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A STOCHASTIC ASSEMBLY SYSTEM WITH RESUME LEVELS

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In small-lot, multi-product and multi-level assembly systems, kitting (accumulating components required for assembly) plays a crucial role in determining system performance, especially when the system operates under a stochastic environment. This paper analyzes the kitting process of a stochastic assembly system, treating it as an assembly-like queue.

Two types of components which are required to complete a kit, independently arrive at the buffers according to Poisson processes. Furthermore, the arrival flow is shut down when the queue size attains to the buffer size until it decreases to a specified level, namely resume level. Using a Markov renewal approach, we derive the exact distribution of the kit completion time interval. The distribution of the sojourn time in the buffer of each component and the loss probability of components are also obtained. Finally, we show the merit of resume levels via numerical examples.

Keywords. Kitting process, assembly-like queue, resume level, kit completion time interval, sojourn time, loss probability.

1. Introduction

The preparation of the set of components to be assembled in the next stage is indispensable in most manufacturing systems. At an assembly point on a production line, components coming from various sources are assembled to make a product. Such queueing system in which service can be rendered only to groups of customers – one from each source – has been studied by several authors as *assembly-like queues*, for example, Bhat (1986), Harrison (1973), Hopp and Simon (1989), Latouche (1981), and Lipper and Sengupta (1986).

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In contrast with the conventional deterministic component flow, the realistic flexible manufacturing system under the stochastic environment has been studied in various aspects recently. In most cases, an object analyzed was the whole assembly system. That is, with terminology by standard queueing theoretic usage, the authors considered component to be assembled as a customer, and assembling group made up of each type of components as a service. In this traditional model, before the server can inaugurate the assembly, he has to wait until the arrivals of all requisite components occur. Thus, the time required to complete the assembly of a set of components involves not only the actual assembling time, but also the time that the server awaits the group of components.

We, therefore, may consider that the conventional stochastic assembly system consists of two processes: "kitting process" and "assembly process". In the kitting process, arriving component is deferred at the buffer until there arrive the necessary components of other types. Then, they are put together to complete a set of components, which we will refer to as a "kit". As soon as a kit is accomplished, it is transmitted to the next stage, assembly process, to be fitted together. Note that the first stage of the assembly system, namely the kitting process, is provided for accumulating the essential components, while the actual assembly is carried out at the assembly process.

Our engrossment concerns with the stochastic behaviour of the kitting process here. We define the kitting process as the process for accumulating components required for assembly system. Actually, this kind of process is formerly treated as a *double-ended queue* problem or *taxi-cab* problem, which has been analyzed by several authors, see Srivastava and Kashyap (1982).

We consider the kitting process which requires two types of components to complete a kit. The arrival of each type of component independently follows Poisson process. Som *et al.* (1994) gave the distribution of kit completion time interval. In that model, however, they did not assume any buffer management scheme. This paper presents a modified kitting process which adopts resume levels on both of the buffers in order to improve the system performance. The arrival flow is shut down once the number of components hits the maximum buffer size until it decreases to a certain value, called the resume level, after which the arrival flow is restored. This kind of arrival flow control can significantly reduce the waiting time in buffer while keeping unchanged or slightly increasing the loss probability.

The models with a resume level have been studied in several papers. For example, Takagi (1985, 1993) analyzed the finite capacity $M/G/1$ queueing system with resume level. Rosenberg *et al.* (1990) discussed the queueing systems with randomly changing arrival rates. To further improve the system performance, a resume level was introduced in the case of finite buffer. Moreover, as a congestion control scheme in communication network, a resume level was added to reduce the time delay as well as multiplexer memory requirements in the packet voice system in the work of Yin *et al.* (1990). It was shown that the packet delay time can be remarkably improved without incurring additional packet loss.

In this paper, we obtain some performance measures of the modified kitting process: the exact distribution of the kit completion time interval, the distribution of the sojourn time in the buffer of each component, and the loss probability of component. Eventually, we show some numerical results.

2. The Model Description

We consider the kitting process which appears in the assembly system. Figure 1 shows the feature of our system. $\mathbf{I}_1$ and $\mathbf{I}_2$ are the buffers for components required to complete a kit, $\mathbf{I}_0$ is the buffer for kits and $\mathbf{M}_0$ is the assembly machine. Two types of components independently arrive at buffers $\mathbf{I}_1$ and $\mathbf{I}_2$ according to Poisson processes with parameters λ_1 and λ_2, respectively. A component arriving at buffer $\mathbf{I}_1$ ($\mathbf{I}_2$) is immediately kitted with one of parts at buffer $\mathbf{I}_2$ ($\mathbf{I}_1$) if it is available. We then refer that a "kit" is composed. If a kit cannot be composed, the processed component is held in buffer $\mathbf{I}_1$ ($\mathbf{I}_2$) and waits for the arrival of "matching" part at buffer $\mathbf{I}_2$ ($\mathbf{I}_1$). Therefore, at any instant, the inventory position is zero for either buffer $\mathbf{I}_1$ or $\mathbf{I}_2$. Once it is composed, a kit is immediately sent to buffer $\mathbf{I}_0$ and waits for receiving the service of $\mathbf{M}_0$. In addition, the followings are assumed.

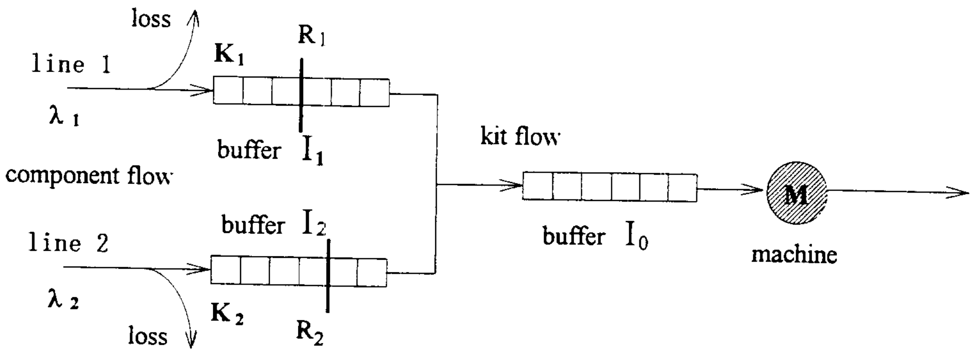

Figure 1. Stochastic assembly system

(A1) The size of buffer $\mathbf{I}_k$ is K_k $(< \infty)$, $k = 1, 2$. Components arriving at buffer $\mathbf{I}_k$ are certainly lost if the buffer is full. On the other hand, buffer $\mathbf{I}_0$ has an infinite waiting station.

(A2) If the queue size attains to the buffer size at buffer $\mathbf{I}_k$, the arrival flow on that line will be shut-down until it decreases to $R_k, 0 \le R_k \le K_k - 1$. Arrivals to buffer $\mathbf{I}_k$ during this shut-down period will be lost. We call R_k as the resume level at buffer $\mathbf{I}_k$.

Som *et al.* (1994) studied the similar kitting process as above, but their model has no resume level. In the case that resume levels R_k are equal to $K_k - 1$, our model coincides with their model. The main purpose of this paper is to show the influence of the resume level upon system performance. We, therefore, consider the stream of arrivals to $\mathbf{I}_0$, i.e. the output of the kitting process, and the sojourn time in the buffer of each components.

To analyze our process, we introduce the following notations. Let $J(t)$ be the state of the buffer management at time t, that is,

$$J(t) = \begin{cases} 0 & \text{if both lines are active (not shut-down),} \\ 1 & \text{if the line 1 is shut-down,} \\ 2 & \text{if the line 2 is shut-down.} \end{cases}$$

In addition, we let $Z(t)$ be the difference between inventory positions of buffers $\mathbf{I}_1$ and $\mathbf{I}_2$ at time t. Since either buffer $\mathbf{I}_1$ or $\mathbf{I}_2$ is empty at any instant, then we can describe the inventory positions of buffers $\mathbf{I}_1$ and $\mathbf{I}_2$ by one-dimensional random variable $Z(t)$. If $Z(t) > (<) 0$, there are no components at buffer $\mathbf{I}_2$ $(\mathbf{I}_1)$, and therefore components at buffer $\mathbf{I}_1$ $(\mathbf{I}_2)$ wait for arrivals of paired components to the buffer $\mathbf{I}_2$ $(\mathbf{I}_1)$. If $Z(t) = 0$, both buffers are empty. Therefore, we can describe the state of the system as the process $(\boldsymbol{J}, \boldsymbol{Z}) = \{(J(t), Z(t))\}$ with the state space $\mathcal{K} = \bigcup_{k=0}^{2} \mathcal{K}_k$ where

$$\begin{aligned} \mathcal{K}_0 &= \{0\} \times \{-K_2 + 1, \dots, -1, 0, 1, \dots, K_1 - 1\}, \\ \mathcal{K}_1 &= \{1\} \times \{R_1 + 1, \dots, K_1 - 1, K_1\}, \\ \mathcal{K}_2 &= \{2\} \times \{-K_2, -K_2 + 1, \dots, -R_2 - 1\}. \end{aligned}$$

Furthermore, for this process, we consider an imbedded process by choosing imbedded epoch τ_n which denotes the composed time of the n-th kit. Let us define $J_n = J(\tau_n^+)$ and $Z_n = Z(\tau_n^+)$ where τ_n^+ means the time just after τ_n. We should note that (J_n, Z_n) cannot take values $(0, -K_2 + 1)$, $(0, K_1 - 1)$, $(1, K_1)$ and $(2, -K_2)$. Moreover, let T_n be the time interval between consecutive kit completion epochs τ_n and τ_{n+1}, i.e., $T_n = \tau_{n+1} - \tau_n$. We call it the kit completion time interval. This interval is considered the interarrival time to the next step in the assembly line system. To obtain the distribution of T_n, we are interested in the behaviour of the output process $(\boldsymbol{J},\boldsymbol{Z},\boldsymbol{T}) = \{(J_n, Z_n, T_n) : n \in \mathrm{N}\}$ with the state space $\mathcal{K}' \times \mathcal{R}$ where $\mathbf{N}$ is the set of all nonnegative integers, $\mathcal{K}' = \mathcal{K} - \{(0, -K_2+1),\ (0, K_1-1),\ (1, K_1),\ (2, -K_2)\}$ and $\mathcal{R} = (0, \infty)$.

For the rest of the paper, we concentrate on the output process $(\boldsymbol{J},\boldsymbol{Z},\boldsymbol{T})$ and derive the distribution of the kit completion interval. Furthermore, we consider the process $(\mathcal{J}, \mathcal{Z})$ and obtain the sojourn time in the buffer of each component and the loss probability of components.

3. The Output Process

In this section, we consider the steady state distribution of the output process $(\boldsymbol{J},\boldsymbol{Z},\boldsymbol{T})$ by using a Markov renewal approach. Further, we derive the distribution of the kit completion interval.

3.1 The steady state distribution

The semi-Markov kernel $Q(\boldsymbol{i},\boldsymbol{j},t)$ for the output process $(\boldsymbol{J},\boldsymbol{Z},\boldsymbol{T})$ is defined by

$$Q(\boldsymbol{i},\boldsymbol{j},t) = \Pr\{(J_{n+1}, Z_{n+1}) = \boldsymbol{j},\ \ T_n \le t \mid (J_n, Z_n) = \boldsymbol{i}\}, \qquad (1)$$

for $\boldsymbol{i} = (i_1, i_2),\ \boldsymbol{j} = (j_1, j_2) \in \mathcal{K}'$ and $t \in \mathcal{R}$. Considering all possible cases and noticing that in case of $Z_n > 0\ (<0)$ the $\mathbf{I}_2$ $(\mathbf{I}_1)$ is empty, these kernels are obtained as follows:

C.1 for $i_1 = 1, R_1 + 1 < i_2 \le K_1 - 1,\ j_1 = 1$ and $j_2 = i_2 - 1$,
or for $i_1 = 1, i_2 = R_1 + 1,\ j_1 = 0$ and $j_2 = R_1$,

$$Q(\boldsymbol{i},\boldsymbol{j},t) = \int_0^t e^{-\lambda_2 u}\lambda_2 du,$$

C.2 for $i_1 = 0, 1 \le i_2 < K_1 - 1,\ j_1 = 0$ and $i_2 - 1 \le j_2 < K_1 - 1$,
or for $i_1 = i_2 = j_1 = 0$ and $1 \le j_2 < K_1 - 1$,

$$Q(\boldsymbol{i},\boldsymbol{j},t) = \int_0^t \frac{(\lambda_1 u)^{j_2 - i_2 + 1}}{(j_2 - i_2 + 1)!}\lambda_2 e^{-(\lambda_1+\lambda_2)u} du,$$

C.3 for $i_1 = 0, 0 \le i_2 < K_1 - 1,\ j_1 = 1$ and $j_2 = K_1 - 1$,

$$Q(\boldsymbol{i},\boldsymbol{j},t) = \int_0^t \sum_{k=K_1-i_2}^{\infty} \frac{(\lambda_1 u)^k}{k!}\lambda_2 e^{-(\lambda_1+\lambda_2)u} du,$$

C.4 for $i_1 = i_2 = 0$ and $j_1 = j_2 = 0$,

$$Q(\boldsymbol{i},\boldsymbol{j},t) = \int_0^t 2\lambda_1\lambda_2 u\ e^{-(\lambda_1+\lambda_2)u} du,$$

C.5 for $i_1 = 2, -K_2 + 1 \leq i_2 < -R_2 - 1,\ j_1 = 2$ and $j_2 = i_2 + 1$,
or for $i_1 = 2, i_2 = -R_2 - 1,\ j_1 = 0$ and $j_2 = -R_2$,

$$Q(\boldsymbol{i}, \boldsymbol{j}, t) = \int_0^t e^{-\lambda_1 u} \lambda_1 du,$$

C.6 for $i_1 = 0, -K_2 + 1 < i_2 \leq -1,\ j_1 = 0$ and $-K_2 + 1 < j_2 \leq i_2 + 1$,
or for $i_1 = i_2 = j_1 = 0$ and $-K_2 + 1 < j_2 \leq -1$,

$$Q(\boldsymbol{i}, \boldsymbol{j}, t) = \int_0^t \frac{(\lambda_2 u)^{-j_2 + i_2 + 1}}{(-j_2 + i_2 + 1)!} \lambda_1 e^{-(\lambda_1 + \lambda_2) u} du,$$

C.7 for $i_1 = 0, -K_2 + 1 < i_2 \leq 0,\ j_1 = 2$ and $j_2 = -K_2 + 1$,

$$Q(\boldsymbol{i}, \boldsymbol{j}, t) = \int_0^t \sum_{k = K_2 + i_2}^{\infty} \frac{(\lambda_2 u)^k}{k!} \lambda_1 e^{-(\lambda_1 + \lambda_2) u} du,$$

C.8 for all other $(\boldsymbol{i}, \boldsymbol{j}, t)$,

$$Q(\boldsymbol{i}, \boldsymbol{j}, t) = 0.$$

Using expressions given above, we study the steady state probabilities of the process $(\boldsymbol{J}, \boldsymbol{Z}) = \{(J_n, Z_n)\}$ and have the following theorem.

Theorem 1. Let us denote the steady distributions as $\{\pi(\boldsymbol{i}); \boldsymbol{i} \in \mathcal{K}'\}$ for the process $(\boldsymbol{J}, \boldsymbol{Z})$, then it is given as follows.

Case A. For $\rho \neq 1$, the steady state probabilities are given by

$$\begin{aligned}
\pi(0, i) &= C_\alpha \frac{1 - \rho^{1+i-K_1}}{\rho^{-R_1} - \rho^{-K_1}}\ , && R_1 + 1 \leq i \leq K_1 - 2, \\
\pi(0, i) &= C_\alpha \rho^{i+1}\ , && 1 \leq i \leq R_1, \\
\pi(0, 0) &= C_\alpha (1 + \rho)\ , && \\
\pi(0, i) &= C_\alpha \rho^i\ , && -R_2 \leq i \leq -1, \\
\pi(0, i) &= C_\alpha \frac{\rho^{K_2+i} - \rho}{\rho^{K_2} - \rho^{R_2}}\ , && -(K_2 - 2) \leq i \leq -(R_2 + 1), \\
\pi(1, i) &= C_\alpha \frac{\rho - 1}{\rho^{-R_1} - \rho^{-K_1}}\ , && R_1 + 1 \leq i \leq K_1 - 1, \\
\pi(2, i) &= C_\alpha \frac{\rho - 1}{\rho^{K_2} - \rho^{R_2}}\ , && -(K_2 - 1) \leq i \leq -(R_2 + 1),
\end{aligned}$$

where $\rho = \lambda_1/\lambda_2$ and

$$C_\alpha = \frac{(\rho^{-R_1} - \rho^{-K_1})(\rho^{K_2} - \rho^{R_2})}{(K_1 - R_1)(\rho^{K_2+1} - \rho^{R_2+1}) - (K_2 - R_2)(\rho^{-R_1} - \rho^{-K_1})} .$$

Case B. For $\rho = 1$, the steady state probabilities are given by

$$\begin{aligned}
\pi(0,i) &= C'_\alpha \frac{K_1 - 1 - i}{K_1 - R_1} , && R_1 + 1 \le i \le K_1 - 2, \\
\pi(0,i) &= C'_\alpha , && -R_2 \le i \le -1,\ 1 \le i \le R_1, \\
\pi(0,i) &= C'_\alpha \frac{K_2 - 1 + i}{K_2 - R_2} , && -(K_2 - 2) \le i \le -(R_2 + 1), \\
\pi(0,0) &= 2C'_\alpha , \\
\pi(1,i) &= \frac{C'_\alpha}{K_1 - R_1} , && R_1 + 1 \le i \le K_1 - 1, \\
\pi(2,i) &= \frac{C'_\alpha}{K_2 - R_2} , && -(K_2 - 1) \le i \le -(R_2 + 1),
\end{aligned}$$

where

$$C'_\alpha = \frac{2}{K_1 + K_2 + R_1 + R_2 + 2} .$$

Proof. The direct argument goes as follows. The Markov chain $(\boldsymbol{J}, \boldsymbol{Z})$ has the transition probabilities defined by

$$\begin{aligned}
Q(\boldsymbol{i}, \boldsymbol{j}) &= \Pr\{(J_{n+1}, Z_{n+1}) = \boldsymbol{j} \mid (J_n, Z_n) = \boldsymbol{i}\} \\
&= \lim_{t \to \infty} Q(\boldsymbol{i}, \boldsymbol{j}, t), \qquad \boldsymbol{i}, \boldsymbol{j} \in \mathcal{K}'.
\end{aligned}$$

After some calculations for $Q(\boldsymbol{i}, \boldsymbol{j}, t)$, these are given by

$$Q(\boldsymbol{i}, \boldsymbol{j}) = \begin{cases}
1, & \text{C.1 and C.5}, \\
(1-\nu)\nu^{j_2 - i_2 + 1}, & \text{C.2}, \\
\nu^{K_1 - i_2}, & \text{C.3}, \\
2\nu(1-\nu), & \text{C.4}, \\
\nu(1-\nu)^{-j_2 + i_2 + 1}, & \text{C.6}, \\
(1-\nu)^{K_2 + i_2}, & \text{C.7}, \\
0, & \text{otherwise},
\end{cases}$$

where the domains correspond to cases C.1 - 8 for $Q(\boldsymbol{i},\boldsymbol{j},t)$ and $\nu = \lambda_1/(\lambda_1+\lambda_2)$. Using these transition probabilities, the steady state distribution $\{\pi(\boldsymbol{i});\ \boldsymbol{i}\in\mathcal{K}'\}$ satisfies the balance equation and normalizing condition:

$$\pi(\boldsymbol{j}) = \sum_{\boldsymbol{i}\in\mathcal{K}'} \pi(\boldsymbol{i})Q(\boldsymbol{i},\boldsymbol{j}), \quad \sum_{\boldsymbol{i}\in\mathcal{K}'} \pi(\boldsymbol{i}) = 1.$$

Therefore, we derive the following set of equations:

$$\begin{aligned}
\pi(0,j) &= \sum_{i=0}^{j+1} \pi(0,i)(1-\nu)\nu^{j+1-i}, \quad 1\le j\le K_1-3 \text{ and } j\ne R_1,\\
\pi(0,-j) &= \sum_{i=0}^{j+1} \pi(0,-i)\nu(1-\nu)^{j+1-i}, \quad 1\le j\le K_2-3 \text{ and } j\ne R_2,\\
\pi(0,R_1) &= \pi(1,R_1+1) + \sum_{i=0}^{R_1+1} \pi(0,i)(1-\nu)\nu^{R_1+1-i},\\
\pi(0,-R_2) &= \pi(2,-(R_2+1)) + \sum_{i=0}^{R_2+1} \pi(0,-i)\nu(1-\nu)^{R_2+1-i},\\
\pi(0,0) &= \nu\pi(0,-1) + 2\nu(1-\nu)\pi(0,0) + (1-\nu)\pi(0,1),\\
\pi(0,K_1-2) &= \sum_{i=0}^{K_1-2} \pi(0,i)(1-\nu)\nu^{K_1-1-i},\\
\pi(0,-(K_2-2)) &= \sum_{i=0}^{K_2-2} \pi(0,-i)\nu(1-\nu)^{K_2-1-i},\\
\pi(1,K_1-1) &= \sum_{i=0}^{K_1-2} \pi(0,i)\nu^{K_1-i},\\
\pi(2,-(K_2-1)) &= \sum_{i=0}^{K_2-2} \pi(0,-i)(1-\nu)^{K_2-i}.
\end{aligned}$$

$$\pi(1,K_1-1) = \pi(1,K_1-2) = \cdots = \pi(1,R_1+1),$$

$$\pi(2,-(K_2-1)) = \pi(2,-(K_2-2)) = \cdots = \pi(2,-(R_2+1)).$$

To solve equations above, we have to consider two cases, i.e., $\rho\ne 1$ and $\rho = 1$, and get steady state probabilities $\pi(\boldsymbol{i})$ in the theorem. □

Let Π_0 be the probability that both buffers $\mathbf{I}_1$ and $\mathbf{I}_2$ are empty, and Π_k the probability that buffer $\mathbf{I}_k$ is not empty for $k = 1, 2$, at just after kit completion, then these are given by

$$\Pi_0 = \pi(0,0),$$
$$\Pi_1 = \sum_{i=1}^{K_1-2} \pi(0,i) + \sum_{i=R_1+1}^{K_1-1} \pi(1,i),$$
$$\Pi_2 = \sum_{i=1}^{K_2-2} \pi(0,-i) + \sum_{i=R_2+1}^{K_2-1} \pi(2,-i).$$

Thus, from Theorem 1, the following is immediately.

Corollary 2. Π_k, $k = 0, 1, 2$, are given as follows.

Case A. For $\rho \neq 1$, these are written as

$$\Pi_0 = C_\alpha(1+\rho),$$
$$\Pi_1 = C_\alpha \rho\left(\frac{\rho}{1-\rho} + \frac{K_1 - R_1}{\rho^{-R_1} - \rho^{-K_1}}\right),$$
$$\Pi_2 = C_\alpha\left(\frac{1}{\rho-1} - \frac{K_2 - R_2}{\rho^{K_2} - \rho^{R_2}}\right).$$

Case B. For $\rho = 1$, these are written as

$$\Pi_0 = 2C'_\alpha,$$
$$\Pi_1 = \frac{C'_\alpha}{2}(K_1 + R_1 - 1),$$
$$\Pi_2 = \frac{C'_\alpha}{2}(K_2 + R_2 - 1).$$

3.2 The kit completion time interval

Let $D(t)$ be the distribution function of the kit completion time interval, that is, $D(t) = \Pr\{T_n \leq t\}$. Then, from the property of Markov renewal process, we have

$$D(t) = \sum_{\boldsymbol{j} \in \mathcal{K}'} \sum_{\boldsymbol{i} \in \mathcal{K}'} \pi(\boldsymbol{i}) Q(\boldsymbol{i}, \boldsymbol{j}, t). \tag{2}$$

From Theorem 1 in the previous subsection, we get the Laplace-Stieltjes transform $D^*(s)$ of $D(t)$ as follows.

Theorem 3. The Laplace-Stieltjes transform $D^*(s)$ of $D(t)$ is given by

$$D^*(s) = \Pi_1 \frac{\lambda_2}{\lambda_2 + s} + \Pi_2 \frac{\lambda_1}{\lambda_1 + s} + \Pi_0 \left(\frac{\lambda_1}{\lambda_1 + s} + \frac{\lambda_2}{\lambda_2 + s} - \frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2 + s} \right), \quad (3)$$

where Π_0, Π_1 and Π_2 are given in Corollary 2.

Proof. From (2), we have

$$D^*(s) = \sum_{\boldsymbol{j} \in \mathcal{K}'} \sum_{\boldsymbol{i} \in \mathcal{K}'} \int_0^\infty e^{-st} \pi(\boldsymbol{i}) Q(\boldsymbol{i}, \boldsymbol{j}, dt).$$

Using Theorem 1 and expressions of $Q(\boldsymbol{i}, \boldsymbol{j}, t)$, we can get the theorem via a direct calculation. □

In case of $\rho = 1$, the equation (3) is also written as

$$D^*(s) = (1 - 2C'_\alpha) \frac{\lambda}{\lambda + s} + 2C'_\alpha \left(\frac{2\lambda}{\lambda + s} - \frac{2\lambda}{2\lambda + s} \right),$$

where $\lambda = \lambda_1 = \lambda_2$.

Remark. Theorem 3 means that the distribution of the kit completion interval is rewritten as

$$D(t) = \Pi_1(1 - e^{-\lambda_2 t}) + \Pi_2(1 - e^{-\lambda_1 t}) + \Pi_0(1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t}).$$

Note that this is a distribution of phase type

$$D(t) = 1 - \boldsymbol{\alpha} \exp(\boldsymbol{T} t) e,$$

with a representation $(\boldsymbol{\alpha}, \boldsymbol{T})$, where

$$\boldsymbol{\alpha} = (\Pi_0, \Pi_1, \Pi_2),$$

$$\boldsymbol{T} = \begin{pmatrix} -(\lambda_1 + \lambda_2) & \lambda_1 & \lambda_2 \\ 0 & -\lambda_2 & 0 \\ 0 & 0 & -\lambda_1 \end{pmatrix}.$$

From these facts, the distribution of the kit completion interval is asymptotically exponential as $\lambda_1 \to \infty$ or $\lambda_2 \to \infty$. This property has been pointed out for the model without resume levels by Som *et al.* (1994). Further, we should also note that if R_k equal to $K_k - 1$, for $k = 1, 2$, the results as mentioned above agree with the equation (20) in Som *et al.* (1994).

4. The Sojourn Time and the Loss Probability

In this section, we deal with the sojourn time in the buffer of each component and the loss probability of components. For convenience, we call the sojourn time in the buffer of each component as the buffer sojourn time. The main purpose that we provide resume levels in the buffers is to reduce the buffer sojourn time. In order to evaluate the system performance, we need to derive the distribution of the buffer sojourn time.

4.1 The steady state distribution at arbitrary time

To obtain the buffer sojourn time and the loss probability of components, we have to study the steady state probability for the process $(\boldsymbol{J}, \boldsymbol{Z})$, which can be written as

$$p(\boldsymbol{i}) = \lim_{t \to \infty} \Pr\{(J(t), Z(t)) = \boldsymbol{i}\}, \quad \text{for } \boldsymbol{i} = (i_1, i_2) \in \mathcal{K}.$$

This process $(\boldsymbol{J}, \boldsymbol{Z})$ denotes the state of the buffers at any instant and has the state space $\mathcal{K}$ as mentioned in Section 2. Then we have the following.

Theorem 4. The steady state distribution of the process $(\boldsymbol{J}, \boldsymbol{Z})$ is obtained as follows.

Case A. For $\rho \neq 1$, the steady state probabilities are given by

$$\begin{aligned}
p(0,i) &= C_\beta \frac{1-\rho^{i-K_1}}{\rho^{-R_1}-\rho^{-K_1}}\,, && R_1+1 \le i \le K_1-1,\\
p(0,i) &= C_\beta \cdot \rho^i\,, && -R_2 \le i \le R_1,\\
p(0,i) &= C_\beta \frac{\rho^{i+K_2}-1}{\rho^{K_2}-\rho^{R_2}}\,, && -(K_2-1) \le i \le -(R_2+1),\\
p(1,i) &= C_\beta \frac{\rho-1}{\rho^{-R_1}-\rho^{-K_1}}\,, && R_1+1 \le i \le K_1,\\
p(2,i) &= C_\beta \frac{\rho-1}{\rho^{K_2+1}-\rho^{R_2+1}}\,, && -K_2 \le i \le -(R_2+1),
\end{aligned}$$

where

$$C_\beta = \frac{(\rho^{-R_1}-\rho^{-K_1})(\rho^{K_2}-\rho^{R_2})}{(K_1-R_1)(\rho^{K_2+1}-\rho^{R_2+1})-(K_2-R_2)(\rho^{-R_1-1}-\rho^{-K_1-1})}\,.$$

Case B. For $\rho = 1$, the steady state probabilities are given by

$$
\begin{aligned}
p(0,i) &= C'_\beta \frac{K_1 - i}{K_1 - R_1}, \quad R_1 + 1 \le i \le K_1 - 1, \\
p(0,i) &= C'_\beta, \quad -R_2 \le i \le R_1, \\
p(0,i) &= C'_\beta \frac{K_2 + i}{K_2 - R_2}, \quad -(K_2 - 1) \le i \le -(R_2 + 1), \\
p(1,i) &= \frac{C'_\beta}{K_1 - R_1}, \quad R_1 + 1 \le i \le K_1, \\
p(2,i) &= \frac{C'_\beta}{K_2 - R_2}, \quad -K_2 \le i \le -(R_2 + 1),
\end{aligned}
$$

where

$$C'_\beta = \frac{2}{K_1 + R_1 + K_2 + R_2 + 4}.$$

Proof. The set of balance equations for the process $(\boldsymbol{\mathcal{J}}, \boldsymbol{\mathcal{Z}})$ in the steady state is given by

$$
\begin{aligned}
(\lambda_1 + \lambda_2)\, p(0,i) &= \lambda_2\, p(0,i+1) + \lambda_1\, p(0,i-1), \\
&\qquad -(K_2 - 2) \le i \le K_1 - 2,\ i \ne R_1,\ i \ne -R_2, \\
(\lambda_1 + \lambda_2)\, p(0,K_1 - 1) &= \lambda_1\, p(0,K_1 - 2), \\
(\lambda_1 + \lambda_2)\, p(0,-(K_2 - 1)) &= \lambda_2\, p(0,-(K_2 - 2)), \\
(\lambda_1 + \lambda_2)\, p(0,R_1) &= \lambda_2\, p(0,R_1 + 1) + \lambda_1\, p(0,R_1 - 1) \\
&\quad + \lambda_2\, p(1,R_1 + 1), \\
(\lambda_1 + \lambda_2)\, p(0,-R_2) &= \lambda_2\, p(0,-(R_2 - 1)) + \lambda_1\, p(0,-(R_2 + 1)) \\
&\quad + \lambda_1\, p(2,-(R_2 + 1)), \\
p(1,i) &= p(1,i+1), \quad R_1 + 1 \le i \le K_1 - 1, \\
p(2,i) &= p(2,i+1), \quad -(K_2 - 1) \le i \le -(R_2 + 1), \\
\lambda_2\, p(1,K_1) &= \lambda_1\, p(0,K_1 - 1), \\
\lambda_1\, p(2,-K_2) &= \lambda_2\, p(0,-(K_2 - 1)).
\end{aligned}
$$

Using the normalizing condition $\sum_{\boldsymbol{i} \in \mathcal{K}} p(\boldsymbol{i}) = 1$, we obtain the steady state probabilities in the theorem. □

We consider the state distribution encountered by the arriving components. Denote the steady state probability at the arriving epoch of the component of line k provided that it actually enters the system without being lost by $\bar{p}_k(\boldsymbol{i})$ for $\boldsymbol{i} \in \mathcal{K} - \mathcal{K}_k$ and $k = 1, 2$. Due to the Poisson arrivals assumption, we can find the steady state probability observed by the arriving component is identical to one at any instant. Therefore, $\bar{p}_k(\boldsymbol{i})$ is given by

$$\bar{p}_k(\boldsymbol{i}) = \frac{p(\boldsymbol{i})}{1 - \sum_{\boldsymbol{j} \in \mathcal{K}_k} p(\boldsymbol{j})}, \quad k = 1, 2. \tag{4}$$

Using Theorem 4, we get the following by the direct calculations.

Corollary 5. $\bar{p}_k(\boldsymbol{i})$ is given by

$$\bar{p}_k(\boldsymbol{i}) = \begin{cases} C_{\alpha\beta}\rho\, p(\boldsymbol{i}), & \boldsymbol{i} \in \mathcal{K} - \mathcal{K}_1, \quad k = 1, \\ C_{\alpha\beta} p(\boldsymbol{i}), & \boldsymbol{i} \in \mathcal{K} - \mathcal{K}_2, \quad k = 2, \end{cases} \qquad \text{for} \quad \rho \neq 1,$$

$$= \quad C'_{\alpha\beta} p(\boldsymbol{i}), \qquad \boldsymbol{i} \in \mathcal{K} - \mathcal{K}_k, \quad k = 1, 2, \quad \text{for} \quad \rho = 1,$$

where $C_{\alpha\beta} = C_\alpha / C_\beta$ and $C'_{\alpha\beta} = C'_\alpha / C'_\beta$.

4.2 Buffer sojourn times

Let us denote the distribution of the buffer sojourn time at buffer $\mathbf{I}_k$ as $S_k(t)$ $(k = 1, 2)$. In order to derive $S_k(t)$, we need to use Corollary 5.

Theorem 6. The Laplace-Stieltjes transform $S_k^*(s)$ of $S_k(t)$ is given as follows.

Case A. For $\rho \neq 1$,

$$S_1^*(s) = C_\alpha \left[\frac{\lambda_1}{\lambda_2 - \lambda_1 + s} + \frac{\lambda_1}{\rho^{-R_1} - \rho^{-K_1}} \left(\frac{1}{s} - \frac{1}{\lambda_2 - \lambda_1 + s} \right) \left\{ \left(\frac{\lambda_2}{\lambda_2 + s} \right)^{R_1} \right. \right.$$
$$\left. \left. - \left(\frac{\lambda_2}{\lambda_2 + s} \right)^{K_1} \right\} - \frac{K_2 - R_2}{\rho^{K_2} - \rho^{R_2}} + \frac{\lambda_1}{\lambda_2 - \lambda_1} \right],$$

$$S_2^*(s) = C_\alpha \rho \left[\frac{\lambda_2}{\lambda_1 - \lambda_2 + s} + \frac{\lambda_2}{\rho^{K_2} - \rho^{R_2}} \left(\frac{1}{\lambda_1 - \lambda_2 + s} - \frac{1}{s} \right) \left\{ \left(\frac{\lambda_1}{\lambda_1 + s} \right)^{R_2} \right. \right.$$
$$\left. \left. - \left(\frac{\lambda_1}{\lambda_1 + s} \right)^{K_2} \right\} + \frac{K_1 - R_1}{\rho^{-R_1} - \rho^{-K_1}} - \frac{\lambda_2}{\lambda_1 - \lambda_2} \right].$$

Case B. For $\rho = 1$, i.e., $\lambda = \lambda_1 = \lambda_2$,

$$S_1^*(s) = C'_\alpha \left[\frac{K_2 + R_2 + 1}{2} + \frac{\lambda}{s} + \frac{\lambda^2}{(K_1 - R_1)s^2} \left\{ \left(\frac{\lambda}{\lambda + s} \right)^{K_1} - \left(\frac{\lambda}{\lambda + s} \right)^{R_1} \right\} \right],$$

$$S_2^*(s) = C'_\alpha \left[\frac{K_1 + R_1 + 1}{2} + \frac{\lambda}{s} + \frac{\lambda^2}{(K_2 - R_2)s^2} \left\{ \left(\frac{\lambda}{\lambda + s} \right)^{K_2} - \left(\frac{\lambda}{\lambda + s} \right)^{R_2} \right\} \right].$$

Proof. An arriving component to line 1, say C_1, that finds i waiting components in line 1 has to wait for $i+1$ arrivals of components to line 2. On the other hand, C_1 finding any waiting components in line 2 has no sojourn time. In addition, C_1 finding no waiting components has to wait for new arrival of component to line 2. In the same way, we can consider the sojourn time of the component in line 2. Noting the facts above, we obtain the distribution of buffer sojourn time as follows:

$$\begin{aligned} S_1(t) = &\sum_{i=1}^{K_1-1} \bar{p}_1(0,i) \int_0^t \frac{\lambda_2(\lambda_2 x)^i}{i!} e^{-\lambda_2 x} dx + \sum_{i=R_2+1}^{K_2} \bar{p}_1(2,-i) \\ &+ \sum_{i=1}^{K_2-1} \bar{p}_1(0,-i) + \bar{p}_1(0,0)(1 - e^{-\lambda_2 t}), \end{aligned} \tag{5}$$

$$\begin{aligned} S_2(t) = &\sum_{i=1}^{K_2-1} \bar{p}_2(0,-i) \int_0^t \frac{\lambda_1(\lambda_1 x)^i}{i!} e^{-\lambda_1 x} dx + \sum_{i=R_1+1}^{K_1} \bar{p}_2(1,i) \\ &+ \sum_{i=1}^{K_1-1} \bar{p}_2(0,i) + \bar{p}_2(0,0)(1 - e^{-\lambda_1 t}), \end{aligned} \tag{6}$$

From Corollary 5 and the equations (5) and (6), we get the theorem. □

Using Theorem 6, we readily obtain the following.

Corollary 7. The expected buffer sojourn times $\mathrm{E}[S_1]$ and $\mathrm{E}[S_2]$ are given as follows.

Case A. For $\rho \neq 1$,

$$\mathrm{E}[S_1] = C_\alpha \rho \left\{ \frac{\lambda_2}{(\lambda_2 - \lambda_1)^2} - \frac{K_1 - R_1}{\rho^{-K_1} - \rho^{-R_1}} \left(\frac{1}{\lambda_2 - \lambda_1} + \frac{K_1 + R_1 + 1}{2\lambda_2} \right) \right\},$$

$$\mathrm{E}[S_2] = C_\alpha \left\{ \frac{\lambda_1}{(\lambda_1 - \lambda_2)^2} - \frac{K_2 - R_2}{\rho^{K_2} - \rho^{R_2}} \left(\frac{1}{\lambda_1 - \lambda_2} + \frac{K_2 + R_2 + 1}{2\lambda_1} \right) \right\}.$$

Case B. For $\rho = 1$, i.e., $\lambda = \lambda_1 = \lambda_2$,

$$\mathrm{E}[S_1] = \frac{C'_\alpha}{6\lambda}\{K_1^2 + K_1R_1 + R_1^2 + 3(K_1 + R_1) + 2\},$$

$$\mathrm{E}[S_2] = \frac{C'_\alpha}{6\lambda}\{K_2^2 + K_2R_2 + R_2^2 + 3(K_2 + R_2) + 2\}.$$

4.3 Loss probability of component

In this subsection, we evaluate the loss probability of component $P_{loss\ k}$ of line k, $k = 1, 2$. It is clear that the loss probability of line k equals to the probability that line k is shut-down just prior to the arrivals of component to line k. Since the component arrival streams of both lines independently follow Poisson distribution, the steady state probability at any instant equals to one just prior to the arrival of component. Therefore, we can evaluate the loss probability as follows:

$$P_{loss\ k} = \sum_{\boldsymbol{i}\in\mathcal{K}_k} p(\boldsymbol{i}), \quad k = 1, 2.$$

Thus we obtain the following theorem.

Theorem 8. The loss probability $P_{loss\ k}$ is given by

$$P_{loss\ k} = \begin{cases} C_\beta(\rho - 1)\dfrac{K_1 - R_1}{\rho^{-R_1} - \rho^{-K_1}}, & k = 1, \\[2ex] C_\beta(\rho - 1)\dfrac{K_2 - R_2}{\rho^{K_2+1} - \rho^{R_2+1}}, & k = 2, \end{cases} \qquad \text{for} \quad \rho \neq 1,$$

$$P_{loss\ 1} = P_{loss\ 2} = C'_\beta, \qquad \text{for} \quad \rho = 1.$$

5. Numerical Examples

In this section, we present some numerical examples in order to evaluate the effect of resume level on the kitting process. We illustrate the performance of the kitting process at different resume levels and the value of $\rho(= \lambda_1/\lambda_2)$ which is the ratio of the arrival rate of line 1 to the arrival rate of line 2. We consider the case $K_1 = 12$, $K_2 = 10$. In addition, the resume level of line 2 is fixed at 9, while that of line 1 is varied to different values to show the influence of resume level upon the system performance. It is worth noticing that we make no buffer management control in line 2, because the resume level R_2 is assumed to be $K_2 - 1$. Furthermore, the arrival rate of line 2 (λ_2) is assumed to be unity.

We plot various performance measures of the system, i.e., the expected buffer sojourn time, loss probabilities of both types of components, and the expected kit completion interval, versus the parameter ρ in cases of $R_1 = 1, 6$ and 11. Figure 2 and Figure 3 illustrate the expected buffer sojourn time of line 1 and line 2, respectively. The curves in Figure 2 show that resume level R_1, significantly reduces the expected sojourn time of line 1, particularly when ρ is greater than 1. On the other hand, compared with no resume model, one might expect that the sojourn time of line 2 increases. Because, after the restoration of the arrival flow of line 1, there may occur burst arrival of the components of line 2 which will rapidly diminish the components of line 1. Then, the components of line 2 arriving afterwards have to wait for the arrival of the matching component from line 1. However, Figure 3 indicates that the increase of mean buffer sojourn time of line 2 is so small that we cannot ignore it.

Next, Figure 4 illustrates the loss probabilities of both lines. Although some components are lost while there are unoccupied buffers during the shut-down period, it is shown that the difference of the loss probabilities between the cases with and without resume levels is very small.

Finally, Figure 5 shows the expected kit completion interval. It is obvious that system performs almost identically regardless of the value of resume level R_1, in view of the expected kit completion interval. Therefore, we know that the expected kit completion interval is just slightly influenced by the resume level R_1.

These examples show introducing resume levels to the kitting process enables the sojourn time of line 1 to decrease remarkably without making the other system performance change greatly. These results indicate the superiority of this trade-off to achieve the desired system performance.

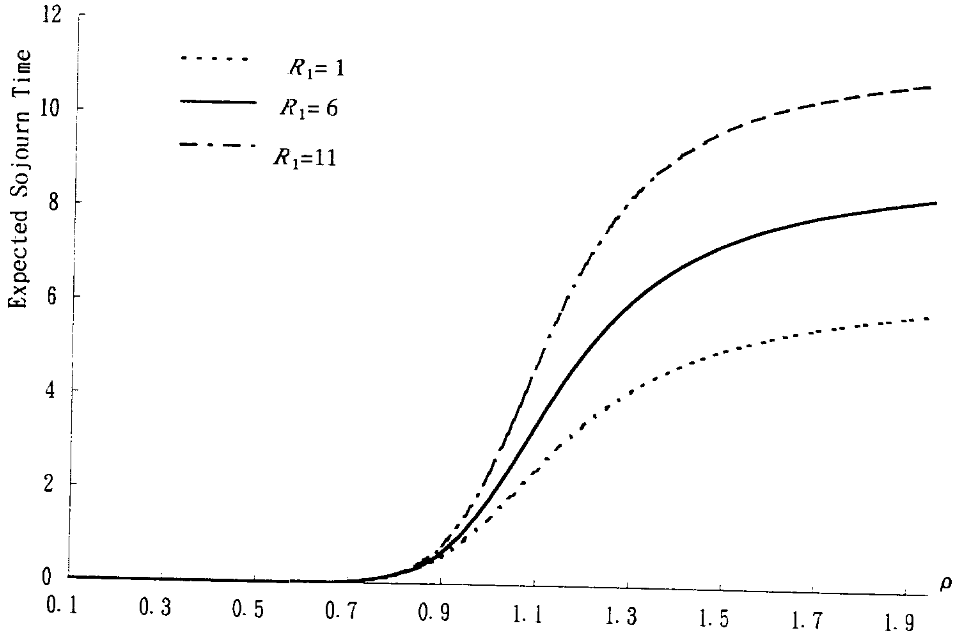

Figure 2: Expected sojourn time of line 1 v.s. ρ

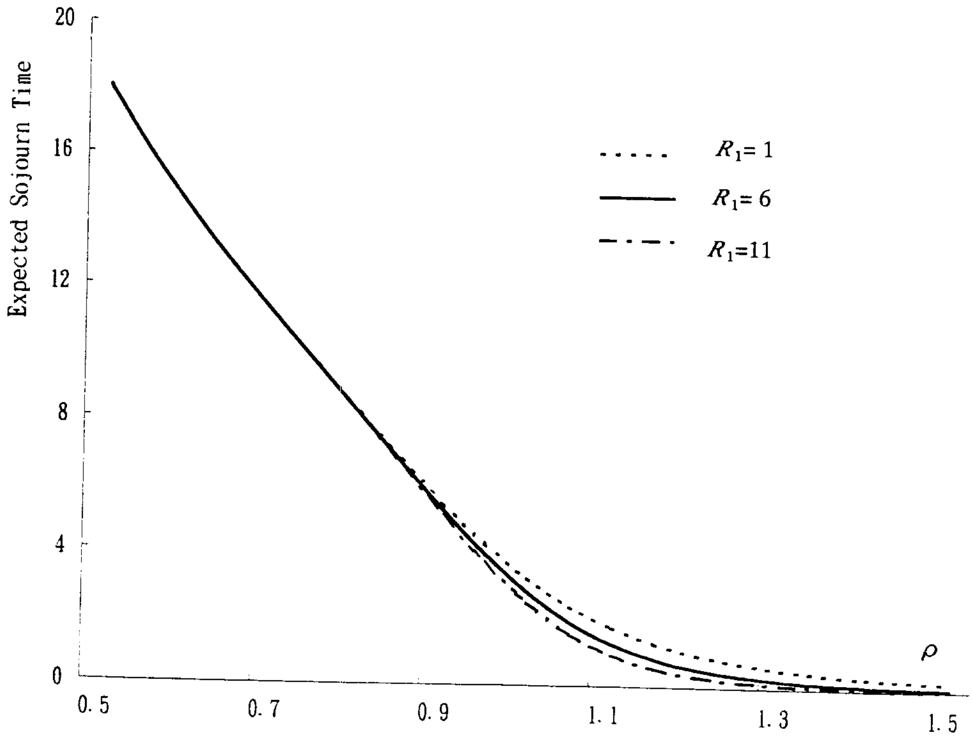

Figure 3: Expected sojourn time of line 2 v.s. ρ

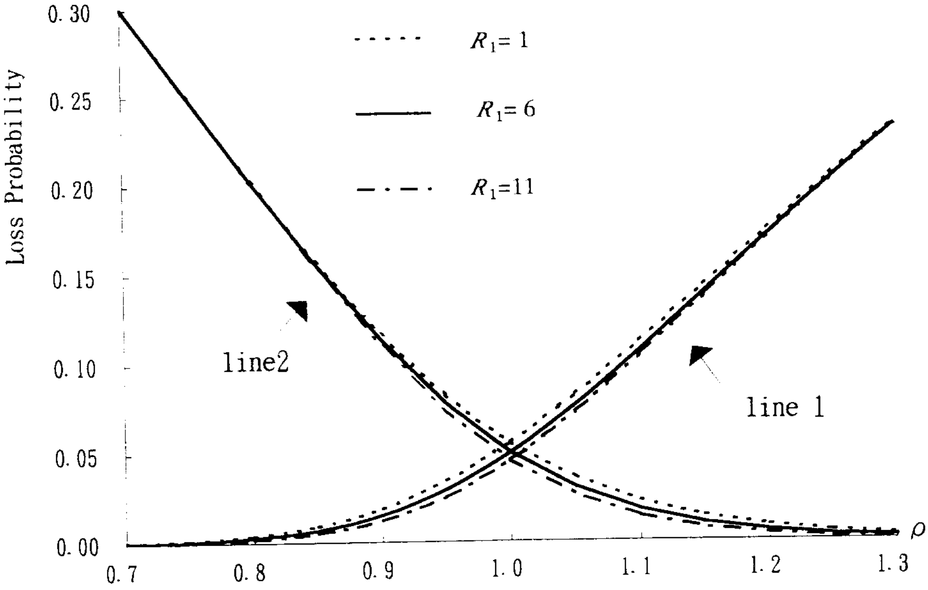

Figure 4: Loss probability v.s. ρ

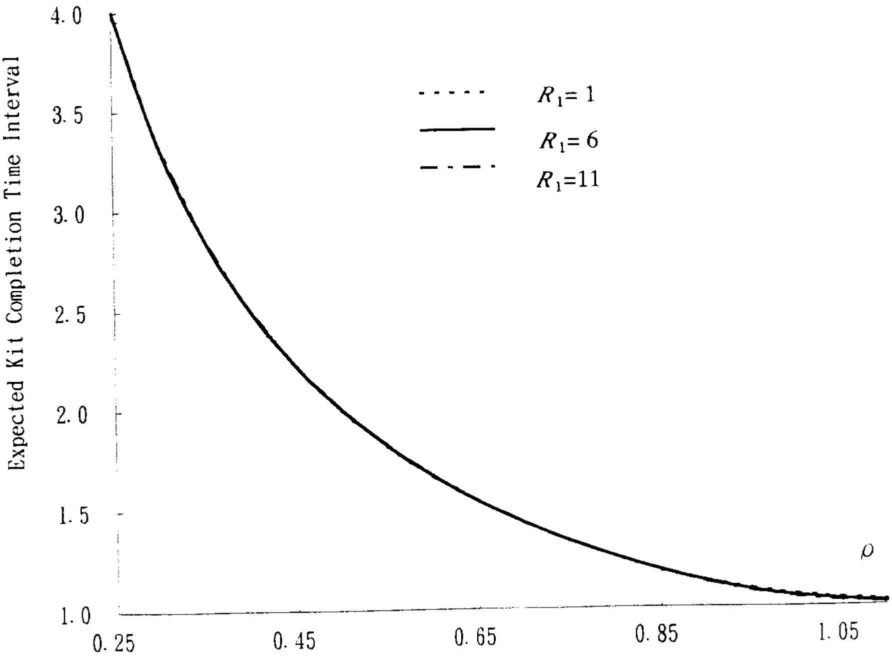

Figure 5: Expected kit completion time interval v.s. ρ

6. Conclusion

We presented a modified kitting process by providing a buffer management scheme on both of the buffers. Using Markov renewal process, we derived the distribution of kit completion intervals. Moreover, to show the improvement of the system performance, we computed the buffer sojourn time and loss probability. Resume levels affect the system by reducing the buffer sojourn time, and increasing loss probability of the component. However, it is shown in the numerical example that, in our system, the loss probability increases slightly while the buffer sojourn time signficantly decreases. That is, resume levels cause a good trade-off between the buffer sojourn time and the loss probability.

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References

Bhat, U. N. (1986), Finite capacity assembly-like queues, *Queueing Systems* 1, 85-101.

Harrison, J. M. (1973), Assembly-like queues, *Journal of Applied Probability* 10, 354-367.

Hopp, W. J. and J. T. Simon (1989), Bounds and heuristics for assembly-like queues, *Queueing Systems* 4, 137-156.

Latouche, G. (1981), Queues with paired customers, *Journal of Applied Probability* 18, 684-696.

Lipper, E. H. and B. Sengupta (1986), Assembly-like queues with finite capacity: bounds, asymptotic and approximations, *Queueing Systems* 1, 67-83.

Rosenberg, C., R. Mazumdar and L. Kleinrock (1990), On the analysis of exponential queueing systems with randomly changing arrival rates: stability conditions and finite buffer scheme with a resume level, *Performance Evaluation* 11, 283-292.

Som, P., W. E. Wilhelm and R. L. Disney (1994), Kitting process in a stochastic assembly system, *Queueing Systems* 17, 471-490.

Srivastava, H. M. and B. R. K. Kashyap (1982), *Special Functions in Queueing Theory and Related Stochastic Process*, Academic Press.

Takagi, H. (1985), Analysis of a finite-capacity M/G/1 queue with a resume level, *Performance Evaluation* 5, 197-203.

Takagi, H. (1993), *Queueing Analysis*, Vol. 2: *Finite Systems*, Elsevier, Amsterdam.

Yin, N., S. Li and T. E. Stern (1990), Congestion control for packet voice by selective packet discarding, *IEEE Transactions on Communications* 38, 674-683.

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