2. PROJECTVOORSTEL – *PROPOSAL*

2.1 Probleemstelling, doelstelling en methodologie en fasering samen met de volledige bibliografische opgave van de geciteerde werken – max. 3 blz. exclusief de bibliografie – *Description of the research proposal including aim, methodology and timing, and bibliography of the cited literature – max. 3 pages. excluding the detailed bibliografic list*

1. Aim

Nowadays manufacturing systems are often composed of multiple in-house fabrication units [8]. The semi-finished products stemming from these units are the input materials for other fabrication units or for assembly lines. Hence, efficient transport of materials between the different stages of the production process is a key issue for overall production cost minimization. This research proposal targets the **performance evaluation of kitting operations** in a production environment. Kitting is a particular strategy for supplying materials to an assembly line. Kitting means that, prior to arriving at an assembly unit, the necessary parts are collected into a specific container, referred to as a "materials kit" or "kit" [1,2,8,11-13]. This contrasts with the more traditional line stocking where the parts are delivered at the assembly line in containers of equal parts [1].

Kitting mitigates storage space requirements at the assembly station since no part inventories need to be kept there. Moreover, parts are placed in proper positions in the container such that assembly time reductions can be realised. Additional benefits include reduced learning time of the workers at the assembly stations and increased quality of the product [11]. The advantages above do not come for free since the kitting operation itself also incurs an additional cost. Kit preparation can be organised in several ways, affecting the associated cost. For example, kits can be picked in a warehouse - several kits can be picked at once - or can be composed along a conveyor belt.

Although kitting is a non-value-adding activity, its application can reduce the overall materials handling time [11]. However, the introduction of a kitting operation in a production process involves a major investment. Therefore **it is important that the impact of the kitting operation on the overall production cost can be assessed prior to the actual introduction of this operation**. The construction and analysis of a number of mathematical models that can assess the performance and cost of kitting operations are the prime research aims of this project.

In particular, this project will construct and analyse queueing models for kitting operations. A kitting operation obviously involves queueing from the vantage point of the different parts. These parts arrive at part inventories, the queues, and “wait” there till they are collected into a kit. The concept of uncertainty being central in queueing theory, a queueing theoretic approach can accurately assess the performance and/or costs of a kitting operation under uncertainty of inventory replenishments and/or product demand. Often, neither the product demand nor the inventory replenishments can be fully controlled such that these are preferably modelled as stochastic processes.

A good queueing model can accurately predict the performance measures of a real system in terms of this system’s parameters. For the kitting operation, the queueing analysis focuses on performance measures such as the throughput of the kits as well as kit earliness (the kit is prepared but not yet required at the assembly line) and kit tardiness (the kit is required at the assembly line but not yet prepared). The stock-out probability, the probability that one of the part inventories is empty such that the kitting operation is disrupted is another important measure. Once the model is constructed and analysed, it does not only allow to assess the cost of the kitting operation but also allows to tune the design parameters (for example the inventory levels) of the kitting operation for overall cost minimisation.

Queueing models that have been studied in literature cannot accurately capture the performance of kitting processes. Therefore, **special purpose queueing models and queueing analysis techniques will be developed** in this project. Although the queueing models are constructed to model kitting processes in particular, the corresponding analysis techniques will be sufficiently general and contribute to the state-of-the-art toolbox of queueing theory. We are confident that these tools will find applications in different areas (e.g. buffers in telecommunication systems) as well.

2. Methodology

Queueing theory uses probabilistic arguments and mathematical techniques to express various stochastic properties of the queueing process in terms of the stochastic properties of the driving processes. Most often, queueing processes can be described as **(Continuous Time) Markov Chains**, albeit with a large but structured state space, such that solution techniques for these chains apply.

Queueing models for kitting processes are rather complicated. Since multiple queues are involved (one for each part in the kit), **the state space of the associated Markov chain is inherently multidimensional**. The state of the Markov chain roughly corresponding to the number of distinct parts in the different inventories, the state space includes all possible part inventory levels. Multidimensionality leads to huge state spaces; this is the state space explosion problem. A second complication is more intricate. Obviously, inventories always have a finite capacity. However, if this capacity is sufficiently large, one may assume that the capacity is infinite. This means that there is always room for arriving parts, which simplifies the analysis. Unfortunately, **the infinite-buffer-capacity assumption is not applicable** for kitting processes. If the capacity is assumed infinite, the model degrades to an instable stochastic model in which some or all of the queues have an unlimited number of parts available all the time with a positive probability. This was already shown in [6] which studies queues with paired customers, an abstraction of a kitting process with two types of parts.

Having established these complications, it is clear that not every queueing model of a kitting operation is mathematically tractable. Complicated assumptions on the statistics of the delivery of components to the kitting process or an overly complicated mathematical description of the dynamics of this process may render such an analysis infeasible. Fortunately, these mathematical tractability constraints are somewhat mitigated if (i) one settles for **efficient numerical procedures** to determine the performance measures instead of closed-form expressions or (ii) one aims for **approximate expressions** for the performance measures. Therefore, the purpose of this proposal is twofold. On the one hand, intricate performance models will be developed which take into account realistic assumptions. In this case, closed-form formulae for the various performance measures of interest are not feasible. Instead, efficient numerical algorithms are developed which generate the performance measures of interest fast. On the other hand, simplified or approximating models will be developed which yield closed-form expressions for the relevant performance measures. These two different approaches to performance modelling are complementary. The accuracy of the approximate results can be assessed by the numerical procedures of the corresponding detailed analysis. Identification of such an approximate closed-form expression for a performance measure then yields a valuable and easy-to-use tool for optimisation of the kitting process without the computational burden of the numerical algorithms.

2.1 Numerical methods

Numerical methods in queueing theory explicitly construct the Markov chain underlying the queueing model. For the kitting process, the multidimensionality of the state space and the inapplicability of the infinite-buffer assumption yield Markov chains with a finite but very large state space.

Standard numerical methods in queueing theory notice that the state space of these Markov chains has some structure; the so-called “generator matrix” is e.g. a tridiagonal or an upper-Hessenberg block matrix. By exploiting this structure, efficient numerical solution techniques can be devised: the so-called **matrix-analytic methods** [7,9]. However, the available solution techniques only focus on two-dimensional Markov chains (one dimension being the level, the other being the phase). This project will investigate if these methods can be adapted to multidimensional Markov chains. One can expect that the generator matrix of the Markov chain is a tridiagonal block matrix whereby the blocks have a tridiagonal block structure as well (and so on).

This proposal also advocates a second numerical approach that may be more appropriate for studying kitting. While the state space of the transition matrix is very large, the number of possible state transitions from any specific state is limited. This means that most of the entries in the generator matrix are zero; the matrix is sparse. In contrast to matrix-analytic methods, **sparse matrix techniques** have hardly been used in queueing theory. Either direct or iterative techniques for sparse matrices can be used. A straightforward iterative solution technique involves repeated multiplication of an initial state vector with a transition matrix that is obtained from the generator matrix by uniformisation. More advanced methods like Krylov-subspace methods [10] (e.g. the generalised minimal residual method) are expected to speed up this approach considerably.

At present, it cannot be predicted which approach will perform best for the kitting processes at hand, both in terms of solution speed and in terms of the numerical stability of the algorithms.

2.2 Approximations

Approximate expressions for performance measures in general follow from simplifications of the mathematical description of the kitting system. Often, these expressions only hold for a particular range of the parameters. For example, a simplification of the mathematical description of the kitting process can be obtained if the corresponding Markov chain is nearly decomposable for some range of the parameters. A chain is nearly decomposable if its state space can be decomposed into several subspaces such that transitions from one subspace to another are not frequent [3].

For kitting systems, some bounds and approximations were already obtained in [11]. This paper however oversimplifies the performance measures; it neglects the actual preparation time of the kits completely. Finding sufficiently accurate approximations is an open research issue that will be investigated in this project. An initial approach will focus on a single queue (a single part inventory) and model the availability of parts in the remaining inventories as on-off processes. The resulting queueing system is a queueing system with server interruptions [4,5].

3. Work packages and timing

For the construction and analysis of the queueing models, a step-by-step approach is followed. Initially, some simplifying assumptions are made which are gradually removed. Such a bottom-up approach allows one to identify the set of analytically feasible models. The following work packages (WP) are defined.

WP 1: Two queue kitting models

A first set of models limits the number of studied part-inventories to two, i.e., there are only two queues. This assumption mitigates the state space explosion problem. Despite the oversimplification, such models can already capture some of the performance issues of a kitting operation. Moreover, these models serve as reference for the following work packages. In particular, the performance measures of interest need to be expressed in terms of the stationary solution of the Markov chain.

WP 2: Multiple queue kitting models

Removing the two-queue assumption, the multidimensionality and associated state space explosion problems come at play. Hence, this work package will explore the limits of numerical solution techniques of the Markov chains involved.

WP 3: Batch-service and batch-arrival queues

The former models assume that kits depart one by one and that the inventories are replenished part by part. In reality, multiple kits may be picked at once and part inventories may be replenished with multiple parts at once. In this work package, the kitting models will be adapted to account for batch departures of kits and batch arrivals of new parts.

WP 4: Controlled kitting models

Kitting typically does not operate independently of the rest of the production process. In this final work package, various external commodities are taken into account. E.g., the levels of the part inventories can trigger production such that replenishments are controlled by the evolution of the kitting operation. In addition, limits can be imposed on the number of available kits. In this case, it is possible that all kits are in use and that the kitting operation is disrupted.

References

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