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The impact of production interruptions in kitting, an analytical study

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Abstract Efficient transport of materials between the stages of the production process is key in the minimization of production costs. The kitting process is an attempt at achieving efficient transport and thus reducing costs. In this paper we discuss the performance of kitting operations in a stochastic assembly system, treating it as an assembly-like queue model. Specially, the impact of production interruptions in the subparts is investigated. To model downtimes, the subparts arrive according to an Interrupted Poisson Process instead of a Poisson Process. The queuing analysis focuses on the calculation of performance measures to compare the models with and without production interruptions.

Unlike previous studies in this domain, we use sparse matrix techniques to define matrices and solve linear equations. Results show that this technique is a valuable queuing theoretic numerical approach for estimating the performance of a kitting process in terms of solution speed and accuracy.

Keywords Kitting process · Assembly-like queue · Continuous Time Markov Chain · Sparse method · Production interruptions · Performance measures

1 Introduction

Nowadays customers put a lot of pressure on the market to afford customized products. This result in the handling of a large number of components in the production systems. The problem of keeping many and varied components is met by applying kitting.

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Nowadays manufacturing systems are often composed of multiple in-house fabrication units [Medbo(2003)]. The semi-finished products stemming from these units are the input materials for other fabrication units or for assembly lines. Hence, efficient transport of materials between the different stages of the production process is a key issue for overall production cost minimization. Therefore the Kitting method was introduced.
Kitting is a particular strategy for supplying materials to an assembly line. Instead of delivering parts at the assembly line in containers of equal parts, in kitting the necessary parts are collected into a specific container, referred to as kit, prior to arriving at an assembly unit [Bozer and McGinnis(1992), Bryznr and Johansson(1995),Medbo(2003),Ramakrishnan and Krishnamurthy(2008), Ramachandran and Delen(2005),Som et al(1994)Som, Wilhelm, and Disney].

Kitting mitigates storage space requirements at the assembly station since no part inventories need to be kept there. Moreover, parts are placed in proper positions in the container such that assembly time reductions can be realized. Additional benefits include reduced learning time of the workers at the assembly stations and increased quality of the product [Ramakrishnan and Krishnamurthy(2008)]. The advantages above do not come for free since the kitting operation itself also incurs additional costs. There are mostly the time and effort for the additional planning of the parts allocation into kits and the kit preparation itself. A storage space will also be essential to store the already prepared kits. Although kitting is a non-value adding activity, its application can reduce the overall materials handling time [Ramakrishnan and Krishnamurthy(2008)]. However, the introduction of a kitting operation in a production process involves a major investment. Therefore it is important to analyse the performance of kitting in a production environment prior to the actual introduction of this operation.

The concept of uncertainty being central in queuing theory, a queuing theoretic approach is used in order to assess the performance of a kitting process under uncertainty of inventory replenishments and/or product demand. Often, neither the product demand nor the inventory replenishment can be fully controlled such that Kitting processes are preferably modelled as stochastic processes. To gain a more realistic insight on the performance of a kitting process in a production environment, temporary interruptions in the production of subparts are taken into account. In this article, special attention will be given to the used queuing analysis technique.

2 Model description

Most authors consider a kitting process as a queuing system in a stochastic environment.

Hopp and Simon (1989) have developed a model that is often used to analyze the performance of an assembly line[Hopp and Simon(1989)]. In their article "Bounds and Heuristics for Assembly-like queues" a model with exponentially distributed processing times and between arrival times distributed according to a Poisson process is described. Out of their model, they deduce boundaries for the capacity of the buffers. This model is mainly based on the model of Lipper and Sengupta (1986)[Lipper and Sengupta(1986)]. The method of Hopp and Simon is easier to implement and the definition of an optimal upper limit of capacity is more accurate, but it is limited to processes with two basic components. The method of Lipper and Sengupta on the other hand, can be applied to more general systems.

Som, Wilhelm, and Disney (1994) consider a kitting process as a delivery system for assembly that is mainly based on the model of Hopp and Simon. An important similarity with our models is the assumption of a finite-buffer-capacity. Of course a buffer has always a finite capacity. However, if the capacity is large enough, we can have a good approximation of a process with a finite capacity on the basis of a model with unlimited capacity. This means that there is always enough room for upcoming parts which simplifies the analysis. Unfortunately, the assumption of an infinite buffer is not valid for kitting processes. If the capacity is assumed to be infinite, then the model will degrade to an unstable stochastic model. This was demonstrated by Latouche (1981) that studied waiting lines with paired customers. We can consider his analysis as an abstraction of a kitting process with two types of parts[Latouche(1981)]. Furthermore, in the article "Assembly-like queues", Harrison (1973) confirms that, to ensure stability in the operations of a kitting process, it is necessary to impose a restriction on the size of the buffer. Under this assumption, the probability to have a certain long-term stock position is equal and independent of the current stock position. We assume that the buffer capacity of the two components is respectively equivalent to C_1 and C_2.

In this article, three mathematical queuing models are defined and analyzed to assess the impact of production interruptions on the performance of kitting. In the first model displayed in figure 1, both components arrive according to a Poisson Process with for both parts a same arrival intensity λ. Two independent input streams arrive at part inventories and wait there till they are collected into a kit. Each component is processed according to an exponential distribution (before kitting) to prepare it for assembly and as mentioned above, we assume a finite buffer capacity for the components. The danger exists that components are denied because the buffers are full, we speak of loss. So when a component is present in the process and the buffer is full, this component is lost.

In the first extensive model, parts of type 1 are subject to interruptions in the production. To model these production breaks, the components arrive according to an Interrupted Poisson Process, abbreviated as IPP. In the queuing analysis, an IPP is a stochastic process in which two states are possible and which one of the two has an intensity equal to zero. This process is divided into two periods, namely the active and inactive period (Heyman and Sobel, 1982). We start with an active period and during this interval there are components arriving according to a Poisson process with intensity ?*. The length of this period is exponentially distributed with mean α^{-1}. At the end of an active period begins a period of inactivity in which components do not arrive, the

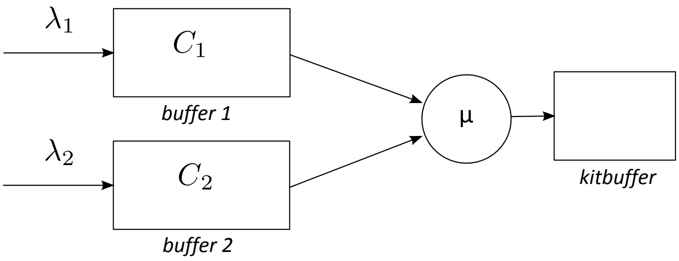

Fig. 1 Basic Model

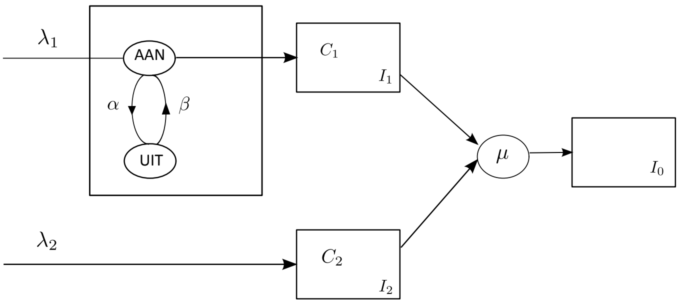

Fig. 2 First Extensive Model

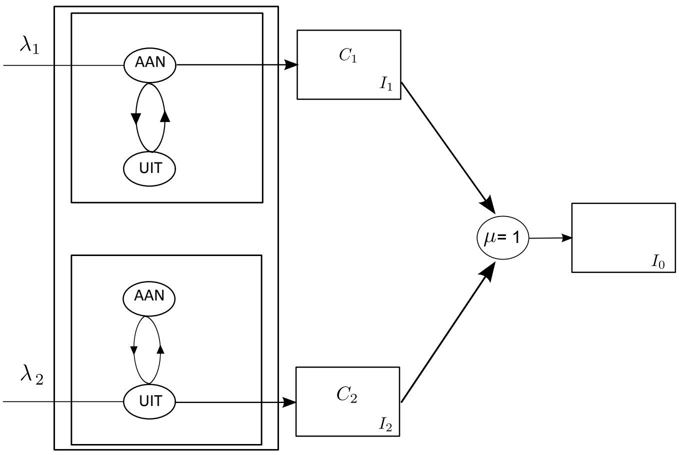

Fig. 3 Second Extensive Model

length is exponentially distributed with mean β^{-1}. At the end of this period begins another new active period and so on. All active and inactive periods are i.i.d. The parameter α (β) describes the intensity to go from an active (inactive) to an inactive (active) period in an infinitesimal time interval.

Finally, in the second extensive model, both part 1 and part 2 suffer from production cuts. The arrival processes are identical and independent of each other. The two Interrupted Poisson processes have the same intensity a and .

In the following section, the kitting process is defined as a (Continuous Time) Markov Chain.

3 Formulation of a Continuous Time Markov Chain

The kitting process modelled as a Markov Chain allows us to determine the probability that a certain state e.g. the number of components in the two part inventories, occurs. Thereafter, this gives us the ability to calculate performance measures such as the average buffer occupancy. Under the Markovian assumptions, we discuss the steps to calculate the performance measures of the process.

3.1 Generator Matrix

We define a stochastic process $X(t)$ as a Markov chain with continuous time parameter where $s, t, u \geq 0$ and all non-negative integer values i, j and r belong to the discrete state space X. It is true that:

$$\mathrm{P[X(t) = j|X(s) = 1, X(u) = r, u \leq s < t] = P[X(t) = j|X(s) = 1].}$$

This definition is based on the Markov property. Suppose now that:

$$\mathrm{p_{ij} \equiv P[X(t) = j|X(s) = i],}$$

where $t \geq s$.

We assume that our Markov chain is homogeneous. A chain is homogeneous if all transition functions $\mathrm{p_{ij}(s,t)}$ depend solely on the difference $(t-s)$ and are independent of the absolute epochs s and t. Transition functions give the probability that a situation will occur given a current state. Among others in the book "Discrete Event Systems" written by Cassandras and Lafortune (2008) [Cassandras and Lafortune(2008)] transition functions satisfy the *Chapman-Kolmogorov equation.*
We prove this by first applying the law of total probability: $\mathrm{P[A] = \Sigma_i P[A|P[B_i].P[B_i]}$.
We consider $[X(u) = r]$ for $s \leq u \leq t$ when the conditional probability of the event $[X(t) = j|X(s) = i]$:

3.2 Stationaire waarschijnlijkheidsvector

The symbol π is similar to the stationary probability vector. This collects vector all stationary state probabilities, i.e. the probabilities that a certain condition occurs when the chain has reached equilibrium. If the time parameter goes to infinity, then its derivative equals zero. The vector is no longer dependent on its elements and converge to a fixed value.

$\pi.\mathbf{Q} = 0$**The multiplication of the stationary probability vector with its generator matrix is equal to zero.**

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