On the effect of movement on the localization accuracy in practical wireless networks

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***Abstract*—The problem of localization involves estimating the position of a user from a number of noisy sensor measurements. In a practical wireless network, these sensor measurements cannot be collected instantaneously and arrive after a certain delay. In a dynamic scenario where the users move around, this delay will render some measurements out-dated and, if not taken into account, have a negative effect on the localization performance. This paper consists of two parts, in the first part we investigate the effect of user movement on the measurement models for ranging. In the second part we use these models to analyze the impact of movement on the accuracy of the position estimate by means of the Cramer-Rao lower bound (CRLB) which bounds the performance of the estimation.**

***Index Terms*—Localization, dynamic, movement, CRLB**

I. Introduction

During the past decade, a multitude of cooperative localization algorithms were presented and analyzed. It has been shown that cooperative localization greatly improves the localization accuracy [1]. However, many of these algorithms concentrate on static networks with no movement. For the few studies where movement is considered, an abstraction is made on how the range measurements are made and collected. However, for dynamic networks with movement of the users, the way the data is collected becomes an important part of the localization problem because the delays present in any practical network, may render part of the data outdated. For the Global Positioning System (GPS) this has never been a problem because measurements are always performed in parallel: the different satellites use different CDMA codes that do not interfere. The receiver knows the codes and has several parallel correlators to track all satellites simultaneously. This parallel approach can be applied in GPS as the satellites are strictly synchronized, such that the ranging is meaningful. Without synchronized anchors it is harder to simultaneously make measurements with all the different sensors, and therefore many current systems employ a sequential measurement scheme.

However, by sequentially measuring the ranges, delays in the network will start to play an important role. The delays originate from different sources; for example

- The radio channel is used for both communications and ranging, and delays due to network congestion can grow large.
- Measurements are deliberately delayed in order to reduce the power consumption of the transceiver or to reduce the network load.

Hence, a measurement can take up to several tens of milliseconds, such that the delays become prominent when measurements with multiple neighboring sensors are made. It is clear that such delays have a great impact on the performance of the localization, especially if a user is moving fast. For single-user (i.e. non-cooperative) tracking other than GPS, the problem of delayed or so-called out-of-sequence measurements (OOSM) has already been well studied, although no optimal solution for the problem has been found yet [2]. For example, in [3], the authors provide an overview of filters, which are adapted to cope with OOSM, and they compare the results with the case where the Kalman and Particle filters do not take into account the presence of OOSM.

For cooperative localization, however, the effect of delays on the localization performance has barely been examined. Yet, it can be expected that the performance degradation will be higher as a result of the cooperation between the users: cooperation typically involves more packets to be sent over the wireless network, and consequently it introduces larger delays. Some studies exist where the MAC delay is evaluated for cooperative localization networks using the IEEE 802.15.4 standard. The MAC delay is examined in [4], and two enhancements were proposed to reduce the delay for localization-specific scenarios. Further, in [5], some lower and upper bounds on the delay were given for cooperative networks. Their results show that for current off-the-shelf ranging equipment, the delays are of the order of seconds. In [6], the MAC delay was examined and the effect on localization was evaluated through simulations. However, in none of the above mentioned works, the effect on the localization accuracy was addressed. In this paper, we will focus on a theoretical analysis of the effect of movement and delays on the accuracy of cooperative localization.

II. Problem formulation

We consider a wireless network with N user nodes that are to be localized and N_{ref} anchor nodes with known positions. In 2D, the node coordinates at timestep k are denoted by column vectors $\mathbf{x}_i^{(k)} = [x_i^{(k)}, y_i^{(k)}]^{\mathrm{T}}$, where for users $i \in \{1, .., N\}$ and for anchors $i \in \{N+1, .., N+N_{\mathrm{ref}}\}$. Extension to 3D is straightforward. Every node i is connected to a number of neighboring nodes (both users and anchors) collected in the set $\mathcal{N}(i)$, with whom ranging can be performed.

In this paper we consider range-based cooperative localization with movement of the users. We concentrate on scenarios

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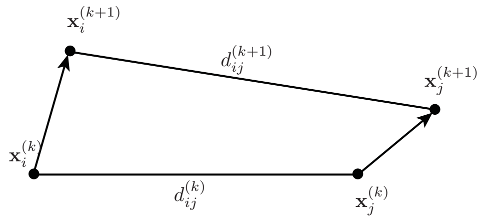

Figure 1. Effect of movement on the distance between two nodes.

with low movement speed and possibly high delays, and where an accurate model for the movement is hard to obtain due to the lack of additional sensors, such as an inertial measurement unit (IMU). These scenarios typically correspond to indoor scenarios. To model the movement of the user we employ the discrete time random walk model:

$$\mathbf{x}_i^{(k+1)} = \mathbf{x}_i^{(k)} + \mathbf{w}_i^{(k)}, \tag{1}$$

for $i \le N$ (i.e. only the users move not the anchors), with $\mathbf{w}_i^{(k)} = [w_{\mathrm{x},i}^{(k)},\, w_{\mathrm{y},i}^{(k)}]^{\mathrm{T}}$ the displacement during the time interval Δt between time steps k and $k+1$. The displacement is modeled as zero mean Gaussian noise: $\mathbf{w}_i \sim \mathcal{N}(\mathbf{0},\, \sigma_{\mathrm{m},i}^2 \mathbf{I}_2)$ with $\mathbf{I}_2$ the 2×2 identity matrix and $\sigma_{\mathrm{m},i}^2$ the variance of movement of the ith user. In the above discrete time model, the noise variance $\sigma_{\mathrm{m},i}^2$ can be related to a maximum[1] user speed $v_{\max}$ as follows: $\sigma_{\mathrm{m},i}^2 = (\Delta t \frac{v_{\max}}{3})^2$. A disadvantage[2] of the discrete time model is the fact that the model is defined for a specific time interval Δt between time steps k and $k+1$. To enforce consistency, the model requires rescaling for a different time interval $\Delta t'$ [7].

III. Effect on Range Measurements

For two-way time-of-arrival (TW-TOA), a distance estimate is obtained by measuring the round-trip delay of a packet. Errors due to movement are introduced at two levels. The first type of error is a result of the movement of the user during the round-trip of the message. For low movement speed, this error will be very small because the round-trip delay is only a fraction of a second. The second type of error is due to a delay in the processing of the measurement. As explained in the introduction, this delay can be large and will therefore be the dominant contribution to the error. Both effects can be modeled in a similar way. Let $d_{ij}^{(k)}$ be the distance between nodes i and j at timestep k:

$$d_{ij}^{(k)} = \sqrt{(x_i^{(k)} - x_j^{(k)})^2 + (y_i^{(k)} - y_j^{(k)})^2}, \tag{2}$$

After a time interval Δt, the nodes will have moved (see Fig. 1). The distance between nodes i and j at timestep $k+1$ can be written as

[1]The probability that the speed of the user is higher than $v_{\max}$ is less than 1% for the random walk model defined in (1).

[2]In a continuous time model there is no inconsistency for different time intervals but the noise variance cannot be related to a physical quantity such as the maximum speed. Because of this we do not employ a continuous model.

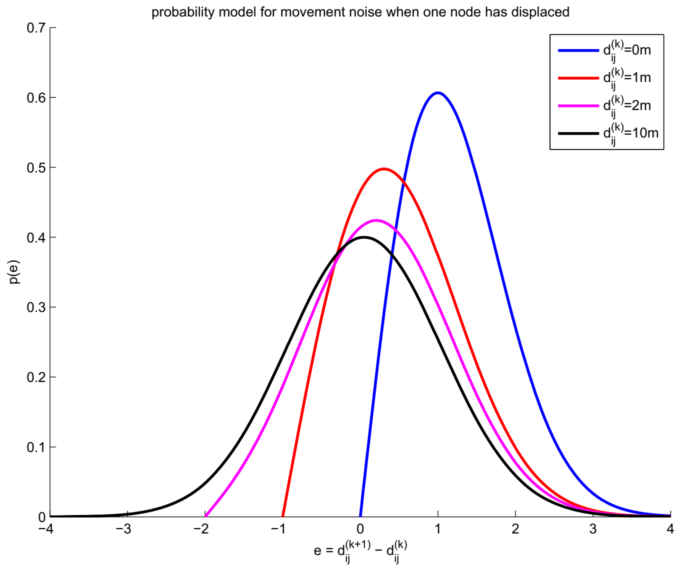

Figure 2. Error on ranging induced by movement.

$$d_{ij}^{(k+1)} = \sqrt{(x^{(k)} - x_j^{(k)} + \Delta x_{ij}^{(k)})^2 + (y^{(k)} - y_j^{(k)} + \Delta y_{ij}^{(k)})^2} \tag{3}$$

where $\Delta x_{ij}^{(k)} = w_{\mathrm{x},i}^{(k)} - w_{\mathrm{x},j}^{(k)}$ and $\Delta y_{ij}^{(k)} = w_{\mathrm{y},i}^{(k)} - w_{\mathrm{y},j}^{(k)}$ are independent random variables distributed as $\mathcal{N}(0,\, \sigma_{\mathrm{m},i}^2 + \sigma_{\mathrm{m},j}^2)$, and represent the displacement of the users from timestep k to $k+1$.

Hence, the new distance $d_i^{(k+1)}$ is a random variable with as distribution a non-central chi distribution with parameter $k = 2$ (which corresponds to a Rice distribution). The probability density function corresponding to the Rice distribution, denoted by $\mathcal{R}(\lambda,\, \sigma)$, is given as

$$p(x|\lambda,\, \sigma) = \frac{x}{\sigma^2} e^{-\frac{(x^2+\lambda^2)}{2\sigma^2}} I_0\left(\frac{\lambda x}{\sigma^2}\right), \tag{4}$$

where I_0 is the 0th order Modified Bessel function of the first kind. This results in the distribution

$$d_{ij}^{(k+1)} \sim \mathcal{R}(d_{ij}^{(k)},\, \sigma_{\mathrm{m},i}^2 + \sigma_{\mathrm{m},j}^2), \tag{5}$$

which is shown in Fig. 2 for different values for $d_{ij}^{(k)}$.

Using this exact formulation leads to an intractable expression for the CRLB. However, we will show that this distribution can be approximated by a Gaussian distribution if $\lambda \gg \sigma$, which represents the case where the movement is small as compared to the distances between the nodes[3]. To prove this we show that the Kullback-Leibler (KL) divergence between the Ricean distribution $p(x)$ and a Gaussian distribution $q(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(\frac{-(x-\lambda)^2}{2\sigma^2})$ goes to zero if $\lambda \gg \sigma$. The KL-divergence can be written as

[3]In general, this requirement will be fulfilled as the time interval that is considered is of the order of a fraction of a second, implying that the movements in this time interval will typically be very small.

$$D_{\mathrm{KL}}(Q\|P) = \mathbb{E}_{q(x)}\left[\ln\left(\frac{q(x)}{p(x)}\right)\right]$$
$$= \mathbb{E}_{q(x)}\left[\ln(\sigma) - \frac{1}{2}\ln(2\pi) - \ln\left(xI_0\left(\frac{\lambda}{\sigma^2}\right)\right) + \frac{\lambda x}{\sigma^2}\right]$$
$$= \ln(\sigma) - \frac{1}{2}\ln(2\pi) + \frac{\lambda^2}{\sigma^2} - \mathbb{E}\left[\ln(x)\right] - \mathbb{E}\left[\ln\left(I_0\left(\frac{\lambda x}{\sigma^2}\right)\right)\right] \tag{6}$$

The modified Bessel function can be approximated for large arguments by the following formula [8, eq. 9.6.7]:

$$I_0(z) \approx \frac{e^z}{\sqrt{2\pi z}} \tag{7}$$

With this approximation we can rewrite the last term in (6) as

$$\mathbb{E}\left[\ln\left(I_0\left(\frac{\lambda x}{\sigma^2}\right)\right)\right]$$
$$\approx \mathbb{E}\left[\frac{\lambda x}{\sigma^2}\right] - \frac{1}{2}\mathbb{E}\left[\ln(x)\right] - \frac{1}{2}\ln\left(2\pi\frac{\lambda}{\sigma^2}\right)$$
$$= \frac{\lambda^2}{\sigma^2} - \frac{1}{2}\mathbb{E}\left[\ln(x)\right] - \frac{1}{2}\ln\left(2\pi\frac{\lambda}{\sigma^2}\right) \tag{8}$$

Inserting (8) in (6) yields the following approximate expression for the KL-divergence when $\lambda \gg \sigma^2$:

$$D_{\mathrm{KL}}(P\|Q) \approx -\frac{1}{2}\mathbb{E}\left[\ln(\mathrm{x})\right] + \frac{1}{2}\ln(\lambda) \tag{9}$$

If the Gaussian distribution $q(x)$ is narrow, i.e., σ is small, we can consider $\ln(x)$ as a constant equal to $\ln(\lambda)$, which makes the KL divergence approximately equal to zero.

We can now use this result to model the error on a distance measurement $d_{ij}^{(k+1)}$:

$$d_{ij}^{(k+1)} \sim \mathcal{N}(d_{ij}^{(k)},\, \sigma^2_{\mathrm{m},i} + \sigma^2_{\mathrm{m},j}), \tag{10}$$

for $d_{ij}^{(k)} \gg \sigma^2_{\mathrm{m},i} + \sigma^2_{\mathrm{m},j}$. In other words, the random walk model (1) simply introduces an extra Gaussian error term in the delayed distance when the distance $d_{ij}^{(k)}$ is large.

IV. Effect on localization

Typically, the localization process can be divided in two steps: a measurement phase and a processing phase. In the measurement phase, range measurements are collected between all nodes in the network. In the processing phase the actual position estimate is computed. In a practical network, the ranging phase cannot collect all range measurements instantaneously, and dynamics of the users result in errors in the range measurements. It is clear that the ranging schedule will have an influence on the delays and consequently, on the errors of the range measurements. In this paper we consider communication and ranging over the network that employs a TDMA scheme following the IEEE 802.15.4 standard that is specifically designed for low rate communication and localization in Personal Area Networks (PAN). In the IEEE standard a slotted carrier sense multiple access - collision avoidance

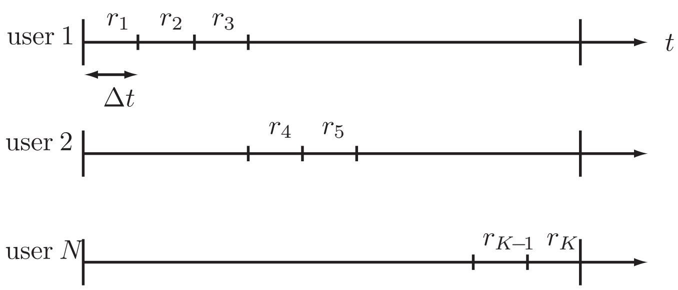

Figure 3. ranging schedule during the measurement phase.

(CSMA-CA) protocol is used to access the channel. In the ranging phase, the users will thus sequentially range with their neighbors.

A. Sequential ranging

In the sequential ranging schedule that we consider, each user makes range measurements with every neighbor, starting with user 1, then user 2, and so on. Let us consider that each range measurement takes an equal amount of time Δt, then at every timestep k, a noisy range measurement r_k is obtained between the nodes i_k and j_k.

$$r_k = d^{(k)}_{i_k,j_k} + v_k, \tag{11}$$

where the noise term v_k is modeled as AWGN with variance σ^2_{r}, and $k = 1..K$ with K the total number of measurements. All measurements are collected in a vector $\mathbf{r} = [r_1,\, r_2,\, \ldots,\, r_K]^{\mathrm{T}}$.

Because we are interested in estimating the user position at the final timestep $k = K$, the measurement need to be related to the distance at the time of this timestep. During the $K - k$ time intervals that the kth measurement is waiting to be processed, both nodes will have moved according to the model in (1), resulting in a combined displacement ϵ_k with respect to the timestep $k = K$:

$$\epsilon_k = \sum_{\ell=k}^{K} \epsilon^{(\ell)}_{i_k} + \sum_{\ell=k}^{K} \epsilon^{(\ell)}_{j_k} \tag{12}$$

where $\epsilon^{(\ell)}_i \sim \mathcal{N}(0,\, \sigma^2_{\mathrm{m},i})$.

Using (12) and the approximation (10), we can write the range measurement r_k with respect to the distance at timestep $k = K$:

$$r_k \approx d^{(K)}_{i_k,j_k} + \epsilon_k + v_k. \tag{13}$$

Because many measurements have common terms in ϵ_k, these measurements are highly correlated. The elements of the covariance matrix $\mathbf{\Sigma_r}$ of the the measurements $\mathbf{r}$ is given by:

$$[\mathbf{\Sigma_r}]_{m,n} = \mathrm{cov}\,(r_m,\, r_n)$$
$$= \mathbb{E}\left[\left(\sum_{\ell=1}^{m}(\epsilon^{(\ell)}_{i_m} + \epsilon^{(\ell)}_{j_m}) + v_m\right)\left(\sum_{\ell=1}^{n}(\epsilon^{(\ell)}_{i_n} + \epsilon^{(\ell)}_{j_n}) + v_n\right)\right]$$

$$= \max(m,n)\cdot\sigma^2_{\mathrm{m},i_m}\Big(\delta(i_m-i_n)+\delta(i_m-j_n)$$
$$+\max(m,n)\cdot\sigma^2_{\mathrm{m},j_m}\Big(\delta(j_m-j_n)+\delta(j_m-i_n)\Big)$$
$$+\delta(m-n)\sigma^2_{\mathrm{r}} \qquad (14)$$

For non-cooperative localization, this covariance matrix has the following peculiar form:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_K^2+\sigma_\mathrm{r}^2 & \cdots & \sigma_K^2 & \sigma_K^2 & \sigma_K^2 \\ \vdots & \ddots & & \cdots & \\ \sigma_K^2 & & \sigma_3^2+\sigma_\mathrm{r}^2 & \sigma_3^2 & \sigma_3^2 \\ \sigma_K^2 & \vdots & \sigma_3^2 & \sigma_2^2+\sigma_\mathrm{r}^2 & \sigma_2^2 \\ \sigma_K^2 & & \sigma_3^2 & \sigma_2^2 & \sigma_1^2+\sigma_\mathrm{r}^2 \end{bmatrix}, \qquad (15)$$

with $\sigma_k^2 = k\sigma_\mathrm{m}^2$.

B. Performance bound

To evaluate the effect of the delays we will calculate the CRLB which gives a lower bound on the mean square error of the position estimate[4]. Consider the estimation of all user coordinates, aggregated in a vector $\boldsymbol{\theta} = [\mathbf{x}_1^\mathrm{T}, \mathbf{x}_2^\mathrm{T}, ..., \mathbf{x}_N^\mathrm{T}]^\mathrm{T}$. It is well-known [9] that the covariance matrix of the unbiased estimate $\hat{\boldsymbol{\theta}}$ is bounded by the CRLB

$$\mathbb{E}_{\hat{\boldsymbol{\theta}}}\Big[(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})^\mathrm{T}\Big] \succeq \mathbf{F}^{-1}, \qquad (16)$$

where $\mathbb{E}_{\hat{\boldsymbol{\theta}}}[\,\cdot\,]$ denotes the expectation over the distribution of $\hat{\boldsymbol{\theta}}$, the symbol $\succeq$ denotes positive-semidefinite inequality and $\mathbf{F}^{-1}$ is the inverse of the Fisher information matrix. Consequently, the mean squared error (MSE) of the position estimate is bounded below by $\mathrm{trace}\left(\mathbf{F}^{-1}\right)$.

Given the relation between parameters and measurements in (13), we can write the Fisher information matrix $\mathbf{F}$ as [9]:

$$\mathbf{F} = \mathbf{G}^\mathrm{T}\mathbf{F_r}\mathbf{G}, \qquad (17)$$

with $\mathbf{G}$ the $K\times 2N$ Jacobian of the transformation $r_k(\boldsymbol{\theta}) = \|\mathbf{x}_{i_k}-\mathbf{x}_{j_k}\|$ and $\mathbf{F_r}$ the Fisher information of the measurements $\mathbf{r}$. It is well-known that the Fisher information matrix for normally distributed inputs is equal to the inverse of its covariance matrix. Hence we can write the $K\times K$ matrix $\mathbf{F_r}$ as: $\mathbf{F_r} = \boldsymbol{\Sigma}_\mathbf{r}^{-1}$. The Jacobian $\mathbf{G}$ is given by:

$$\mathbf{G} = \frac{\partial\mathbf{r}}{\partial\boldsymbol{\theta}^\mathrm{T}} = \begin{bmatrix} \frac{\partial r_1}{\partial x_1} & \frac{\partial r_1}{\partial y_1} & \cdots & \frac{\partial r_1}{\partial x_N} & \frac{\partial r_1}{\partial y_N} \\ \frac{\partial r_2}{\partial x_1} & \frac{\partial r_2}{\partial y_1} & \cdots & \frac{\partial r_2}{\partial x_N} & \frac{\partial r_2}{\partial y_N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\partial r_K}{\partial x_1} & \frac{\partial r_K}{\partial y_1} & \cdots & \frac{\partial r_K}{\partial x_N} & \frac{\partial r_K}{\partial y_N} \end{bmatrix} \qquad (18)$$

[4]It should be noted that for the true measurement models given in the previous section the CRLB cannot be calculated because the regularity conditions are not satisfied. However, by using the approximate models it is possible to calculate the CRLB.

Defining $\mathbf{u}_k \triangleq \frac{\mathbf{x}_{i_k}-\mathbf{x}_{j_k}}{\|\mathbf{x}_{i_k}-\mathbf{x}_{j_k}\|}$, the kth row of $\mathbf{G}$ is given by

$$\frac{\partial r_k}{\partial\boldsymbol{\theta}^\mathrm{T}} = \left[\mathbf{0}_{1\times 2(i_k-1)},\, \mathbf{u}_k^\mathrm{T},\, \mathbf{0}_{1\times 2(j_k-i_k-1)},\, -\mathbf{u}_k^\mathrm{T},\, \mathbf{0}_{1\times 2(N-j_k)}\right] \qquad (19)$$

whenever the corresponding range measurement r_k is between two users ($i_k,\, j_k \le N$), and

$$\frac{\partial r_k}{\partial\boldsymbol{\theta}^\mathrm{T}} = \left[\mathbf{0}_{1\times 2(i_k-1)},\, \mathbf{u}_k^\mathrm{T},\, \mathbf{0}_{1\times 2(N-i_k)}\right], \qquad (20)$$

when the corresponding range measurement r_k is between a user and an anchor ($i_k \le N$ and $j_k > N$).

Notice that equations (17), (18), and (14) correspond to the normal CRLB as in [10], when there is no delay ($\Delta t = 0$) or no movement ($\sigma_{\mathrm{m,i}} = 0,\ \forall i$).

V. Simulation results

We base our simulations on the specifications of the commercially available range modules from Timedomain Inc. [11]. From the datasheet, it follows that the radio range of the devices can be chosen from 35m to 350m (outdoor), and the corresponding measurement delay varies between 6.5ms to 132ms, respectively[5]. From [4], [5], we know that this delay increases as a consequence of the MAC protocol. Therefore, we consider delays in our simulations that vary between $\Delta t = 0$ms and $\Delta t = 200$ms. We will assume that all range measurements take an equal amount of time Δt. For the noise on the range measurements we use a standard deviation of σ_r =0.05cm, in accordance with the available hardware.

A. Impact on non-cooperative localization

We first evaluated the effect of delays on localization in a network with a single user and multiple anchors, spread out evenly in a circle around the user (optimal anchor placement). We assumed the user makes range measurements with the anchors in a random order. The maximum user speed as defined in Section II is set to $v_\mathrm{max} = 0.0014$m/s (or 5km/h).

From Fig. 4 it follows that, when there is no delay, we obtain the classical result that the lower bound on the root mean squared error (RMSE) of the position estimate decreases as the number of anchors increases. However, when there is a delay in the reception of the range measurements, the addition of anchors does not necessarily decrease the error on the position estimate. More specifically, for a delay larger than 100ms, it becomes counter-productive to use more than 4 anchors. We can conclude that when a user is moving and delays in the network are large, using more anchors than strictly necessary has an adverse effect on the localization accuracy because the range measurements between the user and the anchors become outdated. However, for other reasons such as robustness against non line-of-sight (which is not considered in this paper), it may still be advisable to use more anchors than necessary.

[5]The increasing delay results from a longer transmitted sequence required for an increased radio range. The distance itself only has a negligible effect on the delay resulting from the high speed of light

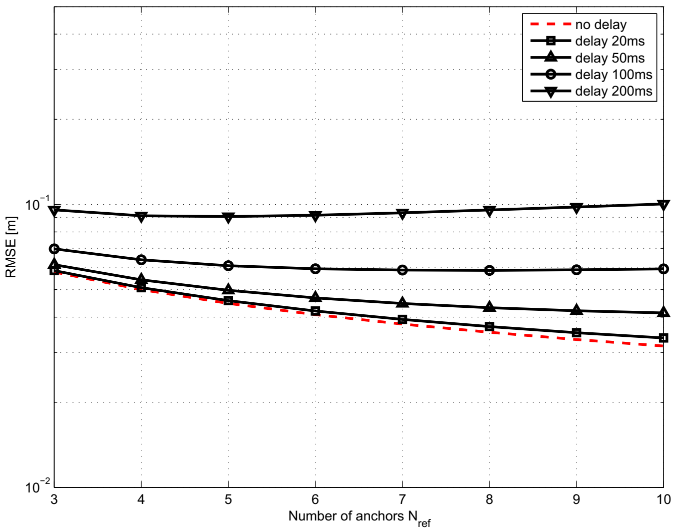

Figure 4. Effect of movement (max 5km/h) on the accuracy of non-cooperative localization, given for an increasing number of anchors.

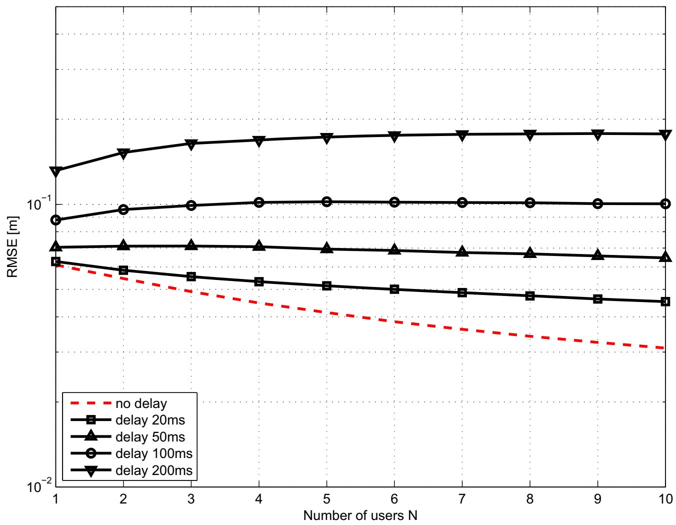

Figure 5. Effect of movement (max 5km/h) on the accuracy of cooperative localization, given for an increasing number of users.

B. *Impact on cooperative localization*

For the simulations on cooperative localization, we increased the number of mobile users in a 20m×20m area from $N = 1$ to $N = 10$. We placed $N_{\text{ref}} = 4$ anchors in the corners of the considered area and distributed all users randomly. We assumed that all nodes are within radio-range of each other within this area and have the same maximum movement speed $v_{\max,i} = 0.0014$m/s for $\forall i$. To average out the effect of the user position, 1000 Monte-Carlo trials were performed for each different parameter. The lower bound on the RMSE, provided by the CRLB, is shown in Fig. 5. Note that the case with $N = 1$ corresponds to the non-cooperative scenario. For an increasing number of users, we observe the classical result that cooperation increases the estimation accuracy when no delays are present. From the same figure, however, it is apparent that when delays are present the beneficial effect of cooperation reduces up to a point where cooperation actually degrades performance ($\Delta t \geq 100$ms). It must be noted that, in order to reach the CRLB, the correlations between measurements should be taken into account. If correlations are neglected, worse performance can be expected.

VI. Conclusions

Intuitively, it is clear that when a mobile user is moving, the delays of the different range measurements can have a significant impact on the localization accuracy. In this paper, we make a theoretical analysis of this effect. First we analyzed the error model of the range measurements under this setting and showed that, if we consider a random walk of the user, the error due to movement can be modeled as Gaussian. Using this result, we computed the Cramer-Rao Lower bound using delayed measurements for cooperative localization. From this analysis we conclude that the delays degrade the performance and that increasing the number of anchors or users in the network can adversely affect the localization accuracy.

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