How to reach equations (7), (9), (11), and (12).
Can you tell me, under what assumptions they were derived?

1 Equation (7)

In section 4 of the paper, the system contents in terms of packets is analyzed, not at start slots as in the previous section, but at arbitrary slots. This means we select any slot \( k \) and look at the system contents \( u_{1,k} \) and \( u_{2,k} \) of both classes at the beginning of that slot. Their distribution is given by the pgf (probability generating function)

\[
U_k(z_1, z_2) = E[z_1^{u_{1,k}} z_2^{u_{2,k}}].
\]

As stated in the paper, the server is one of three possible states during the arbitrary slot \( k \) : it is idle, it is serving a class 2 packet or it is serving a class 1 packet. We can thus distinguish the following three events (see also fig. 1)

1. The server can be found doing nothing in slot \( k \), i.e. the system is completely empty at the beginning of this slot :

\[
u_{1,k} = u_{2,k} = 0,
\]

and therefore, no packet can be selected for service. This even is called “no service”.

2. The server is busy serving a class 2 packet during slot \( k \). This can only be the case if at the beginning of the last “start slot” (this is start slot \( l \), in which the service of the currently served packet started) there were no class 1 packets in the system. Otherwise, a class 1 packet would have gone into service at that time instead of a class 2 packet, because of the priority scheduling. On the other hand, obviously, at least one class 2 packet must have been present, since apparently, it was possible to select one for service. So in this event, which is called “service class 2 packet”, we have

\[
n_{1,l} = 0, n_{2,l} > 0.
\]

3. A class 1 packet is being served during slot \( k \). As such, one must have been present at the beginning of this service time, i.e. at the beginning of the last start slot :

\[
n_{1,l} > 0.
\]

This event is called “service class 1 packet”.

These three events form what is called a complete family. In general, the events \( Y_1, Y_2, \ldots, Y_n \) constitute a complete family if they are mutually exclusive (\( Y_i \cap Y_j = \emptyset, 0 \leq i, j \leq n \)) and exhaustive (\( \bigcup_{i=1}^n Y_i = \Omega \)). This means that precisely one of the events \( Y_n \) will always occur. This definition implies that

\[
\text{Prob}[Y_i, Y_j] = \begin{cases} 
0 & \text{if } i \neq j, \\
\text{Prob}[Y_i] & \text{if } i = j;
\end{cases}
\]
and also that \( \Pr[Y_1] + \ldots + \Pr[Y_n] = 1 \). Using the notation \( \mathbb{E}[X \{ Y \}] \triangleq \mathbb{E}[X \mid Y] \Pr[Y] \) introduced in the paper, we can condition the expected value of \( X \) on the events \( Y_n \) as follows:

\[
\mathbb{E}[X] = \sum_{i=0}^{+\infty} i \Pr[X = i] = \sum_{i=0}^{+\infty} i \left( \sum_n \Pr[X = i \mid Y_n] \right)
\]

\[
= \sum_n \sum_{i=0}^{+\infty} i \Pr[X = i \mid Y_n] \cdot \Pr[Y_n]
\]

\[
= \sum_n \mathbb{E}[X \mid Y_n] \Pr[Y_n]
\]

\[
= \sum_n \mathbb{E}[X \{ Y_n \}].
\]

It is clear that the three events “no service”, “service class 2 packet” and “service class 1 packet” indeed are a complete family (the server can be found in exactly one of these three states at an arbitrary time instant). Thus, we can write

\[
U_k(z_1, z_2) = \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_1^{\mu_2,k} \{ \text{no service} \}]
\]

\[
+ \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_2^{\mu_2,k} \{ \text{service class 2 packet} \}]
\]

\[
+ \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_2^{\mu_2,k} \{ \text{service class 1 packet} \}]
\]

\[
= \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_2^{\mu_2,k} \{ u_{1,k} = u_{2,k} = 0 \}]
\]

\[
+ \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_2^{\mu_2,k} \{ n_1,l = 0, n_2,l > 0 \}]
\]

\[
+ \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_2^{\mu_2,k} \{ n_1,l > 0 \}],
\]

as was explained above. This is the first line of (7), which can also be written as

\[
U_k(z_1, z_2) = \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_2^{\mu_2,k} \mid \text{no service}] \cdot \Pr[\text{no service}]
\]

\[
+ \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_2^{\mu_2,k} \mid \text{service class 2 packet}] \cdot \Pr[\text{service class 2 packet}]
\]

\[
+ \mathbb{E}[\zeta_1^{\mu_1,k} \zeta_2^{\mu_2,k} \mid \text{service class 1 packet}] \cdot \Pr[\text{service class 1 packet}].
\]

The question is now: what are the probabilities with which each of the above events can occur? Remember that slot \( k \) is an arbitrary slot, it is randomly picked out of all the slots on the time axis. First of all, we can split up these slots in two groups: the “idle” slots, during which the server is idle (and the system empty), and the “busy slots”, during which a packet is in service. The probability that a random slot \( k \) is idle, is equal to the probability that no packets were present in the system at the beginning of that slot, i.e.

\[
\Pr[u_{1,k} = u_{2,k} = 0] = U_k(0, 0).
\]

Obviously, the probability of slot \( k \) being a busy slot is then \( 1 - U_k(0, 0) \). Secondly, we note that there are two kinds of busy slots, according to the class of the packet that is in service. That is for \( j = 1, 2 \):

\[
\Pr[\text{service class } j \text{ packet (in slot } k)] = \Pr[\text{service class } j \text{ packet (in slot } k) \mid k \text{ is busy slot}] \cdot \Pr[k \text{ is busy slot}].
\]

\[
\rho_j / \rho = \rho_j / \rho_1 + \rho_2.
\]

Indeed, on the whole, the load imposed on the server originating from class \( j \) traffic is \( \rho_j / \rho = \lambda_j \mu_j \). Hence, the probability that the server is found serving a class \( j \) packet is

\[
\frac{\rho_j}{\rho_1 + \rho_2} = \frac{\rho_j}{\rho}.
\]

All this finally yields equation (7).
2 Equation (9)

Equation (9) relates the distribution $U_k(z_1, z_2)$ of the system contents of both classes at an arbitrary slot $k$ to the distribution $N_l(z_1, z_2)$ of the system contents at the last start slot $l$, i.e. at the beginning of the slot in which the service started of the packet that is served during slot $k$. Indeed, if we consider slot $k$, then the service of the currently served packet has been going on for some time before slot $k$ (see fig. 2). The already elapsed service time of the packet being served in slot $k$ is called $s^+_k$. The arrivals during this time then contribute to the system contents as $(j = 1, 2)$:

$$u_{j,k} = n_{j,l} + \sum_{i=1}^{s^+_k} a_{j,k-i}.$$  

Substituting this into the conditional expected values of (8), we find the first line of (9), which can be simplified considering the following. In the “service class 2 packet” event we find:

$$E\left[\sum_{i=1}^{s^+_k} a_{1,k-i} z_{2}^{n_{2,l}+s^+_k a_{2,k-i}} | n_{1,l} = 0, n_{2,l} > 0\right]$$

$$= E\left[\prod_{i=1}^{s^+_k} \left(\frac{z_{1}^{a_{1,k-i}} z_{2}^{a_{2,k-i}}}{z_{1}^i z_{2}^i}\right) \cdot z_{2}^{n_{2,l}} | n_{1,l} = 0, n_{2,l} > 0\right]$$

$$= E\left[\left(A(z_{1}, z_{2})\right)^{s^+_k} \right] \cdot E\left[z_{2}^{n_{2,l}} | n_{1,l} = 0, n_{2,l} > 0\right]$$

$$= S^+_{2,k}(A(z_{1}, z_{2})) \cdot \sum_{i=0}^{+\infty} z_{2}^i \cdot \text{Prob}[n_{2,l} = i | n_{1,l} = 0, n_{2,l} > 0]$$

$$(s^+_k \text{ is the elapsed service time of a class 2 packet here})$$

$$= S^+_{2,k}(A(z_{1}, z_{2})) \cdot \sum_{i=0}^{+\infty} z_{2}^i \cdot \frac{\text{Prob}[n_{2,l} = i, n_{1,l} = 0, n_{2,l} > 0]}{\text{Prob}[n_{1,l} = 0, n_{2,l} > 0]}$$

$$= S^+_{2,k}(A(z_{1}, z_{2})) \cdot \frac{\text{Prob}[n_{1,l} = 0, n_{2,l} = i]}{\text{Prob}[n_{1,l} = 0, n_{2,l} > 0]}$$

$$= S^+_{2,k}(A(z_{1}, z_{2})) \cdot \frac{N_i(0, z_2) - N_i(0, 0)}{N_i(0, 1) - N_i(0, 0)}.$$  

And almost the same goes for the “service class 1 packet” event:

$$E\left[z_{1}^{n_{1,l}+s^+_k a_{1,k-i} a_{2,k-i}} | n_{1,l} > 0\right]$$

$$= E\left[\prod_{i=1}^{s^+_k} \left(\frac{z_{1}^{a_{1,k-i}} z_{2}^{a_{2,k-i}}}{z_{1}^i z_{2}^i}\right) \cdot z_{1}^{n_{1,l}} z_{2}^{n_{2,l}} | n_{1,l} > 0\right]$$

$$= E\left[\left(A(z_{1}, z_{2})\right)^{s^+_k} \right] \cdot E\left[z_{1}^{n_{1,l}} z_{2}^{n_{2,l}} | n_{1,l} > 0\right]$$

$$= S^+_{1,k}(A(z_{1}, z_{2})) \cdot \sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} z_{1}^i z_{2}^j \cdot \text{Prob}[n_{1,l} = i, n_{2,l} = j | n_{1,l} > 0]$$

$$(s^+_k \text{ is the elapsed service time of a class 1 packet here})$$

$$= S^+_{1,k}(A(z_{1}, z_{2})) \cdot \frac{\sum_{i=0}^{+\infty} \sum_{j=0}^{+\infty} z_{1}^i z_{2}^j \cdot \text{Prob}[n_{1,l} = i, n_{2,l} = j, n_{1,l} > 0]}{\text{Prob}[n_{1,l} > 0]}.$$
\[ S_{1,k}^{+}(A(z_1, z_2)) \cdot \frac{\sum_{i=1}^{\infty} \sum_{j=0}^{\infty} z_1^i z_2^j \cdot \text{Prob}[n_{1,l} = i, n_{2,l} = j]}{\text{Prob}[n_{1,l} > 0]} \]
\[ = S_{1,k}^{+}(A(z_1, z_2)) \cdot \frac{N_l(z_1, z_2) - N_l(0, z_2)}{1 - N_l(0, 1)}. \]

These two results, together with (8) give the last line of (9).

3 Equation (11)

It is clear from (9) that if we want to determine \( U(z_1, z_2) \), we need to know the value of \( U(0, 0) \), i.e. the probability of having an empty system at an arbitrary slot. For this, it is explained in the paper how we can use the (already known) steady-state distribution of \( n_{1,l} \) and \( n_{2,l} \), the system contents at arbitrary start slots:

\[ U(0, 0) = \lim_{k,l \to \infty} \text{Prob}[n_{1,l} = n_{2,l} = 0 \mid \text{slot } k \text{ is a start slot}] \cdot \text{Prob}[\text{slot } k \text{ is a start slot}]. \]

First, concerning the first factor, we see that the condition can be dropped:

\[ \lim_{k,l \to \infty} \text{Prob}[n_{1,l} = n_{2,l} = 0 \mid \text{slot } k \text{ is a start slot}] = \lim_{l \to \infty} \text{Prob}[n_{1,l} = n_{2,l} = 0] = N(0, 0) \]

Actually, this is not so straightforward as it may seem. To understand this, we can make the following consideration. If \( k \) is a start slot, then slot \( l \) is the same as slot \( k \). So, choosing an arbitrary slot \( k \) that is also a start slot, is actually the same as directly choosing an arbitrary start slot. Moreover, this way all start slots are taken into account only once. This means that the statistics of (a) taking an arbitrary slot that is also a start slot is the same as the statistics of (b) taking an arbitrary start slot.

Secondly, the probability \( \text{Prob}[\text{slot } k \text{ is a start slot}] \), can be obtained by distinguishing the start slots in the same way as we previously did for the arbitrary slots. For \( k \to \infty \):

\[ \text{Prob}[\text{slot } k \text{ is start slot}] = \text{Prob}[k \text{ is a start slot and idle}] + \text{Prob}[k \text{ is a start slot and serving a class 2 packet}] + \text{Prob}[k \text{ is a start slot and serving a class 1 packet}] \]

So again, we have three different cases:

1. If a slot is idle, it is certainly a start slot, therefore

\[ \text{Prob}[\text{slot } k \text{ is a start slot and idle}] = \text{Prob}[\text{slot } k \text{ is idle}] = U(0, 0). \]

2. The probability that an arbitrary class 2 busy slot is also a start slot is \( 1/\mu_2 \), because on the whole, each packet stays in the server for an average time of \( \mu_2 \) slots. Hence, there is a fraction of \( 1/\mu_2 \) start slots among the class 2 busy slots:

\[ \text{Prob}[k \text{ is a start slot and serving a class 2 packet}] = \text{Prob}[k \text{ is a start slot}] \cdot \text{Prob}[k \text{ is a class 2 busy slot}] \]
\[ = \frac{1}{\mu_2} \cdot (1 - U(0, 0)) \frac{\rho_2}{\rho}. \]

3. Likewise,

\[ \text{Prob}[k \text{ is a start slot and serving a class 1 packet}] = \frac{1}{\mu_1} \cdot (1 - U(0, 0)) \frac{\rho_1}{\rho}. \]

\[ = S_{1,k}^{+}(A(z_1, z_2)) \cdot \frac{\sum_{i=1}^{\infty} \sum_{j=0}^{\infty} z_1^i z_2^j \cdot \text{Prob}[n_{1,l} = i, n_{2,l} = j]}{\text{Prob}[n_{1,l} > 0]} \]
\[ = S_{1,k}^{+}(A(z_1, z_2)) \cdot \frac{N_l(z_1, z_2) - N_l(0, z_2)}{1 - N_l(0, 1)}. \]
Summing these, we have that

\[
\text{Prob}\left[\text{slot } k \text{ is start slot}\right] = U(0, 0) + \frac{1 - U(0, 0)}{\mu_2} \rho_2 + \frac{1 - U(0, 0)}{\mu_1} \rho_1
\]

This result then leads to the first line of (11). From which, we find

\[
U(0, 0) = \frac{N(0, 0) \left( \frac{\rho_1}{\mu_1 \rho} + \frac{\rho_2}{\mu_2 \rho} \right)}{1 - N(0, 0) \left( 1 - \frac{\rho_1}{\mu_1 \rho} - \frac{\rho_2}{\mu_2 \rho} \right)}
\]

Now fill in the quantity \(N(0, 0)\) given by (5). This finally gives

\[
U(0, 0) = 1 - \rho.
\]

4 Equation (12)

The steady-state version \((k, l \to \infty)\) of (9) is

\[
U(z_1, z_2) = U(0, 0) + (1 - U(0, 0)) \left[ \frac{\rho_2}{\rho} S_2^+ (A(z_1, z_2)) \frac{N(0, z_2) - N(0, 0)}{N(0, 1) - N(0, 0)} + \frac{\rho_1}{\rho} S_1^+ (A(z_1, z_2)) \frac{N(z_1, z_2) - N(0, z_2)}{1 - N(0, 1)} \right]
\]

All subexpressions herein can be obtained from the analysis so far. For ease of writing, I will sometimes use the following abbreviated notations \((j = 1, 2)\):

\[
\begin{align*}
A &\triangleq A(z_1, z_2) \\
A^Y &\triangleq A(Y(z_2), z_2) \\
S_j &\triangleq S_j(A(z_1, z_2)) \\
S_j^Y &\triangleq S_j(A(Y(z_2), z_2))
\end{align*}
\]

Step by step, we will now go through the necessary calculations to obtain (12) from the above expression for \(U(z_1, z_2)\):

- With the abbreviations given above, the expressions (6) and (3) for \(N(z_1, z_2)\) and \(N(0, z_2)\) respectively, become

\[
N(z_1, z_2) = N(0, 0) \frac{z_1 (z_2 A - S_2) + S_2^Y (S_1 - z_1 A) + A^Y (z_1 S_2 - z_2 S_1)}{(z_1 - S_1)(z_2 - S_2)},
\]

\[
N(0, z_2) = N(0, 0) \frac{z_2 A^Y - S_2^Y}{z_2 - S_2^Y}.
\]

- From this we find

\[
N(z_1, z_2) - N(0, z_2)
= N(0, 0) \frac{z_1 (z_2 A - S_2) + S_2^Y (S_1 - z_1 A) + A^Y (z_1 S_2 - z_2 S_1)}{(z_1 - S_1)(z_2 - S_2)} - N(0, 0) \frac{z_2 A^Y - S_2^Y}{z_2 - S_2^Y}
= N(0, 0) \frac{z_1 (z_2 A - S_2) + S_2^Y (S_1 - z_1 A) + A^Y (z_1 S_2 - z_2 S_1) - (z_2 A^Y - S_2^Y)(z_1 - S_1)}{(z_1 - S_1)(z_2 - S_2)}
\]

5
\[ N(0, z_2) - N(0, 0) = N(0, 0) \frac{z_2 A v - z_2}{z_2 - S_2'}. \]

- To get \( N(0, 1) \) from \( N(0, z_2) \), we need to apply de l’Hôpital’s rule (using the full notation here):

\[
N(0, 1) = \lim_{z_2 \to 1} N(0, z_2)
\]
\[
= \lim_{z_2 \to 1} N(0, 0) \frac{z_2 A(Y(z_2), z_2) - S_2(A(Y(z_2), z_2))}{z_2 - S_2(Y(z_2), z_2)}
\]
\[
= \lim_{z_2 \to 1} N(0, 0) \frac{A(Y(z_2), z_2) + z_2 a(z_2) - S_2(A(Y(z_2), z_2)) a(z_2)}{1 - S_2'(A(Y(z_2), z_2)) a(z_2)},
\]

with

\[
a(z_2) = \frac{d}{dz_2} A(Y(z_2), z_2) = \frac{\partial}{\partial z_1} A(Y(z_2), z_2) Y'(z_2) + \frac{\partial}{\partial z_2} A(Y(z_2), z_2),
\]

and, for \( z_2 \to 1 \),

\[
a(1) = \frac{\partial}{\partial z_1} A(1, 1) \cdot Y'(1) + \frac{\partial}{\partial z_2} A(1, 1)
\]
\[
A'_1(1) = \lambda_1 \quad A'_2(1) = \lambda_2
\]
\[
= A'_1(1) \cdot \frac{\lambda_2 \mu_1}{1 - \rho_1} + A'_2(1)
\]
\[
= \lambda_1 \cdot \frac{\lambda_2 \mu_1}{1 - \rho_1} + \lambda_2. \quad \text{(from (4))}
\]

So, with \( z_2 \to 1 \),

\[
N(0, 1) = N(0, 0) \frac{A(1, 1) + a(1) - S'_2(1) a(1)}{1 - S'_2(1) a(1)}
\]
\[
= N(0, 0) \frac{1 + a(1) - \mu_2 a(1)}{1 - \mu_2 a(1)} \quad \text{(Remember, } A(1, 1) = 1, S'_1(1) = \mu_1 \text{ and } S'_2(1) = \mu_2 \text{ )}
\]
\[
= N(0, 0) \frac{1 + (1 - \mu_2)(\lambda_1 \frac{\lambda_2 \mu_1}{1 - \rho_1} + \lambda_2)}{1 - \mu_2(\lambda_1 \frac{\lambda_2 \mu_1}{1 - \rho_1} + \lambda_2)}
\]
\[
= \ldots
\]
\[
= N(0, 0) \frac{1 + \lambda_2 - \rho}{1 - \rho}.
\]

- From the above, it also follows that

\[
N(0, 1) - N(0, 0) = N(0, 0) \left( \frac{1 + \lambda_2 - \rho}{1 - \rho} - 1 \right) = N(0, 0) \frac{\lambda_2}{1 - \rho},
\]

\[ \]
\[ 1 - N(0, 1) = 1 - N(0, 0) \frac{1 + \lambda_2 - \rho}{1 - \rho} = \ldots = \frac{\lambda_1}{1 - \rho + \lambda_1 + \lambda_2}. \]

- Finally, from (10), we have for \( j = 1, 2 \):

\[
S_j^+ (A(z_1, z_2)) = \frac{S_j(A(z_1, z_2)) - 1}{S_j^+(1)(A(z_1, z_2) - 1)} \quad \left( \text{for } N(0, 0) \text{ is given by (5)} \right).
\]

With the above results, \( U(z_1, z_2) \) becomes:

\[
U(z_1, z_2) = U(0, 0) + (1 - U(0, 0)) \left\{ \frac{1 - \rho}{\rho} \right\} \left[ \frac{S_2 - 1}{\mu_2 (A - 1)} \frac{1 - \rho + \lambda_1 + \lambda_2}{\lambda_1} \cdot N(0, 0) \frac{z_2 A^Y - S_2^Y}{z_2 - S_2^Y} \right.
\]

\[
+ \frac{1 - \rho}{\rho} S_1 - 1 \frac{1 - \rho + \lambda_1 + \lambda_2}{\lambda_1} \cdot N(0, 0) \frac{(A - 1) z_1 (z_2 - S_2^Y) + (A^Y - 1) (z_1 S_2 - z_2 S_1 - z_1 z_2 + z_2 S_1)}{(z_1 - S_1)(z_2 - S_2^Y)} \]

\[
= (1 - \rho) \left[ 1 + \frac{S_2 - 1}{\mu_2 (A - 1)} \frac{z_2 A^Y - S_2^Y}{z_2 - S_2^Y} \right.
\]

\[
+ \frac{S_1 - 1}{\mu_1 (A - 1)} \cdot \frac{(A - 1) z_1 (z_2 - S_2^Y) + (A^Y - 1) (z_1 S_2 - z_2 S_1 - z_1 z_2 + z_2 S_1)}{(z_1 - S_1)(z_2 - S_2^Y)} \]

\[
= \frac{1 - \rho}{(A - 1)(z_1 - S_1)(z_2 - S_2^Y)} \left[ (A - 1)(z_1 - S_1)(z_2 - S_2^Y) + (S_2 - 1)(z_2 A^Y - z_2)(z_1 - S_1) \right.
\]

\[
+ (S_1 - 1)(A - 1)(z_1 - S_1)(z_2 - S_2^Y) + (A^Y - 1)(z_1 S_2 - z_2 S_1 - z_1 z_2 + z_2 S_1) \]

\[
= \ldots
\]

\[
= (1 - \rho) \left[ \frac{S_1(z_1 - 1)}{z_1 - S_1} + \frac{A^Y - 1}{A - 1} \frac{z_1 z_2 (S_1 - S_2 + 1) + z_1 z_2 (S_2 - S_1) + z_2 S_1 (1 - S_2)}{(z_1 - S_1)(z_2 - S_2^Y)} \right].
\]

This is exactly equation (12).

Can I proceed deriving the system contents of the same system, assuming geometrically distributed service time, i.e. using the same approach in the paper "Analysis of packet delay in a GI-Geo-1 preemptive resume priority buffer" for the same authors? or will I have difficulty doing this?

5 Preemptive vs non preemptive server discipline

The analysis in this paper is valid for general service times, i.e. the pgf’s \( S_1(z) \) and \( S_2(z) \) are not further specified. In the paper “Analysis of a GI-Geo-1 preemptive resume priority buffer”, the
model is not the same as here. To begin with, the paper deals with geometrically distributed service times only. Another, and more important, difference is the fact that the server discipline is preemptive instead of non-preemptive, which will make the analysis slightly less complicated.

Even if you would assume geometric distributions for the service times in the non-preemptive case (i.e., in the model discussed above), the results for the system contents would still be different from those in the preemptive case. For instance, in case of the preemptive discipline, the behavior of the class 1 packets are not affected by that of the class 2 packets:

\[
U_1(z) = (1 - \rho_1) \frac{S_1(A_1(z))(z - 1)}{z - S_1(A_1(z))}
\]

Since under no circumstance the class 1 packets have to wait for service completion of a class 2 packet, the result for \(U_1(z)\) is exactly the same as for a simple GI-G-1 queue without classes. This is quite different in the non-preemptive case. Here, class 1 packets will sometimes have to wait when the server is busy with a class 2 packet. Even if a class 1 packet is available for service, it will not interrupt the server if it is busy with a class 2 packet (that is what “non-preemptive” means). Therefore, in this case, the system behavior of the class 1 packets is influenced by the behavior of the class 2 packets, whereas this was not so in the preemptive case. This complication leads to a more difficult analysis and a different result for \(U_1(z)\). Indeed, it can be seen that equation (13) still contains \(S_2(z)\) and \(A_2(z)\).

So, in general, you cannot use the results of the preemptive model to obtain those of the non-preemptive one. Not even for geometric service times.
Figure 1

1. "no service"

2. "service class 2 packet"

3. "service class 1 packet"
Figure 2

\[ q_{jk-3} \quad q_{jk-2} \quad q_{jk-1} \quad k \quad h+1 \quad \ldots \]

\[ S_{k}^{+} \]

Service time of a packet