terms at the decision unit, which degrade the signal-to-noise ratio and thus deteriorate the performance of the communication system.

Further we derived an analytical expression for the bit error rate for QPSK transmission with imperfect channel estimation. For both uncoded SIMO communication and Alamouti space-time block coded MIMO communication

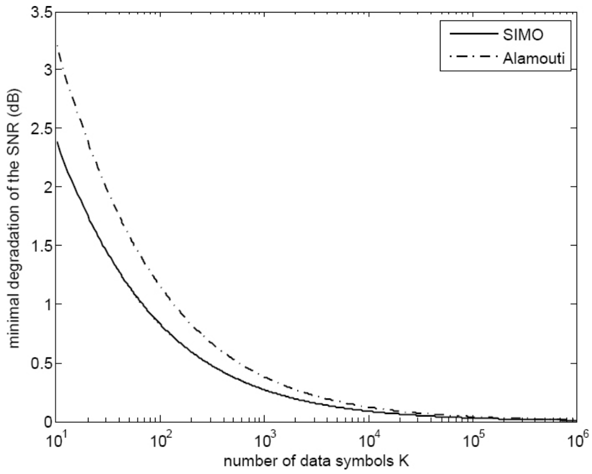

Figure 1: Minimal degradation of the SNR (in dB).

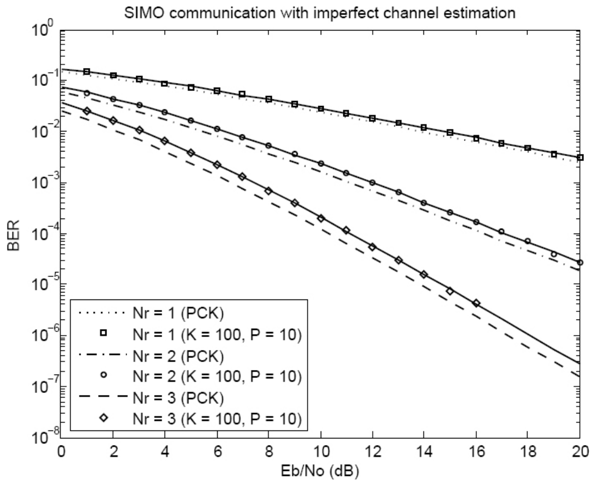

Figure 2: Bit error rate for uncoded SIMO.

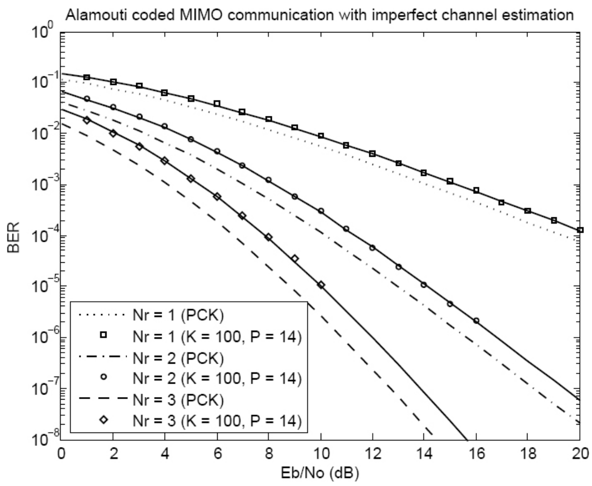

Figure 3: Bit error rate for Alamouti space-time block code.

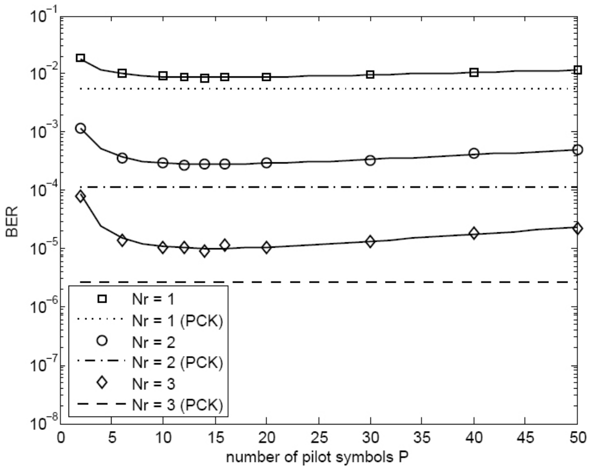

Figure 4: Bit error rate versus number of pilot symbols for Alamouti coded MIMO communication ($K = 100$, $E_b/N_0 = 10dB$).

we found the optimal number of pilot symbols that minimizes the BER degradation, given a number of data symbols. For best performance the number of data symbols in one transmission block should be taken as high as possible, provided that the duration of one block does not exceed the channel coherence time. Selecting the optimal number of pilot symbols turns out not to be a very critical issue.

8. ACKNOWLEDGMENTS

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