Buffer Analysis of a Class of Retransmission Protocols with Variable Feedback Delays

Koen De Turck, Stijn De Vuyst, Sabine Wittevrongel

SMACS Research Group
Department of Telecommunications and Information Processing
Ghent University, Belgium

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Introduction and motivation
- ARQ Retransmission Protocols
- The Classical Variants of ARQ
- Existing ARQ Models for Performance Analysis
- Possible Applications for this Model

Discrete-time Model of the Transmitter Queue
- Modelling Assumptions
- Queue Content on Send Opportunities
- Queue Content on Random Slots
- Extension to Correlated Errors & Feedback Delays

Some Numerical Examples
- Throughput
- Mean Delay
- Mean Queue Content

Conclusions
ARQ retransmission protocols

‘Automatic Repeat reQuest’

Reliable transmission of packets over unreliable channel (mostly layer 2)

- Receiver checks packets for errors and returns ACK or NACK accordingly. Transmitter retransmits packet if necessary.

Diagram:

- TRANSMITTER
- Channel
- RECEIVER

- First transmission of P
- Feedback delay f
- P discarded from queue
- P correctly received
- Waiting
- Transmission
- Resequencing
Classical Variants of ARQ

Stop and Wait ARQ

transmitter \[\cdots 1 2 2 R 3 \cdots\]

channel

receiver \[\cdots 1 2 2 \cdots\]

feedback delay $f$
Classical Variants of ARQ

Go-back-$N$ ARQ (GBN)

- Has window size $N$: maximum number of outstanding acknowledgements
- On NACK of packet $i$, all packets $i, i+1, \ldots$ in window are retransmitted

![Diagram of Go-back-$N$ ARQ (GBN)]
Classical Variants of ARQ

Selective Repeat ARQ (SR)

- Only the NACKed packets are retransmitted
- Most efficient, but requires resequencing!
Discrete-time models for ARQ performance analysis

Many kinds of models with specific focus:

- Classical ARQ
- Various channel error models: uncorrelated, correlated
- Multi-copy ARQ (send $m$ copies of a packet)
- Limited number of retransmissions per packet (time-outs)
- ARQ in combination with FEC: ‘Hybrid ARQ’
- …

Studied performance characteristics:

- Throughput (usually)
- Transmitter or receiver queueing behaviour (sometimes)

However, authors almost always consider fixed feedback delay $f$!

$\implies$ We consider a GBN and SR queueing model of the transmitter in case of variable feedback delay $f$

$\sim f$: iid random variables with pgf $F(z)$

$\sim f$: correlated random variables
Motivation

Possible situations where accounting for variable feedback delay is important

- Mobile links with multipath propagation (layer 2)

- Networks with dynamic and/or multipath routing (layer 3,4)
  Error control over virtual networks, overlay networks, tunnelling, ad hoc networks, ...
Modelling assumptions

Transmitter queue:
- Discrete-time queue with $\infty$ capacity
- $a$: # arrivals per slot: iid with pgf $A(z)$

Joint transmission and acknowledgement:
- At most $N$ packets are sent together in one slot
- $N$: window size
- Receiver returns only one joint ACK/NACK for the whole window
- $f$: feedback delay of a (joint) transmission: iid with pgf $F(z)$
- $p$: error probability of a single packet

Send opportunity:
A slot in which the transmitter is not waiting for acknowledgements and there are more than $n^*$ packets waiting
- $n^*$: send threshold ($< N$)
Modified GBN and SR

For example, $N = 5$ and $n^* = 1$:
Modified GBN and SR

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For example, $N = 5$ and $n^* = 1$:
Queue content on send opportunities

Number of arrivals in a feedback delay:

- $\alpha$: # arrivals during $f \sim$ has pgf $\alpha(z) = F(A(z))$

Number of required retransmissions in a window:

- $r$: # of packets in the transmitted window that contain an error distribution depends on protocol (GBN or SR)
  
  $r(i, n) = \text{Prob}[i \text{ out of } n \text{ packets must be resent}]$
  
  $\sim$ has pgf $R_n(z)$

For GBN: $R_n(z) = (1-p)^n + pz \frac{(1-p)^n - z^n}{1-p - z}$

For SR: $R_n(z) = (1-p + pz)^n$

$(0 \leq n \leq N)$
Queue content on send opportunities

- \( v_k \): queue content on \( k \)th send opportunity \( \sim \) pmf \( v(k) \)

\[
v_{k+1} = \begin{cases} 
  v_k - N + r + \alpha, & \text{when } v_k \geq N, \\
  r + \alpha & \text{when } n^* < v_k < N, \\
  v_k + a & \text{when } v_k \leq n^*. 
\end{cases}
\]

- Let \( V(z) \) be the equilibrium distribution of \( v_k \), then

\[
V(z) = \frac{1}{z^N - R_N(z)\alpha(z)} \sum_{i=0}^{N-1} v(i) \left( z^N f_i(z) - z^i R_N(z)\alpha(z) \right)
\]

with

\[
f_i(z) = \begin{cases} 
  z^i A(z), & \text{when } 0 \leq i \leq n^*, \\
  \alpha(z) R_i(z), & \text{when } n^* < i < N,
\end{cases}
\]
Queue content on random slots

- $u$: queue content in random slot $\sim$ pgf $U(z)$ and pmf $u(k)$

- Consider a large number $Q$ of send opportunities

This corresponds to

$$Q \left( \text{Prob}[v \leq n^*] + \mathbb{E}[f] \text{Prob}[v > n^*] \right) = Q\beta \text{ slots}$$

If $k \leq n^*$, then

$$u(k) = \frac{Qv(k)}{Q\beta} = \frac{v(k)}{\beta}$$

If $k > n^*$, then

$$u(k) = \frac{Q\sum_{j=n^*+1}^{k} v(j) \mathbb{E}[\# \text{ slots with } k \text{ packets}|j \text{ packets at start fd}]}{Q\beta}$$
Queue content on random slots

The equilibrium distribution of $u_k$ is then

$$U(z) = \frac{\sum_{k=0}^{n^*} v(k)z^k + \mathbb{E}[f]\left(V(z) - \sum_{k=0}^{n^*} v(k)z^k\right)L(z)}{\text{Prob}[v \leq n^*] + \mathbb{E}[f]\text{Prob}[v > n^*]}$$

with

$$L(z) = \tilde{F}(A(z)) = \frac{A(z)(F(A(z)) - 1)}{\mathbb{E}[f](A(z) - 1)}$$
Extension to correlated errors & feedback delays

Markovian background process

- In one of $M$ states (finite)
- Transition probabilities
  \[ q(i, j) = \text{Prob}[\text{BG state } i \text{ in slot } k+1 | \text{BG state } j \text{ in slot } k] \]

Both the packet error probability and the length of the feedback delay can be assumed to depend on the state of a background process.
Extension to correlated errors & feedback delays

State-dependent error probabilities

- In state $i$, the error probability is $p_i$.
- $R_n(z)$ is extended to an $M \times M$ diagonal matrix:

$$[R_n(z)]_{ii} = R_n(z)|_{p=p_i}$$

State-dependent feedback delay distribution

- On send opportunities we define

$$f(i, j, k) = \text{Prob}[f \text{ is } k \text{ slots, next state is } j \mid \text{current state is } i]$$

- $F(z)$ is extended to an $M \times M$ matrix:

$$[F(z)]_{ij} = \sum_{k=0}^{\infty} f(i, j, k)z^k$$
Extension to correlated errors & feedback delays

Queue content at send opportunities

- On send opportunities, define the vector

\[
[V(z)]_i = \sum_{n=0}^{\infty} z^n \text{Prob}[\text{queue content } n \text{ and BG state is } i]
\]

and

\[
V(z) = \sum_{n=0}^{\infty} v(n) z^n
\]

We find:

\[
V(z) = \sum_{n=0}^{N-1} v(n) \left( z^N f_n(z) - z^n R_N(z) F(A(z)) \right) \left( z^N I - R_N(z) F(A(z)) \right)^{-1}
\]

All \(MN\) unknown probabilities can be found by cancelling out the zeroes of \(z^N I - R_N(z) F(A(z))\)!
Throughput vs the mean feedback delay

\[ \eta = \frac{N - R'_N(1)}{E[f]} \]

The throughput \( \eta \) as a function of \( E[f] \) for \( N=5 \)

- error probability \( p=0.2 \) (GBN and SR)
- error probability \( p=0.4 \) (GBN and SR)
Mean delay vs. the mean feedback delay

Mean packet delay $E[d]$ as a function of $E[f]$ for $N=5$ and error probability $p=0.2$. Geometric arrivals and load $= E[a]/\eta = 0.8$

- $f$ geometrically distributed (GBN and SR)
- $f$ constant (GBN and SR)
Mean queue content vs. the mean feedback delay

Mean queue content $E[u]$ as a function of $E[f]$ for $N = 5$ and error probability $p = 0.2$. Geometric arrivals and load $= 0.8$

- $f$ geometrically distributed (GBN and SR)
- $f$ constant (GBN and SR)
Mean queue content $E[u]$ as a function of $E[f]$ for $N=5$ and error probability $p=0.2$. Geometric arrivals with fixed arrival intensity 0.1

- $f$ geometrically distributed (GBN and SR)
- $f$ constant (GBN and SR)
Mean queue content \( E[u] \) as a function of the load for \( N = 5 \) and \( p = 0.2 \). The mean feedback delay is fixed \( (E[f] = 7) \)

- \( f \) geometrically distributed (GBN and SR)
- \( f \) constant (GBN and SR)
Mean queue content $E[u]$ as a function of the arrival intensity for $N = 5$ and $p = 0.2$. The mean feedback delay is fixed ($E[f] = 7$) and arrivals are geometric.

- $f$ geometrically distributed (GBN and SR)
- $f$ constant (GBN and SR)
Conclusions

We studied the impact of a variable feedback delay on ARQ performance

- Modified GBN and SR with send threshold \( n^* \), a window \( N \) and joint acknowledgement
- Subsequent transmissions have iid feedback delay
- fixed packet error probability

Queue content distribution at send opportunities and random slots

Background states
We also considered the case where the feedback delays and error probabilities are Markov modulated

Observation from examples:
- Variability of the feedback delay clearly degrades queueing performance