UNIVERSITEIT GENT
TELIN

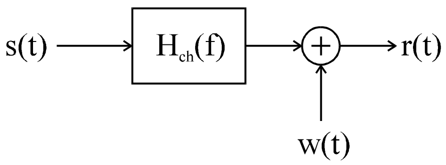
Equalization in digital communication

Prof. Marc Moeneclaey

UNIVERSITEIT GENT
TELIN

Intersymbol interference (1)

Dispersive (frequency-selective) channel



$H_{ch}(f)$: channel transfer function (often unknown)
w(t) : AWGN

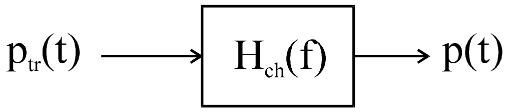
UNIVERSITEIT GENT
TELIN

Intersymbol interference (2)

Linear modulation

$$s(t) = \sqrt{E_s} \sum_k a_k \, p_{tr}(t - kT)$$

$$r(t) = \sqrt{E_s} \sum_m a_m \, p(t - mT) + w(t)$$



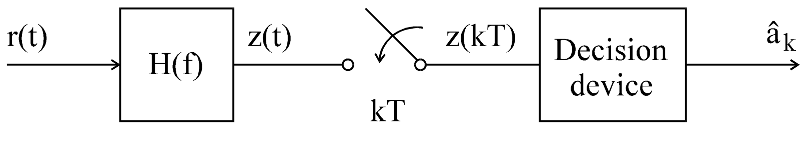
$$P(f) = H_{ch}(f) P_{tr}(f)$$

3

UNIVERSITEIT GENT
TELIN

Intersymbol interference (3)

Receiver



$$z(kT) = \underbrace{\sqrt{E_s} a_k \, g(0)}_{\text{useful term}} + \underbrace{\sqrt{E_s} \sum_{m \neq 0} a_{k-m} \, g(mT)}_{\text{intersymbol interference term (ISI)}} + \underbrace{n(kT)}_{\text{noise term}}$$

$$G(f) = H(f)P(f) = H(f) H_{ch}(f) P_{tr}(f)$$

4

UNIVERSITEIT GENT
TELIN

Intersymbol interference (4)

Receiver (cont.)

$$\frac{P_{noise}}{P_{useful}} = \frac{N_0}{E_s|g(0)|^2} \int_{-\infty}^{+\infty} |H(f)|^2 \, df$$

is minimum for $H(f) = P_{tr}^*(f)H_{ch}^*(f)$ (matched filter)

$$\frac{P_{ISI}}{P_{useful}} = \frac{1}{|g(0)|^2} \sum_{m \neq 0} |g(mT)|^2$$

is zero when G(f) fulfills the Nyquist criterion

In general, the matched filter does not yield zero ISI

5

UNIVERSITEIT GENT
TELIN

Intersymbol interference (5)

Eye diagram

Eye diagram shows $\mathrm{Re}\left[\sum_m a_m g(t - mT)\right]$

as a function of time for all possible data sequences

Eye diagram is periodic in t with period T

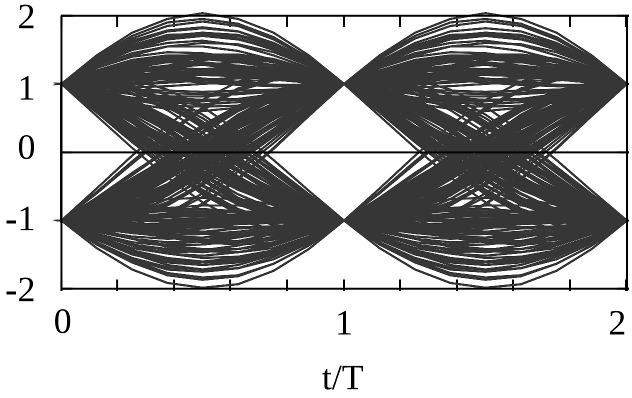
M-PAM : g(t) has duration LT
⇒ eye diagram has M^L lines per period

6

UNIVERSITEIT GENT
TELIN

Intersymbol interference (6)

Eye diagram 20 % cosine rolloff pulse

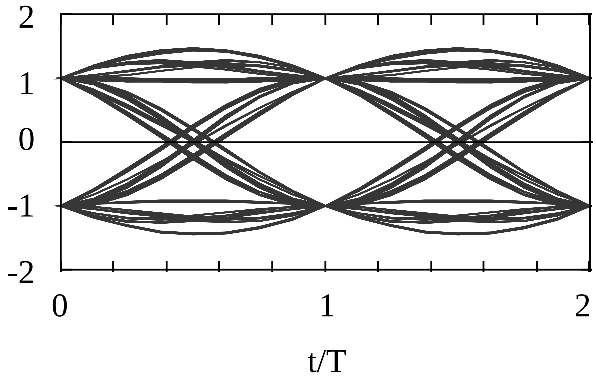


7

UNIVERSITEIT GENT
TELIN

Intersymbol interference (7)

Eye diagram 50 % cosine rolloff pulse

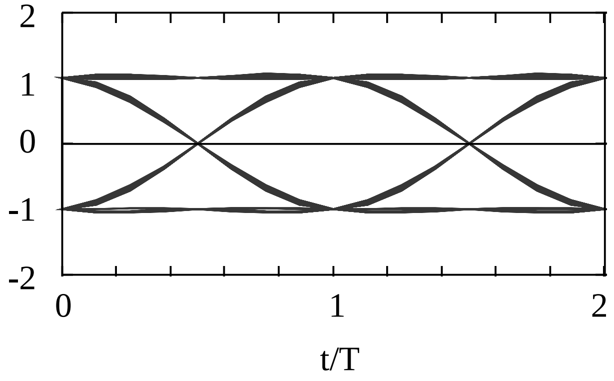


8

UNIVERSITEIT GENT
TELIN

Intersymbol interference (8)

Eye diagram 100 % cosine rolloff pulse



9

UNIVERSITEIT GENT
TELIN

Intersymbol interference (9)

Scatter diagram

For a given sampling instant kT, scatter diagram shows

$$\sum_m a_m g(kT - mT) = a_k g(0) + \sum_{m \neq 0} a_{k-m} g(mT)$$

in the complex plane for all possible data sequences

Constellation of M points : g(t) has duration LT

$\Rightarrow$ scatter diagram has M^L points

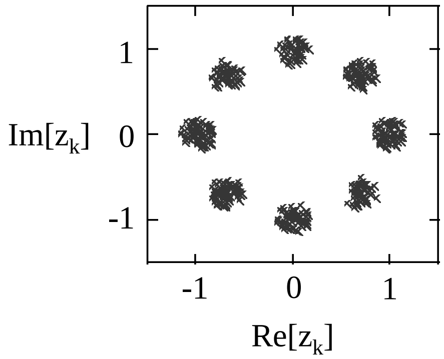
10

UNIVERSITEIT GENT
TELIN

Intersymbol interference (10)

Scatter diagram (cont.)

50 % cosine rolloff pulse, T/10 sampling error, 8-PSK



11

UNIVERSITEIT GENT
TELIN

Intersymbol interference (11)

Intersymbol interference power

$$\sum_{m=-\infty}^{+\infty} g(mT)\exp(-j2\pi fmT) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} G\left(f+\frac{k}{T}\right) = G_{fld}\left(e^{j2\pi fT}\right)$$

"folding" of G(f) yields $G_{fld}(\exp(j2\pi fT))$ which is periodic in f with period 1/T

12

UNIVERSITEIT GENT
TELIN

Intersymbol interference (12)

Intersymbol interference power (cont.)

Notation :

$$\left\langle X\left(e^{j2\pi fT}\right)\right\rangle = T\int_{\frac{-1}{2T}}^{\frac{1}{2T}} X\left(e^{j2\pi fT}\right)df$$

(averaging over interval (-1/(2T), 1/(2T))

13

UNIVERSITEIT GENT
TELIN

Intersymbol interference (13)

Intersymbol interference power (cont.)

$$g(mT) = \left\langle G_{fld}(e^{j2\pi fT})\exp(j2\pi fmT)\right\rangle$$

$$g(0) = \left\langle G_{fld}\left(e^{j2\pi fT}\right)\right\rangle$$

$$\sum_m |g(mT)|^2 = \left\langle \left|G_{fld}\left(e^{j2\pi fT}\right)\right|^2\right\rangle$$

14

UNIVERSITEIT GENT
TELIN

Intersymbol interference (14)

Intersymbol interference power (cont.)

$$\frac{P_{ISI}}{P_{useful}} = \frac{\left\langle \left| G_{fld}\left(e^{j2\pi fT}\right) - \left\langle G_{fld}\left(e^{j2\pi fT}\right) \right\rangle \right|^2 \right\rangle}{\left| \left\langle G_{fld}\left(e^{j2\pi fT}\right) \right\rangle \right|^2}$$

$$= \frac{\text{power of fluctuation of } G_{fld}\left(e^{j2\pi fT}\right) \text{ about its mean}}{\left| \text{mean of } G_{fld}\left(e^{j2\pi fT}\right) \right|^2}$$

UNIVERSITEIT GENT
TELIN

Intersymbol interference (15)

Intersymbol interference power (cont.)

Zero ISI is obtained when

$$G_{fld}\left(e^{j2\pi fT}\right) = \text{constant} \qquad \text{(Nyquist criterion)}$$

“Equalization” : elimination (or reduction) of ISI

UNIVERSITEIT GENT
TELIN

Intersymbol interference (16)

Example

Transmit pulse : P(f) = 50% square-root cosine rolloff

Channel : direct path + reflection

$$H_{ch}(f) = 1 + b\exp(-j2\pi f\tau)$$

reflection : delay τ, magnitude b (0<b<1)

$$\text{ripple [dB]} = 20\log\left(\frac{1+b}{1-b}\right)$$

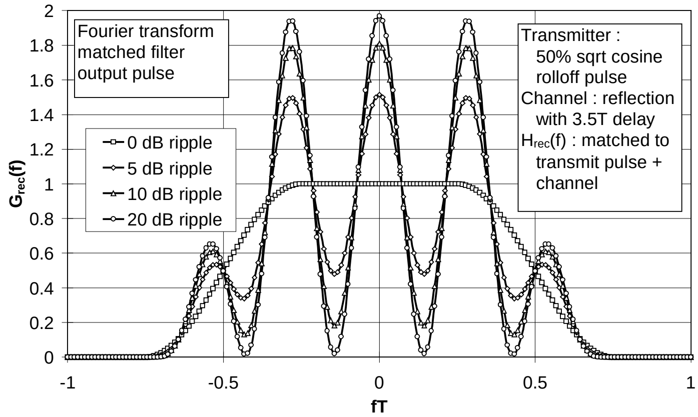
Receive filter : matched to transmit pulse + channel

$$H_{rec}(f) = P^*(f)H^*_{ch}(f)$$

UNIVERSITEIT GENT
TELIN

Intersymbol interference (17)

Example (cont.)

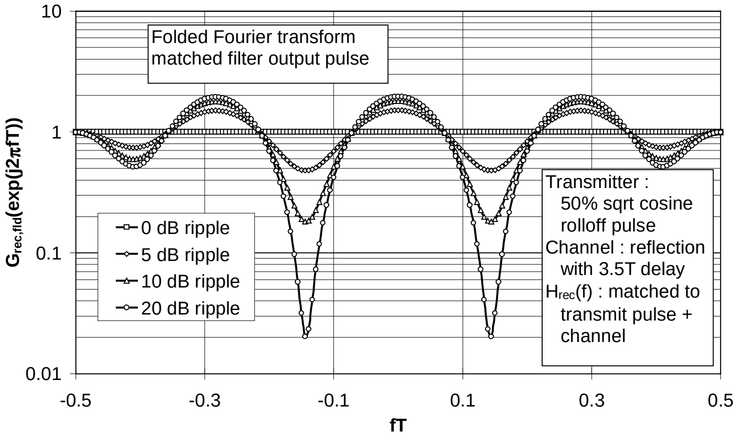


UNIVERSITEIT GENT

TELIN

Intersymbol interference (18)

Example (cont.)



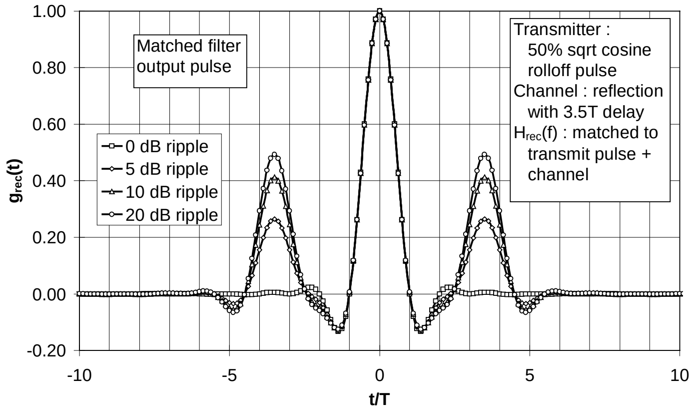
19

UNIVERSITEIT GENT

TELIN

Intersymbol interference (19)

Example (cont.)

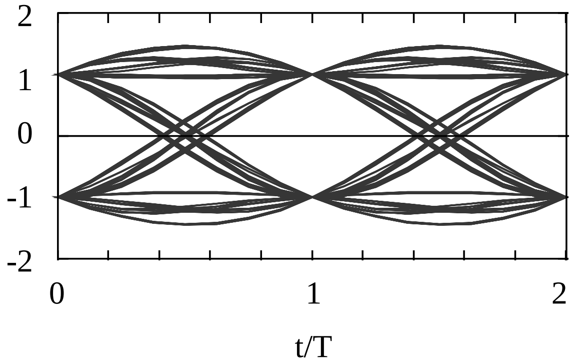


20

Intersymbol interference (20)

Example (cont.)

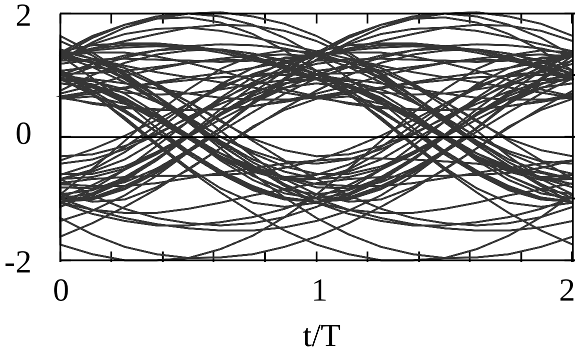
Eye diagram at matched filter output (0 dB ripple)



Intersymbol interference (21)

Example (cont.)

Eye diagram at matched filter output (5 dB ripple)

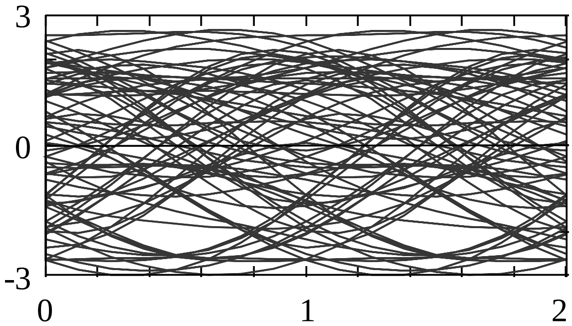


UNIVERSITEIT GENT
TELIN

Intersymbol interference (22)

Example (cont.)

Eye diagram at matched filter output (10 dB ripple)



UNIVERSITEIT GENT
TELIN

Intersymbol interference (23)

Example (cont.)

Ripple [dB]	$\frac{P_{ISI}}{P_{useful}}$ [dB]
0	$-\infty$
5	-9.96
10	-6.03
15	-4.84
20	-4.47

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (ZF-LE) (1)

Equalizer structure

Decompose the receive filter as

$$H(f) = \underbrace{H_{rec}(f)}_{\substack{\text{analog}\\ \text{prefilter}}}\ \underbrace{C(\exp(j2\pi fT))}_{\text{linear equalizer}}$$

$$C\left(e^{j2\pi fT}\right) = \sum_{m=-\infty}^{+\infty} c_m \exp(j2\pi fmT)$$

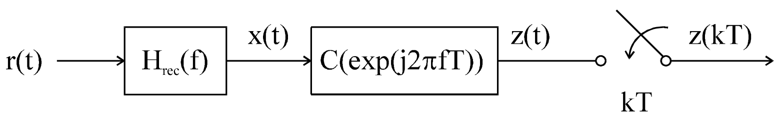
c_m : m-th equalizer tap

25

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (2)

Equalizer structure (cont.)



$$z(t) = \sum_{m=-\infty}^{+\infty} c_m x(t - mT) \qquad z(kT) = \sum_{m=-\infty}^{+\infty} c_m x(kT - mT)$$

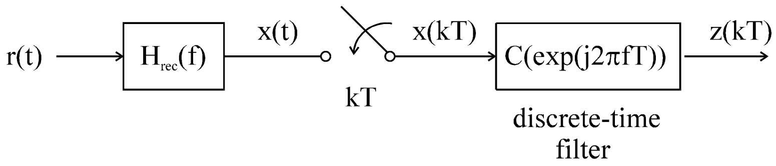
26

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (3)

Equalizer structure (cont.)

$$z(kT) = \sum_{m=-\infty}^{+\infty} c_m x(kT - mT)$$

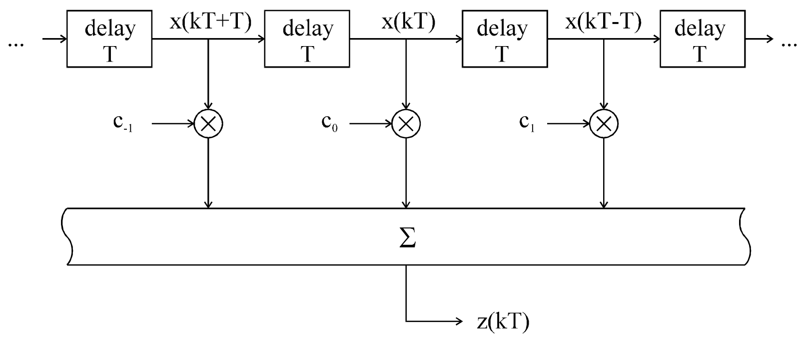


27

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (4)

Equalizer structure (cont.)



28

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (5)

Equalizer transfer function

The zero-forcing linear equalizer should be selected such that the resulting $G_{fld}(\exp(j2\pi fT))$ is constant ("equalized"), in order to eliminate all ISI

$$G(f) = \underbrace{P_{tr}(f)H_{ch}(f)H_{rec}(f)}_{G_{rec}(f)}\ \underbrace{C(\exp(j2\pi fT))}_{\substack{\text{periodic in f,}\\ \text{period 1/T}}}$$

$$G_{fld}\left(e^{j2\pi fT}\right) = G_{rec,fld}\left(e^{j2\pi fT}\right)C\left(e^{j2\pi fT}\right)$$

29

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (6)

Equalizer transfer function (cont.)

$$\Rightarrow C\left(e^{j2\pi fT}\right) = \frac{1}{G_{rec,fld}\left(e^{j2\pi fT}\right)}$$

The ZF-LE is not unique : different choice of $H_{rec}(f)$ yields a different equalizer and a different P_{noise}/P_{useful}

30

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (7)

Optimum zero-forcing linear equalizer

H(f) minimizes P_{noise}/P_{useful}, under the restriction of zero ISI

Solution : $$H(f) = \underbrace{P_{tr}^{*}(f) H_{ch}^{*}(f)}_{\text{matched filter}} \underbrace{C(\exp(j2\pi fT))}_{\text{linear equalizer}}$$

$$C\left(e^{j2\pi fT}\right) = \frac{1}{G_{rec,fld}\left(e^{j2\pi fT}\right)}$$

$$G_{rec}(f) = \left|P_{tr}(f) H_{ch}(f)\right|^2$$

31

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (8)

Noise enhancement

As H(f) for the optimum ZF-LE is different from the matched filter, the equalizer eliminates ISI at the expense of an increased noise level

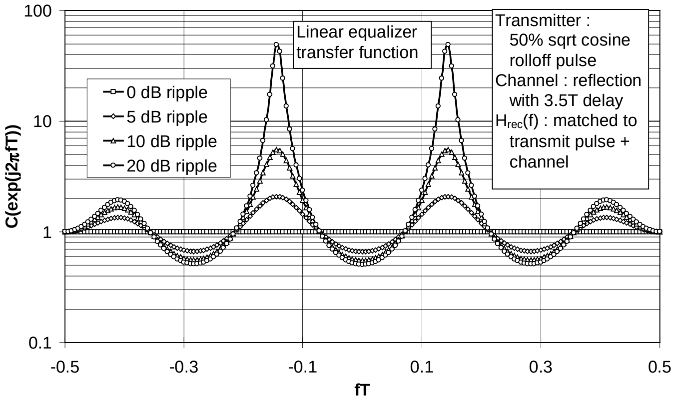
$$\left(\frac{P_{noise}}{P_{useful}}\right)_{\substack{\text{matched}\\ \text{filter}}} < \left(\frac{P_{noise}}{P_{useful}}\right)_{ZF-LE}$$

32

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (9)

Example Channel : direct path + reflection

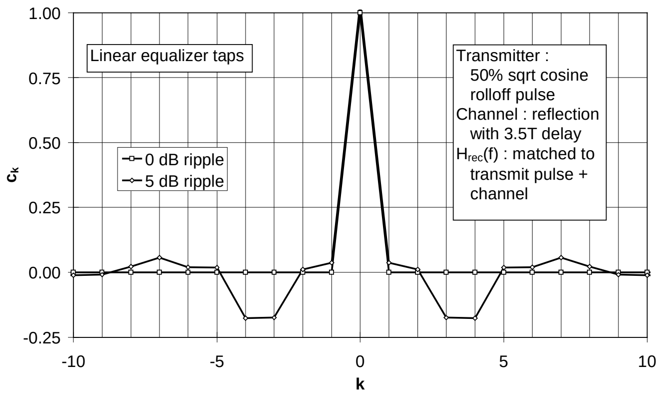


33

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (10)

Example (cont.)

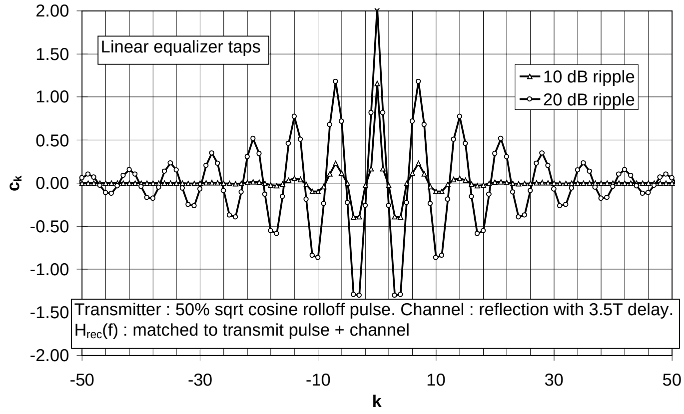


34

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (11)

Example (cont.)

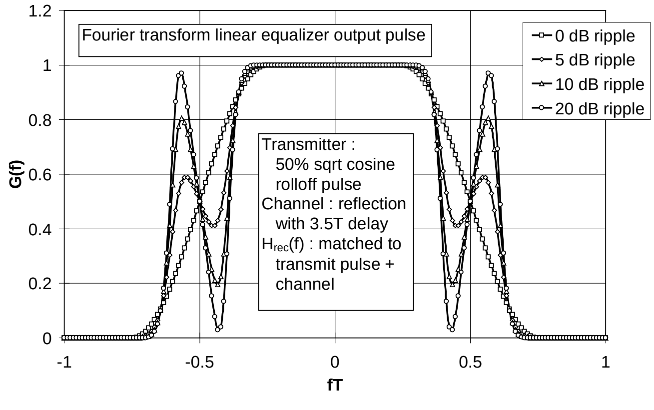


35

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (12)

Example (cont.)



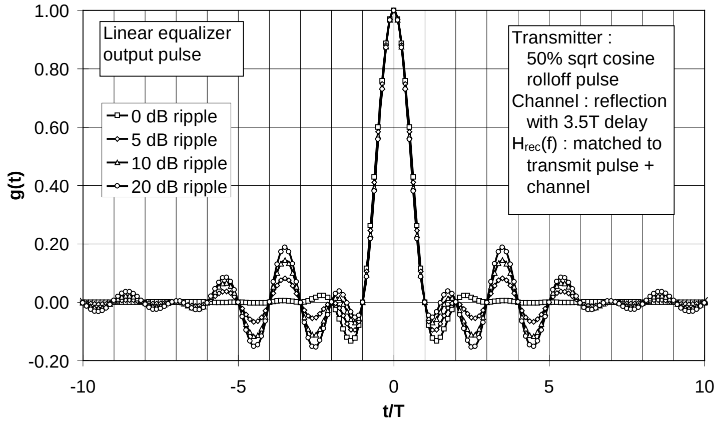
36

UNIVERSITEIT GENT

TELIN

Zero-forcing linear equalizer (13)

Example (cont.)



37

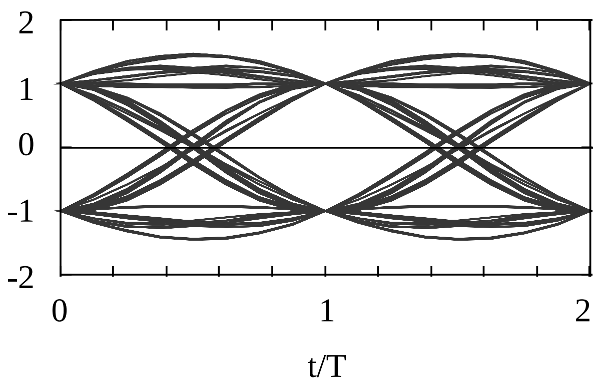
UNIVERSITEIT GENT

TELIN

Zero-forcing linear equalizer (14)

Example (cont.)

Eye diagram at equalizer output (0 dB ripple)



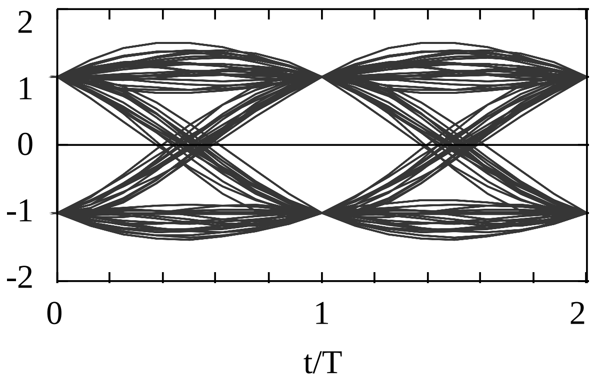
38

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (15)

Example (cont.)

Eye diagram at equalizer output (5 dB ripple)



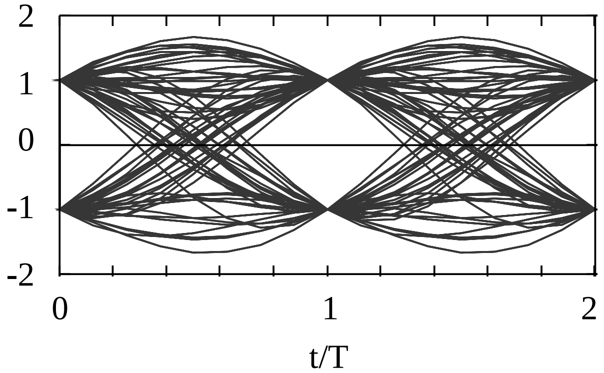
39

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (16)

Example (cont.)

Eye diagram at equalizer output (10 dB ripple)



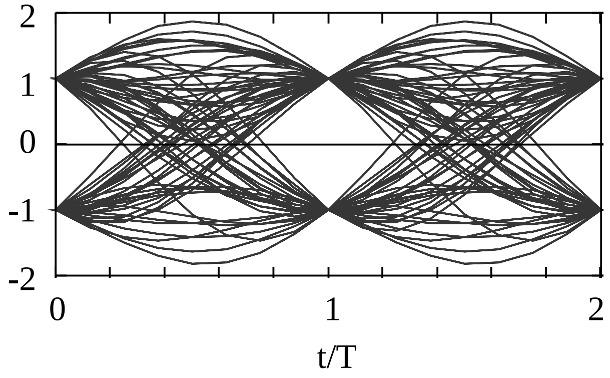
40

UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (17)

Example (cont.)

Eye diagram at equalizer output (20 dB ripple)



UNIVERSITEIT GENT
TELIN

Zero-forcing linear equalizer (18)

Example (cont.)

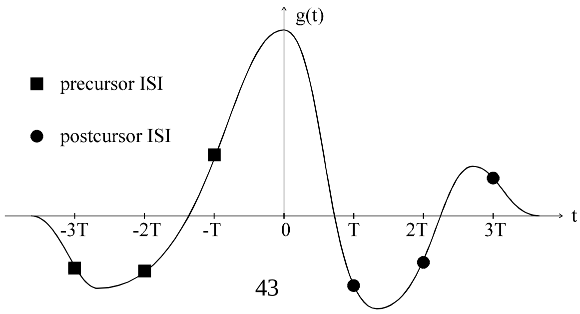
Ripple [dB]	Noise enhancement [dB] (ZF-LE)
0	0
5	0.49
10	1.7
15	3.36
20	5.33

Zero-forcing decision-feedback equalizer (ZF-DFE) (1)

UNIVERSITEIT GENT
TELIN

Equalizer structure

$$\mathrm{ISI}(kT) = \underbrace{\sqrt{E_s}\sum_{m>0} a_{k-m}\, g(mT)}_{\substack{\text{postcursor ISI}\\ \text{(past symbols)}}} + \underbrace{\sqrt{E_s}\sum_{m<0} a_{k-m}\, g(mT)}_{\substack{\text{precursor ISI}\\ \text{(future symbols)}}}$$

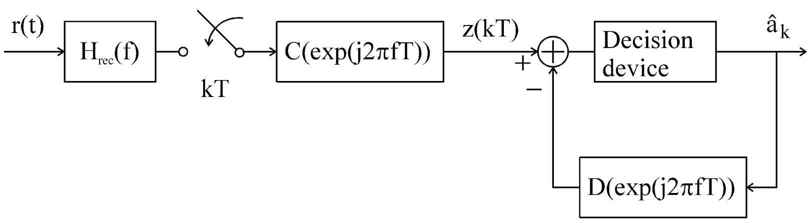


Zero-forcing decision-feedback equalizer (2)

UNIVERSITEIT GENT
TELIN

Equalizer structure (cont.)

Forward equalizer C(exp(j2πfT)) eliminates precursor ISI
Feedback equalizer D(exp(j2πfT)) is a causal filter, using past decisions to cancel postcursor ISI



UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (3)

Equalizer transfer function

$$C\left(e^{j2\pi fT}\right) = \sum_{m=-\infty}^{+\infty} c_m e^{j2\pi fT} \qquad D\left(e^{j2\pi fT}\right) = \sum_{m=1}^{+\infty} d_m e^{-j2\pi fT}$$

$$G_{fld}\left(e^{j2\pi fT}\right) = G_{rec,fld}\left(e^{j2\pi fT}\right) C\left(e^{j2\pi fT}\right)$$

Requirement for zero ISI :

$$G_{fld}\left(e^{j2\pi fT}\right) = 1 + D\left(e^{j2\pi fT}\right)$$

45

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (4)

Equalizer transfer function (cont.)

$$\Rightarrow C\left(e^{j2\pi fT}\right) = \frac{1 + D\left(e^{j2\pi fT}\right)}{G_{rec}\left(e^{j2\pi fT}\right)}$$

The ZF-DFE is not unique : different choice of $H_{rec}(f)$ and D(exp(j2πfT)) yields a different equalizer and a different P_{noise}/P_{useful}

46

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (5)

Optimum zero-forcing decision-feedback equalizer

H(f) minimizes P_{noise}/P_{useful}, under the restriction of zero precursor ISI

Solution

$$H(f) = \underbrace{P_{tr}^{*}(f)H_{ch}^{*}(f)}_{\text{matched filter}}\ \underbrace{C(\exp(j2\pi fT))}_{\text{forward equalizer}}$$

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (6)

Optimum zero-forcing decision-feedback equalizer (cont.)

$$G_{rec}(f) = |H_{ch}(f)|^2 |P(f)|^2 > 0$$

$$\Rightarrow g_{rec}(t) = g_{rec}^{*}(-t) \Rightarrow g_{rec}(mT) = g_{rec}^{*}(-mT)$$

$$G_{rec,fld}(z) = \sum_{m=-\infty}^{+\infty} g_{rec}(mT)z^{-m} = G_{rec,fld}^{*}\left(\frac{1}{z^{*}}\right)$$

poles and zeroes of $G_{rec,fld}(z)$ occur in pairs :
z_p and $1/z_p^*$ (poles), z_n and $1/z_n^*$ (zeroes)

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (7)

Optimum zero-forcing decision-feedback equalizer (cont.)

"Spectral factorization" : $G_{rec,fld}(z) = K\,B(z)\,B^*\left(\frac{1}{z^*}\right)$

K>0

$$B(z) = \frac{\prod_i \left(1 - z_{n,i} z^{-1}\right)}{\prod_j \left(1 - z_{p,j} z^{-1}\right)} = 1 + \sum_{m=1}^{+\infty} b_m z^{-m} \quad \text{(causal)}$$

poles and zeroes inside unit circle

$$B^*\left(\frac{1}{z^*}\right) = \frac{\prod_i \left(1 - z_{n,i}^* z\right)}{\prod_j \left(1 - z_{p,j}^* z\right)} = 1 + \sum_{m=1}^{+\infty} b_m^* z^{m} \quad \text{(anti-causal)}$$

poles and zeroes outside unit circle

49

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (8)

Optimum zero-forcing decision-feedback equalizer (cont.)

$$C\left(e^{j2\pi fT}\right) = \frac{1}{K\,B^*\left(e^{j2\pi fT}\right)} \qquad \text{(anti-causal)}$$

$$\Rightarrow G_{fld}\left(e^{j2\pi fT}\right) = B\left(e^{j2\pi fT}\right) \qquad \text{(causal)}$$

$$D\left(e^{j2\pi fT}\right) = B\left(e^{j2\pi fT}\right) - 1 \qquad \text{(causal)}$$

50

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (9)

Noise enhancement

As H(f) for the optimum ZF-DFE is different from the matched filter, the equalizer eliminates ISI at the expense of an increased noise level. The noise enhancement is less than for the optimum ZF-LE

$$\left(\frac{P_{noise}}{P_{useful}}\right)_{\substack{\text{matched}\\ \text{filter}}} < \left(\frac{P_{noise}}{P_{useful}}\right)_{ZF-DFE} < \left(\frac{P_{noise}}{P_{useful}}\right)_{ZF-LE}$$

51

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (10)

Error propagation in DFE

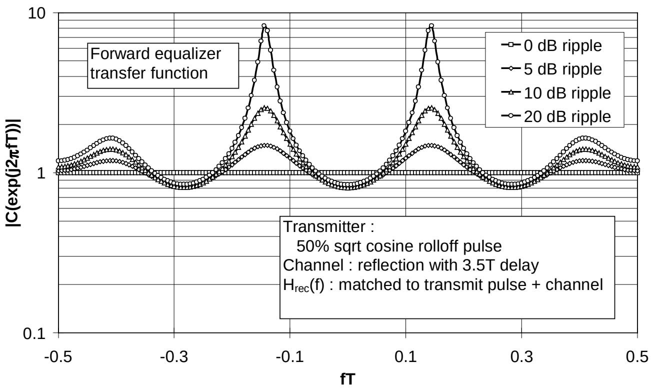
Cancellation of postcursor ISI assumes correct symbol decisions. Erroneous symbol decisions enhance postcursor ISI, which in turn may give rise to additional decision errors

52

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (11)

Example Channel : direct path + reflection

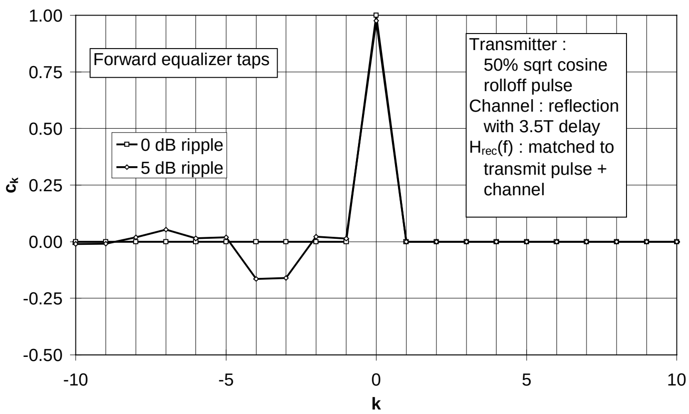


53

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (12)

Example (cont.)

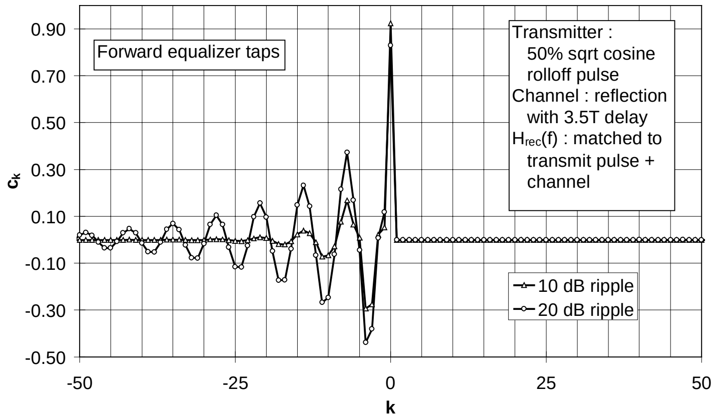


54

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (13)

Example (cont.)

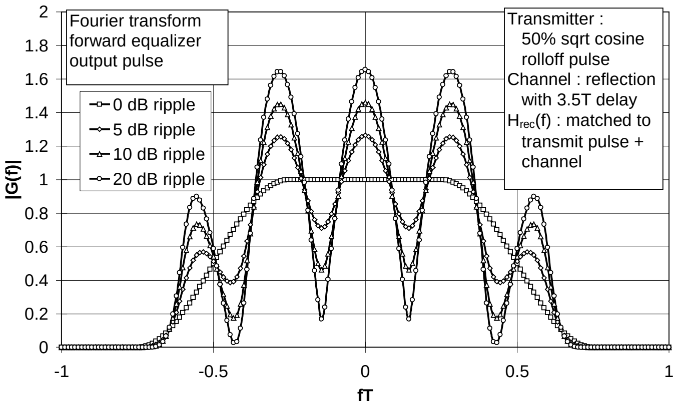


55

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (14)

Example (cont.)

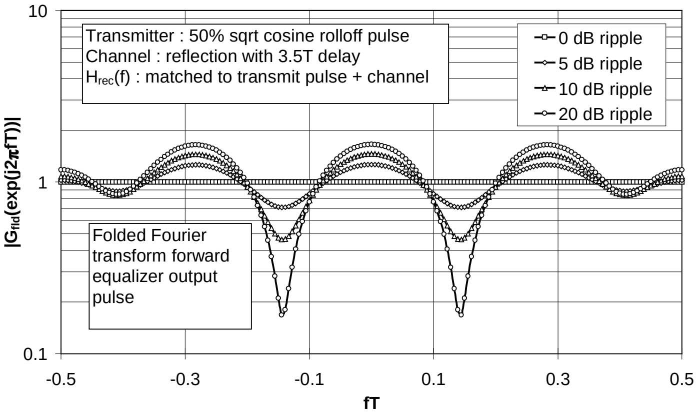


56

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (15)

Example (cont.)

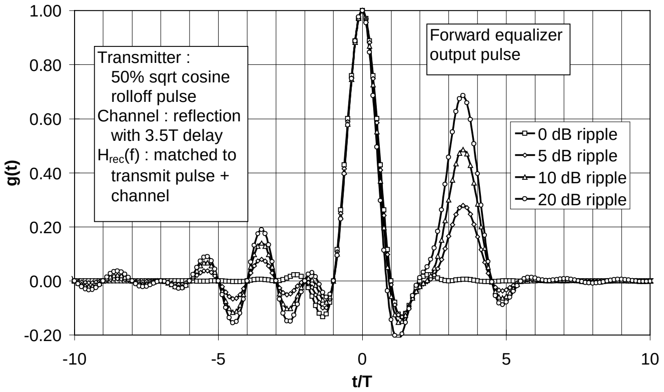


57

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (16)

Example (cont.)

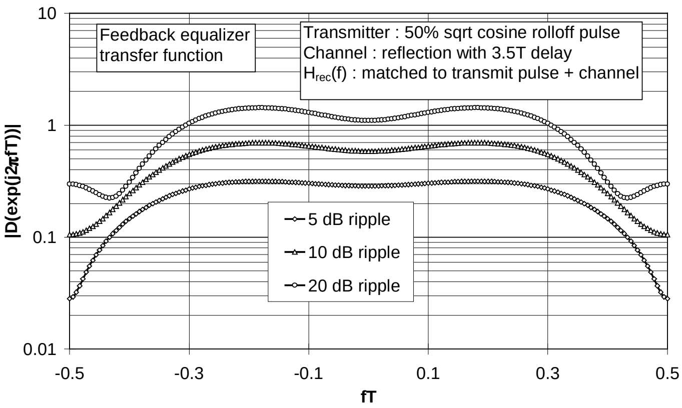


58

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (17)

Example (cont.)

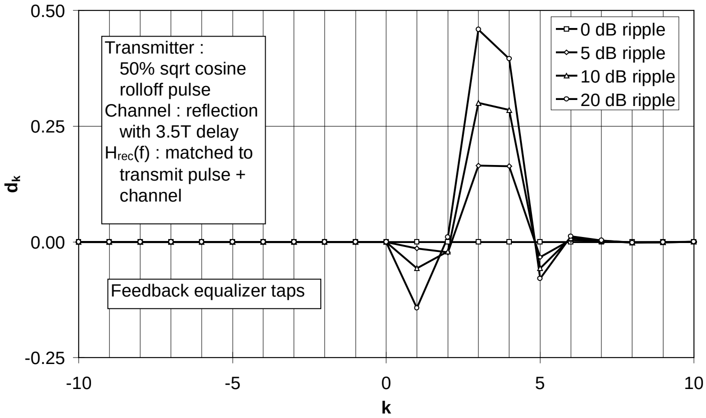


59

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (18)

Example (cont.)



60

UNIVERSITEIT GENT
TELIN

Zero-forcing decision-feedback equalizer (19)

Example (cont.)

Ripple [dB]	Noise enhancement [dB] (ZF-DFE)
0	0
5	0.24
10	0.71
15	1.14
20	1.44