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Kitting in multi-echelon, multi-product assembly systems with parts substitutable

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The kitting problem in multi-echelon, multi-product assembly systems with parts substitutable is to allocate on-hand stock and expected future deliveries to kits to minimize total cost—including job earliness, job tardiness, and in-process holding cost—while considering shop capacity and subassembly precedence restrictions, and parts being substitutable. This class of problem is NP-hard. When dealing with a large instance encountered in industry, in the worst case, it may not be possible to obtain an optimal solution in a reasonable time. In this paper, rules for using part substitutes along with heuristic procedure are presented. Computational results demonstrate that the proposed heuristic outperforms others tested. It is expected that the new heuristic can be applied in resolving large-scale kitting problems encountered in industry to allocate available resources near optimality to enhance schedule performance and to lower the total cost of operating multi-echelon, multi-product assembly systems.

1. Introduction

In multi-echelon, multi-product assembly systems, a 'kit' or a set of components (i.e. parts) that comprise a subassembly is 'collected' (or 'kitted') from stock on hand and then 'released' to initiate an assembly process. In spite of diligent attempts to ensure part availability, on-hand stock may fall short of requirements, because of unexpected events such as yield losses or transportation delays. Stock deficiencies disrupt assembly schedules, leading to job tardiness and high subassembly holding costs.

Decision alternatives arise daily in the kitting process. On any given day, components on hand may be sufficient for only some kits. To which kits should available components be assigned? How should the promised deliveries be allocated to kits? How should part substitutes be used to meet assembly schedules?

This kitting problem is especially serious in the electronic industry for the following reasons:

(1) A kit (e.g. a circuit card) usually requires numerous different components. Many of which (e.g. semi-conductors) may have rather variable part delivery lead times due to in-process yields, increasing the likelihood of shortages,
(2) Electronics components are rather costly, so safety stocks are usually reduced to minimal levels,
(3) The electronics assembly system is typically composed of two or three echelons. For instance, consider a three-echelon assembly. At echelon 3,

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electronic components are kitted and assembled into circuit cards. In turn, at echelon 2 circuit cards are kitted along with other components to produce subassemblies. Finally, at echelon 1 subassemblies are assembled into end products, so that kitting delays are particularly disruptive, because they affect the coordination of material flow from one level to the next, and

(4) Substitutes are commonly used in the electronics industry. Substitution may be one-directional, bi-directional, or product-dependent. Using substitutes carefully may enhance schedule performance. However, using substitutes carelessly may cause stock deficiencies and disrupt assembly schedules. (For example, subassemblies of some jobs may not be allowed to use part p (i.e. part of type p) as a substitute for part p'. If most of the supply of p' is used to substitute for part p, some assemblies cannot be kitted, resulting in a poor or even infeasible schedule).

Although the kitting problem can be eliminated by increasing safety stocks, such an alternative is not attractive, due to the high cost of carrying additional inventory. Another way to resolve this problem is to efficiently allocate components to kits, considering those currently available, anticipated future deliveries promised by vendors, and using substitutes. In addition, shop capacity must be considered. The kitting problem is thus to allocate on-hand stock and anticipated future deliveries to kits to minimize total cost—including job earliness, job tardiness, and in-process holding cost—while considering shop capacity and subassembly precedence restrictions, and part substitution.

It is known that this class of problems is NP-hard (Mazzola and Neebe 1986). When dealing with a large instance encountered in industry, in the worst case, it may not be possible to obtain an optimal solution in a reasonable time. The purpose of this research is to extend the research of Chen and Wilhelm (1993, 1994 and Wilhelm *et al.* 1996) and thus to develop rules for using part substitutes and a heuristic procedure with capability relative to runtime and solution quality, so that the available resources can be allocated to near optimality for large-scale, industrial applications. The proposed heuristic is empirically evaluated in comparison with other heuristics and how each is affected by a set of factors.

This research is justified by the prevalence of assembly operations, which constitute a very high percentage of total manufacturing operations, and by the prevalence of kitting, especially in the electronics industry. Thus we expect that industry can benefit from this research.

The objectives of this research are:

(1) A new model for the multi-echelon, multi-product kitting problem.
(2) A technique for determining when to use substitutes, and
(3) A heuristic for large-scale, industry applications.

A new model structure for the multi-echelon, multi-product kitting problem, which may be used to incorporate a broad set of problem features is addressed by objective (1). Since using part substitutes carelessly may cause stock deficiencies and disrupt assembly schedules, objective (2) can be achieved by developing rules and/or constraints for using substitutes. It is known that the kitting problem is NP-hard. A good heuristic is thus needed for large-scale, industrial applications. Objective (3) can be achieved by a heuristic which can demonstrate its capability relative to run time and solution quality through empirical testing. Upon fulfilling the research objectives, available resources can be allocated near optimally to enhance schedule performance and to lower the total cost of operating multi-echelon, multi-product assembly systems.

The body of this paper is organized in five sections. A literature review which encompasses the kitting and related problems is presented in § 2. A new model for this type of kitting problem is discussed in § 3. The heuristic is detailed in § 4. Computational results are reported in § 5. Conclusions and suggestions for future research are discussed in § 6.

2. Literature review

Although the kitting problem seems to be very important in industry, it appears that not much research on it has been documented. Irastorza and Deane (1974) and Roderick (1990) developed quantitative methods to prescribe the release of jobs into a job shop. Gershwin *et al.* (1984) and Han and McGinnis (1980) devised methods for the release of jobs into manufacturing cells. The literature on project scheduling treats resources as renewable, non-renewable, or doubly constrained (Slowinski 1981). None of these treatments allows resources (e.g. parts) to be delivered over the duration of the horizon and allows them to be used in any later period. Dietrich (1990) developed a heuristic based on linear programming for resource deployment decisions in a single-level, multi-product, multi-period manufacturing network. The resources she dealt with were very similar to parts in the kitting problem, i.e. they can be accumulated from one period to another and are referred to as 'accumulated' resources.

The most closely related studies are by Chen and Wilhelm (1993, 1994, and Wilhelm *et al.* 1996) who developed two optimal approaches—one by using Lagrangian relaxation and the other using a cutting plane method—and one heuristic to resolve the kitting problem for small-lot, multi-echelon, multi-product assembly systems. Their optimizing approaches and heuristic perform quite well. However, they did not consider that parts may be substituted.

Other than these related studies, industrial schedulers tend to apply one of two heuristics, namely, 'dedicated' or 'compete', to quickly obtain a feasible kitting schedule (Chen and Wilhelm 1993). In both heuristics, kits are ranked according to priority for the allocation of available parts. The priority of a kit is, in general, determined according to the due date of its associated end product. In the 'compete' procedure, parts (no matter whether they are originally required parts or substitutes) are assigned to a kit only when all the parts required by that kit are available. When two kits compete for the same parts on hand, the priority kit has a better opportunity to be assigned parts. No partial kits can be released into assembly. In the 'dedicated' procedure, parts are allocated to the priority kit once they are delivered. Whenever a kit is completed, it is then released to begin the assembly process.

Although industrial schedulers may use these two heuristics to quickly obtain a kitting schedule, the solution quality may not be quite acceptable. Another alternative is to employ a heuristic based on linear programming. However, it is suspected that when constraints are very 'dense' (i.e. each variable appears in many constraints and every constraint contains many decision variables), optimal solutions will consist of many decision variables with fractional values. In this situation, a rounding procedure may have to be applied repeatedly, leading to computational inefficiency as well as poor solution quality.

3. The new model

A new model for the kitting problem, a revision of the one presented by Chen and Wilhelm (1993) is presented in this section so that a precise mathematical structure describing the problem can be considered. Parts that may be substituted are commonly found in industry. For a pair of parts p and p', three types of substitutions are considered in this paper.

(1) Substitution may be one-directional (e.g., part p may be substituted for part p', but part p' may not be substituted for part p),
(2) substitution may be bi-directional (e.g. part p may be substituted for part p' and vice versa), and
(3) substitution may be product-dependent (e.g., part p may be substituted for part p' in product A, but part p may not be substituted for part p' in product B).

To accommodate substitutions, binary decision variables are formulated as:

$$X_{ijd} = \begin{cases} 1 \text{ if subassembly } ij \text{ uses all originally required parts and is kitted on day } d \\ 0 \text{ otherwise} \end{cases}$$

$$X'_{ijd} = \begin{cases} 1 \text{ if subassembly } ij \text{ uses any substitutes and is kitted on day } d \\ 0 \text{ otherwise,} \end{cases}$$

$$Y_{cpijd} = \begin{cases} 1 \text{ if subassembly } ij \text{ uses } c \text{ units of originally required part } p \text{ and is kitted} \\ \text{on day } d \\ 0 \text{ otherwise.} \end{cases}$$

$$Y'_{cp'ijd} = \begin{cases} 1 \text{ if subassembly } ij \text{ uses } c \text{ units of substitutes (part } p') \text{ and is kitted day } d \\ 0 \text{ otherwise.} \end{cases}$$

Let $\Theta(p)$ be the set of subassemblies that originally require part p or may use part p as substitutes, $\theta(i,j)$ be the set of parts was originally required by subassembly ij, and $\theta'(i,j)$ be the set of parts that can be used as substitutes for subassembly ij. Following Chen and Wilhelm (1993), the kitting problem with parts substitution may now be formulated as

Problem P_0:

$$\text{minimize}\, Z = \sum_j \sum_{i \in I(j)} \sum_d C_{ij}(d)(X_{ijd} + X'_{ijd}) \tag{1}$$

subject to

$$\sum_j \sum_{i \in SS_{(s,j)}} (X_{ijd} + X'_{ijd}) \leq R_{sd} \quad \text{for } d = 1, \ldots, \mathrm{D}; s = 1, \ldots, S \tag{2}$$

$$\sum_j \sum_{i \in \Theta(p)} \sum_{t=1}^{d} \left(\sum_{c=1}^{q_{ijp}} c Y_{cpijt} + \sum_{c=1}^{q_{ijp}} c Y'_{cpijt} \right) \leq \mathrm{N}_{pd} \quad \text{for } d = 1, \ldots, D-1; p \in P \tag{3}$$

$$\sum_j \sum_{i \in \Theta(p)} \sum_{t=1}^{D} \left(\sum_{c=1}^{q_{ijp}} c Y_{cpijt} + \sum_{c=1}^{q_{ijp}} c Y'_{cpijt} \right) = \mathrm{N}_{\mathrm{PD}} \quad \text{for } p \in \mathrm{P} \tag{4}$$

$$\sum_d (d + \sigma_{mj})(X_{mjd} + X'_{mjd}) \leq \sum_d (X_{ijd} + X'_{ijd}) \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j); m \in \mathrm{M}(i,j) \tag{5}$$

$$\sum_d (X_{ijd} + X'_{ijd}) = 1 \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j) \tag{6}$$

$$X_{ijd} = 0 \text{ or } 1 \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j); d = 1, \ldots, \mathrm{D} \tag{7}$$

$$X'_{ijd} = 0 \text{ or } 1 \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j); d = 1, \ldots, \mathrm{D} \tag{8}$$

$$Y_{cpijd} = 0 \text{ or } 1 \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j); d = 1, \ldots, \mathrm{D}; p \in \theta(i,j); c = 0, \ldots, q_{ijp} \tag{9}$$

$$Y'_{cpijd} = 0 \text{ or } 1 \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j); d = 1, \ldots, \mathrm{D}; p' \in \theta'(i,j); c = 0, \ldots, q_{ijp} \tag{10}$$

$$X'_{ijd} \geq Y'_{cpijd} \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j); d = 1, \ldots, \mathrm{D}; p' \in \theta'(i,j); c = 1, \ldots, q_{ijp} \tag{11}$$

$$Y'_{cp'ijd} = Y_{(q_{ijp}-c)pijd} \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j); d = 1, \ldots, \mathrm{D}; p' \in \theta'(i,j); p \in \theta(i,j); \\ c = 0, \ldots, q_{ijp} \tag{12}$$

$$\sum_{p \in \theta(i,j)} Y_{q_{ijp}pijd} \leq |\theta(i,j)| - 1 + X_{ijd} \quad \text{for } j \in \mathrm{J}; i \in \mathrm{I}(j); d - 1, \ldots, \mathrm{D}; p \in \theta(i,j) \tag{13}$$

Objective function (1) minimizes total cost which consists of job earliness cost, job tardiness cost, and subassembly holding cost. Shop capacity restrictions are enforced by constraint (2), where R_{sd} is measured in terms of the number of kits that can be released into shop s each day. Constraint (3) assures that the kitting schedule is based on part availability, considering the cumulative delivery of parts on days $t = 1, \ldots, d-1$. Equation (4) is based on the assumption that the cumulative number of part p available over the kitting horizon is equal to the total number of part p required to kit all subassemblies. Inequality (5) imposes precedence relationships among subassemblies. Constraint (6) insures that each subassembly is kitted exactly once during the planning horizon, using either all originally required parts or any part substitutes. Constraints (7)–(10) require decision variables to be binary integers. Inequality (11) assures that $X'_{ijd} = 1$ if subassembly ij uses part substitutes and is kitted on day d. Equation (12) indicates that subassembly ij must use c units of part p' as substitutes if it only uses $(q_{ijp} - c)$ units of originally required part p, and Inequality (13) assures that $X_{ijd} = 1$ if subassembly ij use all originally parts and is kitted on day d.

4. The heuristic

The heuristic is discussed in this section. The average performance of this heuristic is compared empirically with two heuristics commonly used in industry, 'dedicated' and 'compete', and with the heuristic of Chen and Wilhelm (1993) in the next section. In order to develop a good heuristic for this type of kitting problem, five goals must be achieved.

(1) Determine a good kitting sequence for jobs,
(2) Avoid incurring job tardiness costs as much as possible,
(3) Avoid incurring job earliness costs as much as possible,
(4) Avoid incurring subassembly holding costs as much as possible, and
(5) Use right quantity of substitutes at the right time.

Since cost parameters and the sojourn time (processing time + queuing time) required to complete different job are not the same, kitting jobs according to earliest due date first may not always result in a solution of high quality. In this heuristic, we use a sequence based on earliest-due-date-first as well as one similar to that of Chen and Wilhelm (1993) to determine a kitting sequence. In the first $|\mathrm{J}|$ passes, jobs are scheduled according to increasing order of slack $[\mathrm{SLACK}_j = \gamma_j + 1 - \max_{mj \in SE(E_j,j)} \sum_{aj \in \mathrm{PH}(mj,1j)} \sigma_{aj}]$. Subassemblies of a selected job are then sequenced with the earliest kitting due date first $[\mathrm{KITDUE}(ij) = \gamma_j + 1 - \sum_{aj \in \mathrm{PH}(ij,1j)} s_{aj}]$. At the end of each of the first |J| passes, the sequence position (for the (|J| + 1) pass) of one job with the lowest potential tardiness cost is determined. After the first |J| passes, jobs are scheduled according to ORDER (i.e., sequence position) in the $(|J|+1)^{th}$ pass.

To achieve goal (2), subassemblies (and assemblies) must be kitted as early as possible. However, this may lead to high job earliness and subassembly holding costs (which may be incurred when a subassembly is completed and waiting for its immediate successor to be kitted, either because there are not enough parts to kit its immediate successor or because the associated subassemblies used to assemble its immediate successor are not completed yet) and thus conflicts with goals (3) and (4). So, each time when all subassemblies of a selected job are scheduled, the kitting days for all subassemblies of that selected job are right shifted (if possible) to avoid job earliness and subassembly holding costs.

To achieve goal (5), rules for determining whether to use substitutes must be developed. Let part p' (called superior parts) be the substitutes for part p (called inferior parts) and vice versa. Then, jobs (i.e. customer orders) may be divided into three categories:

(1) jobs that can use part p' as substitute for part p, but cannot use part p as substitute for part p',
(2) jobs that can use part p' as substitute for part p and vice versa, and
(3) jobs that cannot use part p' as substitute for part p and vice versa.

Let Upd be the number of part p allocated on day d so far, $U_{pd} = \sum_{t=1}^{d}$ Upt, $A_{pd} = N_{PD} - U_{PD} - min_{t\in\{d,d+1,\ldots,D\}}\{N_{pt} - U_{pt}\}$, IA[IB] be the total number of part $p[p']$ (originally) required to complete customer orders in category (1), IC [ID] be the total number of part $p[p']$ (originally) required to complete customer orders in category (2), and IE [IF] be the total number of part $p[p']$ (originally) required to complete customer orders in category (3). In this paper, we develop the following rules to determine when to use substitutes:

Rule 1. Subassemblies should use their originally required parts as many times as possible,
Rule 2. Subassemblies should not use substitutes until kitting delays begin to affect the coordination of material flows,
Rule 3. On any given day d, if Ap′d $\geq IB + IF - ALLT(p')$, jobs in categories (1) or (2) may use available part p' as substitutes for part p, in which ALLT(p') is the sum of part p' allocated to the jobs that required part p' and cannot use part p as substitutes (i.e. jobs in categories (1) and (3)), (14)
Rule 4. On any given day d, if Ap′d $< IB + IF - ALLT(p')$, the quantity $IB + IF - ALLT(p')$ − Ap′d of the available part p' must be reserved for those subassemblies that originally require part p' and cannot use part p as substitutes. The remaining stock on hand of part p' can be used as substitutes,
Rule 5. On any given day d, if $Ap_d < IE - ALLT(p)$, the quantity $IE - ALLT(p) - Ap_d$ of the available part p must be reserved for those subassemblies of jobs in category (3). The rest of part p can be used by subassemblies of jobs in categories (1) and (2), (15)
Rule 6. On any given day d, if $Ap_d \geq IE - ALLT(p)$, jobs in category (2) may use available part p as substitutes for part p',

Rule 7. On any given day d, if $\mathrm{Ap_d} < IE - ALLT(p)$, the quantity $IE - ALLT(p) - \mathrm{Ap_d}$ of available part p must be reserved for those subassemblies that require part p and cannot use part p' as substitutes. The rest of part p can be used as substitutes for part p', and

Rule 8. On any given day d, if $\mathrm{Ap'd} < IB + IF - ALLT(p')$, the quantity $IB + IF - ALLT(p') - \mathrm{Ap'd}$ of available part p' must be reserved for those subassemblies that require part p' and cannot use use part p as substitutes. The rest of part p' can be used by subassemblies of jobs in category (2).

Rule (1) states that subassemblies should use their originally required parts whenever possible to avoid incurring poor or even infeasible schedules. Rule (2) indicates that subassemblies which can use part p' as substitutes for part p should not do so until kitting delays affect the coordination of material flows. For example, consider a three-echelon assembly system. If subassembly 2 (at echelon 2) is kitted on day 9, then it is not necessary for subassembly 3 (also at echelon 2) to use part p' as substitutes so that it could be kitted at day 9 (assuming both subassemblies have the same planned lead time).

On day d and after, the accumulated available quantity of part p' is equal to the left hand side of inequality (14), and the quantity of part p' still required by jobs that cannot use part p as substitutes is equal to the right hand side of Inequality (14). Rule (3) states that if the accumulated available quantity of part p' is greater than or equal to the quantity of part p' still required by jobs that cannot use part p as substitutes, no restriction on the use of part p' is necessary. On the contrary, if inequality (14) be not true, rule (4) assures that enough part p' is reserved for the use by jobs that cannot use part p as substitutes.

On day d and after, the accumulated available quantity of part p is equal to the left hand side of inequality (3), and the quantity of part p still required by jobs that cannot use part p' as substitutes is equal to the right hand side of inequality (15). If inequality (15) be true, rule (5) assures that enough part p is reserved for the use by jobs that cannot use part p' as substitutes. Similarly, rules (6), (7), and (8) assure that enough part $p[p']$ is reserved for the use by jobs that cannot use part $p'[p]$ as substitutes.

The heuristic (H1) is now detailed.

Step 0. Initialization

0-A. Set $\Pi_1 = \Pi_2 = \mathrm{J}$, $\mathrm{POSITION} = |\mathrm{J}|$, $FLAG1 = 1$, $FLAG2 = 0$, $ALLT(p) = 0$, for every $p \in \mathrm{P}$.

0-B. For each $j \in \mathrm{J}$, set

$$\mathrm{SLACK}_j = \gamma_j + 1 - \max_{mj \in SE(E_j,j)} \sum_{aj \in \mathrm{PH}(mj,1j)} \sigma_{aj}, \mathrm{KITDUE}(1,j) = \gamma_j + 1 - \sigma_{1j}$$

and $\mathrm{KITDUE}(i,j) = \mathrm{KITDUE}(k,j) - \sigma_{ij}$ for each $ij \in \mathrm{A}(1,j)$, in which $kj \in \mathrm{K}(i,j)$.

Step 1. Schedule all jobs in set Π_1

1-A. If $FLAG2 = 0$, select $j^* = argmin_{j \in \Pi_1}\{\mathrm{SLACK}\}$, breaking ties by selecting the one with the smallest order quantity first (breaking any second level tie by selecting the job not allowed to use substitutes and breaking any third level tie by selecting the job that uses the least total number of A^+ parts). Otherwise, select $j^* = argmin_{j \in \Pi_1}\{\mathrm{ORDER}(j)\}$.

1-B. Select subassembly (i^*, j^*) as the one with the earliest KITDUE, breaking ties by selecting the one that uses the least total number of A^+ parts. Set KITDAY(i^*, j^*) as early as possible without violating rules (3)–(8) and constraints (2)–(4). The right hand sides of constraints (2)–(4) are updated whenever a kitting schedule for a subassembly is determined; so is ALLT(p).
1-C. Select subassembly (i^*, j^*) as the one with the latest KITDAY(i, j). Break the first ties by selecting subassembly $(i^*, j^*) = \text{argmax}_{i \in I(j^*)}$ $\{\text{KITDUE}(i, j^*)\}$ and break any second level tie by selecting subassembly $(i^*, j^*) = \text{argmax}_{i \in I(j^*)}\{H_{ij}\}$.

1-C-1. For $i^* = 1$
If KITDAY$(1, j^*) <$ KITDUE$(1, j^*)$, shift KITDAY$(1, j^*)$ as close to (but not later than) KITDUE$(1, j^*)$ as possible, without violating (2)–(4) and rules (1)–(2). Otherwise, KITDAY$(1, j^*)$ is not changed.

1-C-2. For $i^* > 1$
If KITDAY$(i^*, j^*) <$ KITDAY$(k, j^*) - \sigma_{i^*j^*}$, in which $k \in K(i^*, j^*)$, set KITDAY(i^*, j^*) as close to (but not later than) KITDAY$(k, j^*) - \sigma_{i^*j^*}$ as possible, observing (2)–(4) and rules (1)–(2). Otherwise, KITDAY(i^*, j^*) is not changed.

1-D. Set $\Pi_1 \leftarrow \Pi_1 \backslash \{j^*\}$. If $\Pi_1 \neq \phi$, go to step 1. Otherwise $(\Pi_1 = \phi)$, set $D = \max_j \text{KITDAY}(1, j)$ and go to step 1-E.
1-E. If FLAG1 = 1, set FLAG1 = 0, record the kitting schedule and calculate the resulting total cost. If FLAG2 = 0, go to step 2. Otherwise, (FLAG2 = 1), got to step 3.

Step 2. Determine the ORDER of jobs for the adjusted schedule.
2-A. For $j \in \Pi_2$, calculate $TC_j = T_j \max\{0, D + \sigma_{1j} - \gamma_j - 1\}$.
2-B. Select the job with $j+ = \mathit{argmin}_{j \in \Pi_2}\{TC_j\}$, breaking ties by setting $j+ = \mathit{argmax}_{j \in \Pi_2}\{Q_j\}$ (breaking any second level tie by selecting the one allowed to use substitutes and breaking any third level tie by selecting the one that uses the largest total number of A^+ parts). Update $\Pi_2 \leftarrow \Pi_2 \backslash \{j+\}$ Set ORDER(j+) = POSITION, and POSITION ← POSITION − 1.
2-C. If POSITION > 1, set FLAG = 0, $\Pi_1 = \Pi_2$ and go to step 1. Otherwise, (POSITION = 1) proceed to step 2-D.
2-D. Let j+ be the remaining job in Π_2ORDER$(j+) = 1, \Pi_1 = \Pi_2 = J$, FLAG1 = 1, FLAG2 = 1, reinitialize constraints (2)–(4) and ALLT(p), for every $p \in P$, and go to step 1.

Step 3. Determine the final schedule
If TCOST1 < TCOST2, the kitting schedule based on earliest-due-date-first is the final schedule; otherwise, the kitting schedule based on ORDER is the final schedule. Stop.

To begin, Π_1, Π_2, POSITION, FLAG1, FLAG2, and ALLT(p) are initiatives (step 0-A) and each job slack and the kitting due date for each subassembly are calculated in step 0-B. In step 1-A, a job is selected to determine its kitting schedule. In the first $|J|$ passes, the job in Π_1, with the smallest job slack is selected first, breaking ties by selecting the one with the smallest order quantity first (breaking any second level tie by selecting the job not allowed to use substitutes and breaking any third level tie by selecting the job that uses the least total number of A^+ parts). In the $(|J| + 1)$th pass, jobs are selected according to sequence position (i.e. ORDER), which is determined during the first $|J|$ passes.

For the selected job, j^*, the earliest feasible (i.e. without violating constraints (2)–(4) and rules (3)–(8)] kitting schedules for all subassemblies are determined in step 1-B. The subassembly with the earliest KITDUE(i, j^*) among all subassemblies of the selected job j^* is scheduled first. A tie is broken by giving priority to the subassembly that uses the least total number of A^+ parts. In step 1-C, the kitting schedules for all subassemblies (and assembly) of the selected job are refined (if possible) to reduce job earliness cost and subassembly holding cost, and to avoid violating rules (1) and (2). The subassembly with the latest kitting day is rescheduled first, breaking any first level tie by selecting subassembly $(i^*, j^*) = \text{argmax}_{i\in \mathrm{I}(j^*)}$ $\{\text{KITDUE}(i, j^*)\}$ and breaking any second level tie by selecting subassembly $(i^*, j^*) = \text{argmax}_{i\in \mathrm{I}(j^*)}\{\mathrm{H}_{ij^*}\}$. In step 1-C-1, KITDAY$(1, j^*)$ is rescheduled to be as close as possible to $\gamma_j - S_{1j^*} + 1$. In step 1-C-2, KITDAY$(i^*, j^*), i^* \neq 1$, is rescheduled to be as close as possible to the day when its immediate successor requires it (i.e. KITDAY$(k, j^*) - \sigma_{i^*j^*}$), so that its completion does not delay the kitting of its immediate successor. In step 1-D, Π_1 is updated. If all jobs in Π_1 have been scheduled, the kitting horizon (i.e. D) relative to all jobs in Π_1 is determined and the procedure advances to step 1-E. Otherwise, another job is selected to schedule. At the end of the first and the $(|\mathrm{J}| + 1)$th passes, the kitting days of all subassemblies and total costs are recorded in step 1-E. If the adjusted schedule has been obtained, the procedure jumps to step 3 to determine the final schedule. Otherwise, this procedure advances to step 2 to determine the ORDER of another job.

Assuming that it is kitted on day D, the tardiness cost for a job is calculated in step 2-A. In step 2-B, the job $j+$ which would incur the smallest tardiness cost if it were kitted on day D is selected first to determine its sequence position (i.e. ORDER) in the $(|\mathrm{J}| + 1)$th pass, breaking ties by selecting the job $j+ = argmax_{j\in\Pi_2}\{\mathrm{N}_j\}$ (breaking any second level tie by selecting the one allowed to use substitutes and breaking any third level tie by selecting the one that uses the largest total number of A^+ parts). Π_2 and POSITION are also updated in step 2-B. If the sequence positions of more than one job in Π_2 have not been determined, Π_2 is updated and the procedure returns to step 1. Otherwise, POSITION is set to 1 and the procedure proceeds to step 2-D. In step 2-D, the sequence position of job $j+$, whose sequence position has not been determined, is set to 1. Both FLAG1 and FLAG2 are set to 1. Constraints (2)–(4) and ALLT(p) are reinitialized, and the procedure returns to step 1 for the $|\mathrm{J} + 1|$th pass to determine the adjusted schedule. In step 3, the resulting total costs based on earliest-due-date-first and ORDER are compared to determine the final kitting schedule. If TCOST1 < TCOST2, the kitting schedule based on earliest-due-date-first is the final schedule. Otherwise, the kitting schedule based on ORDER is the final schedule. After the final schedule has been determined, the procedure terminates in step 3.

The complexity of the presented procedure is analysed as follows:

Step 1. 1-A: The job with the smallest job slack [or sequence position] is selected first. This takes $O(|\mathrm{J}|log|\mathrm{J}|)$.

1-B: The subassembly with the earliest KITDUE(i, j) is selected first. The complexity of step 1-B is $O(\max_j \{|\mathrm{I}(j)|log|\mathrm{I}(j)|\})$. Since there are $|\mathrm{J}|$ jobs, the complexity is $O(|\mathrm{J}| \max_j \{|\mathrm{I}(j)|log|\mathrm{I}(j)|\})$.

1-C: The subassembly with the latest KITDAY(i, j) is selected first. This also takes $O(\max_j \{|\mathrm{I}(j)|log|\mathrm{I}(j)|\})$. Since there are $|\mathrm{J}|$ jobs, the complexity

is $O(|J| \max_j \{|I(j)| log|I(j)|\})$.
1-D: $D = \max_j \{KITDAY(1, j)\}$. This takes $O(|J|)$.

The complexity of step 1 is $O(\max \{|J| \max_j \{|I(j)| log|I(j)|\}, |J| log|J|\})$.

Step 2. 2-B: the job with the smallest TC_j is determined. This takes $O(|J|)$.

The complexity of step 2 is $O(|J|)$ and the procedure takes $O(\max \{|J| \max_j \{|I(j)| log|I(j)|\}, |J| log|J|\})$ for one pass. Since the heuristic procedure requires $|J| + 1$ passes, the complexity of H1 is

$$O(|J| \max \{|J| \max_j \{|I(j)| log|I(j)|\}, \quad |J| log|J|\}).$$

A numerical example which illustrates the heuristic is given in appendix B.

5. Computational results

In this section, a set of test problems is used to test the computational characteristics of heuristic H1. The solution quality of H1 is compared with H2 (dedicated), H3 (compete), and Chen and Wilhelm's procedure, (note that Chen and Wilhelm's procedure (H4) does not allow parts to be substituted) using 320 test problems. All of the computational experiments were performed on a DX2-66 486PC, running MS-DOS. Procedures which implement the methods discussed in this paper were coded in C.

5.1. *Data sets*

Since the literature contains no 'standard' test data for this type of kitting problem, the test problems were randomly generated to study the effects of the following factors:

(1) The magnitude of |E| (number of echelons),
(2) Part delivery time distributions,
(3) The number of different part types in class A^+ required by subassemblies,
(4) Shop capacities (R_{sd}),
(5) The number of substitution pairs in each echelon,
(6) The proportion of jobs allowed to use substitutes, and
(7) The magnitude of |J| (number of jobs).

It is known that all these factors can affect the size and complexity of the kitting problem. The magnitude of |E| reflects the assembly structure and the part delivery time distribution influences the subassembly kitting. The number of each different part type in the A^+ class required by a subassembly dictates the density of constraint (3). The capacity of each shop reflects number of subassemblies that can be kitted each day and thus affects the competition of kits. The number of substitution pairs in each echelon and the proportion of jobs allowed to use substitutes affect part substitution and thus affect the competition of kits. Finally, the magnitude of |J| affects the size and complexity of the kitting problem.

Based on these factors, 320 test problems were generated to evaluate the heuristics. Parameters v_{ij}, I, σ_{ij}, γ_j, and T_j are given in Table 1 and the customer order quantity (i.e., Q_j) distribution is given in Table 2.

Contributed value (v_{ij})	$= \$200\ Q_j$ if $ij \notin SE(E_j,j)$ and $E_j = 3$ $= \$100\ Q_j$ otherwise
Carrying cost (I)	$= 0{\cdot}20$ \$/\$ in inventory/year
Sojourn time (σ_{ij})	$= 4$ days if $ij \in SE(E_j,j)$ $= 3$ days if $ij \notin SE(E_j - 1, j)$ $= 2$ days if $ij \in SE(E_j - 2, j)$
Due date (γ_j)	$= \max_{mj \in SE(E_j,j)} \{\sum_{aj \in PH(mj,1j)} \sigma_{aj}\} - 1 +$ slack time generated from U[0, 4]
Tardiness cost (T_j)	$= 2H_{1j}$

Table 1. Parameters v_{ij}, R_{sd}, I, σ_{ij}, γ_j, and T_j.

Order quantity (N_j)	30	35	40	45	50
Probability	0·2	0·2	0·2	0·2	0·2

Table 2. Customer order quantity distribution.

5.2. *Results*

Computational results are reported in this subsection. Table 3 lists test parameters and solution values for heuristics H1, H2, H3, and H4. The second column in Table 3 indicates the number of echelons of the binary assembly tree (in the cases E = 3, a total of $|P| = 70$ different A^+ part types were available with 10, 20, and 40 part types used exclusively by echelons 1, 2, and 3, respectively; in the cases of E = 2, a total of $|P| = 60$ different A^+ part types were available with 20 and 40 part types used exclusively by echelons 1 and 2, respectively). The third column illustrates whether part delivery time distribution was, according to Tables 4, ('early') or 5 ('late'). The fourth column indicates the number of different A^+ part types required by each subassembly (in the case of E = 3, different part types required by each subassembly were generated from $U[2,4][U[3,5]]$ for $i \in SE(E-2, j)$, $U[4,8][U[6,10]]$ for $i \in SE(e-1, j)$, and $U[8,16][U[12,20]]$ for $i \in SE(E, j)$, respectively, when PR is 'few' ['many']; in the case of E = 2, different part types required by each subassembly were generated from $U[4,8][U[6,10]]$ for $i \in SE(E-1, j)$, and $U[8,16]U[[12,20]]$ for $i \in SE(E-2, j)$, respectively, when PR is 'few' ['many']. The number of each required part type was then generated from $Q_j\,U[2,4][Q_j\,U[1,2]]$ if $i \in SE(E, j)[i \notin SE(E, j)]$). The fifth column states shop capacities (in the case of E = 3, shop capacities in echelon 1, 2, and 3 were $|J| + 3[|J|/2 + 3]$ $|J|/2 + 3\ [|J|/4 + 3]$ and $|J|/4 + 3\ [|J|/8 + 3]$, respectively, when SC is 'loose' ['tight']; in the case of E = 2 shop capacities in echelon 1 and 2 were $|J|/2 + 3\ [|J|/4 + 3]$ and $|J|/4 + 3\ [|J|/8 + 3]$, respectively, when SC is 'loose' ['tight']). The sixth column indicates the number of substitution pairs (in the case of E = 3, the number of substitution pairs were generated from $U[2,3][U[3,4]]$ for $i \in SE(E-2, j)$, $[U[5,7][U[7,9]]$ for $i \in SE(E-1, j)$, and $U[11,13][U[13,15]]$ for $i \in SE(E, j)$, respectively, when NSP is 'few' ['many']; in the case of E = 2, the number of substitution pairs were generated from $U[5,7][U[7,9]]$ for $i \in SE(E-1, j)$ and $U[11,13](U[13,15])$ for $i \in SE(E, j)$, respectively, when NSP is 'few' ['many']). The seventh column demonstrates the proportion of jobs in

								H1	H2	H3	H4
NO.	NE	PD	PR	SC	NS	PJ	NJ	TC	TC	TC	TC
1	2	T4	F	LS	F	S	40	677	886	779	711
2	2	T4	F	LS	F	S	50	658	813	741	784
3	2	T4	F	LS	F	S	60	912	1237	1096	998
4	2	T4	F	LS	F	S	70	1054	1439	1367	1115
5	2	T4	F	LS	F	S	80	988	1379	1340	1000
6	2	T4	F	LS	F	L	40	677	840	781	711
7	2	T4	F	LS	F	L	50	594	719	746	784
8	2	T4	F	LS	F	L	60	859	1192	1103	998
9	2	T4	F	LS	F	L	70	1004	1317	1276	1115
10	2	T4	F	LS	F	L	80	916	1236	1183	1000
11	2	T4	F	LS	M	S	40	666	781	868	711
12	2	T4	F	LS	M	S	50	727	898	854	784
13	2	T4	F	LS	M	S	60	880	1223	1132	998
14	2	T4	F	LS	M	S	70	1039	1442	1392	1115
15	2	T4	F	LS	M	S	80	909	1439	1281	1000
16	2	T4	F	LS	M	L	40	650	777	882	711
17	2	T4	F	LS	M	L	50	666	805	835	784
18	2	T4	F	LS	M	L	60	832	1101	991	998
19	2	T4	F	LS	M	L	70	988	1291	1289	1115
20	2	T4	F	LS	M	L	80	783	1221	1087	1000
21	2	T4	F	T	F	S	40	677	886	779	711
22	2	T4	F	T	F	S	50	658	813	741	784
23	2	T4	F	T	F	S	60	912	1237	1096	998
24	2	T4	F	T	F	S	70	1054	1439	1367	1115
25	2	T4	F	T	F	S	80	988	1379	1340	1000
26	2	T4	F	T	F	L	40	677	840	781	711
27	2	T4	F	T	F	L	50	594	719	746	784
28	2	T4	F	T	F	L	60	859	1192	1103	998
29	2	T4	F	T	F	L	70	1004	1317	1276	1115
30	2	T4	F	T	F	L	80	916	1236	1183	1000
31	2	T4	F	T	M	S	40	666	781	868	711
32	2	T4	F	T	M	S	50	727	898	854	784
33	2	T4	F	T	M	S	60	880	1223	1132	998
34	2	T4	F	T	M	S	70	1039	1442	1392	1115
35	2	T4	F	T	M	S	80	909	1439	1281	1000
36	2	T4	F	T	M	L	40	650	777	882	711
37	2	T4	F	T	M	L	50	666	805	845	784
38	2	T4	F	T	M	L	60	832	1101	991	998
39	2	T4	F	T	M	L	70	988	1291	1289	1115
40	2	T4	F	T	M	L	80	783	1221	1087	1000
41	2	T4	M	LS	F	S	40	546	723	634	696
42	2	T4	M	LS	F	S	50	822	933	984	867
43	2	T4	M	LS	F	S	60	711	894	874	776
44	2	T4	M	LS	F	S	70	1021	1236	1230	1154
45	2	T4	M	LS	F	S	80	1043	1405	1403	1226
46	2	T4	M	LS	F	L	40	484	590	557	696
47	2	T4	M	LS	F	L	50	782	916	959	867
48	2	T4	M	LS	F	L	60	665	889	916	776
49	2	T4	M	LS	F	L	70	955	1158	1158	1154
50	2	T4	M	LS	F	L	80	999	1337	1373	1226
51	2	T4	M	LS	M	S	40	558	734	696	696
52	2	T4	M	LS	M	S	50	732	900	873	867
53	2	T4	M	LS	M	S	60	735	986	898	776

								H1	H2	H3	H4
NO.	NE	PD	PR	SC	NS	PJ	NJ	TC	TC	TC	TC
54	2	T4	M	LS	M	S	70	1030	1251	1340	1154
55	2	T4	M	LS	M	S	80	1012	1462	1296	1226
56	2	T4	M	LS	M	L	40	510	660	628	696
57	2	T4	M	LS	M	L	50	679	850	825	867
58	2	T4	M	LS	M	L	60	711	908	848	776
59	2	T4	M	LS	M	L	70	954	1189	1340	1154
60	2	T4	M	LS	M	L	80	952	1313	1296	1226
61	2	T4	M	T	F	S	40	546	723	634	696
62	2	T4	M	T	F	S	50	822	933	984	867
63	2	T4	M	T	F	S	60	711	894	874	776
64	2	T4	M	T	F	S	70	1021	1236	1230	1154
65	2	T4	M	T	F	S	80	1043	1405	1403	1226
66	2	T4	M	T	F	L	40	484	590	557	696
67	2	T4	M	T	F	L	50	782	916	959	867
68	2	T4	M	T	F	L	60	665	889	916	776
69	2	T4	M	T	F	L	70	955	1158	1158	1154
70	2	T4	M	T	F	S	80	999	1337	1373	1226
71	2	T4	M	T	M	S	40	558	734	696	696
72	2	T4	M	T	M	S	50	732	900	873	867
73	2	T4	M	T	M	S	60	735	986	898	776
74	2	T4	M	T	M	S	70	1030	1251	1304	1154
75	2	T4	M	T	M	S	80	1012	1462	1296	1226
76	2	T4	M	T	M	L	40	510	676	628	696
77	2	T4	M	T	M	L	50	679	850	825	867
78	2	T4	M	T	M	L	60	711	908	848	776
79	2	T4	M	T	M	L	70	954	1189	1162	1154
80	2	T4	M	T	M	L	80	952	1313	1217	1226
81	2	T5	F	LS	F	S	40	1513	1716	1657	1599
82	2	T5	F	LS	F	S	50	1793	2125	1904	1896
83	2	T5	F	LS	F	S	60	2102	2554	2369	2396
84	2	T5	F	LS	F	S	70	2407	2892	2778	2527
85	2	T5	F	LS	F	S	80	2584	3098	2991	2632
86	2	T5	F	LS	F	L	40	1513	1662	1659	1599
87	2	T5	F	LS	F	L	50	1729	1973	1841	1986
88	2	T5	F	LS	F	L	60	2040	2470	2319	2396
89	2	T5	F	LS	F	L	70	2359	2784	2688	2527
90	2	T5	F	LS	F	L	80	2489	2930	2773	2632
91	2	T5	F	LS	M	S	40	1525	1676	1712	1599
92	2	T5	F	LS	M	S	50	1771	2203	2114	1896
93	2	T5	F	LS	M	S	60	2118	2582	2324	2396
94	2	T5	F	LS	M	S	70	2377	2830	2723	2527
95	2	T5	F	LS	M	S	80	2565	3181	2931	2632
96	2	T5	F	LS	M	L	40	1502	1678	1716	1599
97	2	T5	F	LS	M	L	50	1699	2018	2020	1896
98	2	T5	F	LS	M	L	60	2032	2457	2257	2396
99	2	T5	F	LS	M	L	70	2267	2712	2641	2527
100	2	T5	F	LS	M	L	80	2441	2838	2722	2632
101	2	T5	F	T	F	S	40	1513	1716	1657	1599
102	2	T5	F	T	F	S	50	1793	2125	1904	1896
103	2	T5	F	T	F	S	60	2102	2554	2369	2396
104	2	T5	F	T	F	S	70	2407	2892	2778	2527
105	2	T5	F	T	F	S	80	2584	3098	2991	2632
106	2	T5	F	T	F	L	40	1513	1662	1659	1599

Table 3 (*continued*)

NO.	NE	PD	PR	SC	NS	PJ	NJ	H1 TC	H2 TC	H3 TC	H4 TC
107	2	T5	F	T	F	L	50	1729	1973	1841	1896
108	2	T5	F	T	F	L	60	2040	2470	2319	2396
109	2	T5	F	T	F	L	70	2359	2784	2688	2527
110	2	T5	F	T	F	L	80	2489	2930	2773	2632
111	2	T5	F	T	M	S	40	1525	1676	1712	1599
112	2	T5	F	T	M	S	50	1771	2203	2114	1896
113	2	T5	F	T	M	S	60	2118	2582	2324	2396
114	2	T5	F	T	M	S	70	2377	2830	2723	2527
115	2	T5	F	T	M	S	80	2565	3181	2931	2632
116	2	T5	F	T	M	L	40	1502	1678	1716	1599
117	2	T5	F	T	M	L	50	1699	2018	2020	1896
118	2	T5	F	T	M	L	60	2032	2457	2257	2396
119	2	T5	F	T	M	L	70	2267	2712	2641	2527
120	2	T5	F	T	M	L	80	2441	2838	2722	2632
121	2	T5	M	LS	F	S	40	1298	1457	1343	1431
122	2	T5	M	LS	F	S	50	1669	1915	1883	1767
123	2	T5	M	LS	F	S	60	1706	1992	1939	1807
124	2	T5	M	LS	F	S	70	2287	2578	2486	2453
125	2	T5	M	LS	F	S	80	2571	2977	2962	2786
126	2	T5	M	LS	F	L	40	1205	1421	1313	1431
127	2	T5	M	LS	F	L	50	1661	1919	1822	1767
128	2	T5	M	LS	F	L	60	1661	1945	1913	1807
129	2	T5	M	LS	F	L	70	2199	2491	2457	2453
130	2	T5	M	LS	F	L	80	2533	2926	2849	2786
131	2	T5	M	LS	M	S	40	1336	1547	1418	1431
132	2	T5	M	LS	M	S	50	1635	1831	1840	1767
133	2	T5	M	LS	M	S	60	1644	1983	1855	1807
134	2	T5	M	LS	M	S	70	2196	2523	2419	2453
135	2	T5	M	LS	M	S	80	2424	2991	2727	2786
136	2	T5	M	LS	M	L	40	1277	1504	1403	1431
137	2	T5	M	LS	M	L	50	1626	1813	1813	1767
138	2	T5	M	LS	M	L	60	1576	1892	1746	1807
139	2	T5	M	LS	M	L	70	2093	2414	2285	2453
140	2	T5	M	LS	M	L	80	2333	2784	2610	2786
141	2	T5	M	T	F	S	40	1298	1457	1343	1431
142	2	T5	M	T	F	S	50	1669	1915	1883	1767
143	2	T5	M	T	F	S	60	1706	1992	1939	1807
144	2	T5	M	T	F	S	70	2287	2578	2486	2453
145	2	T5	M	T	F	S	80	2571	2977	2962	2786
146	2	T5	M	T	F	L	40	1205	1421	1313	1431
147	2	T5	M	T	F	L	50	1661	1919	1822	1767
148	2	T5	M	T	F	L	60	1661	1945	1913	1807
149	2	T5	M	T	F	L	70	2199	2491	2457	2453
150	2	T5	M	T	F	L	80	2533	2926	2849	2786
151	2	T5	M	T	M	S	40	1336	1547	1418	1431
152	2	T5	M	T	M	S	50	1635	1831	1840	1767
153	2	T5	M	T	M	S	60	1644	1983	1855	1807
154	2	T5	M	T	M	S	70	2196	2523	2419	2453
155	2	T5	M	T	M	S	80	2424	2991	2727	2786
156	2	T5	M	T	M	L	40	1277	1504	1403	1431
157	2	T5	M	T	M	L	50	1635	1831	1840	1767
158	2	T5	M	T	M	L	60	1644	1983	1855	1807
159	2	T5	M	T	M	L	70	2196	2523	2419	2453

								H1	H2	H3	H4
NO.	NE	PD	PR	SC	NS	PJ	NJ	TC	TC	TC	TC
160	2	T5	M	T	M	L	80	2424	2991	2727	2786
161	3	T4	F	LS	F	S	20	1088	1368	1312	1162
162	3	T4	F	LS	F	S	25	1344	1868	1688	1353
163	3	T4	F	LS	F	S	30	1489	1772	1809	1694
164	3	T4	F	LS	F	S	35	1709	2334	2351	1819
165	3	T4	F	LS	F	S	40	1576	2304	2014	1894
166	3	T4	F	LS	F	L	20	1053	1369	1324	1162
167	3	T4	F	LS	F	L	25	1309	1795	1647	1353
168	3	T4	F	LS	F	L	30	1445	1729	1715	1694
169	3	T4	F	LS	F	L	35	1644	2270	2302	1819
170	3	T4	F	LS	F	L	40	1548	2342	2055	1894
171	3	T4	F	LS	M	S	20	1004	1245	1120	1162
172	3	T4	F	LS	M	S	25	1286	1686	1575	1353
173	3	T4	F	LS	M	S	30	1463	1892	1812	1694
174	3	T4	F	LS	M	S	35	1542	2003	2001	1819
175	3	T4	F	LS	M	S	40	1500	2190	2165	1894
176	3	T4	F	LS	M	L	20	908	1177	978	1162
177	3	T4	F	LS	M	L	25	1186	1674	1500	1353
178	3	T4	F	LS	M	L	30	1463	1807	1807	1694
179	3	T4	F	LS	M	L	35	1467	1848	1862	1819
180	3	T4	F	LS	M	S	40	1503	2030	1993	1894
181	3	T4	F	T	F	S	20	1096	1364	1304	1193
182	3	T4	F	T	F	S	25	1398	1829	1686	1354
183	3	T4	F	T	F	S	30	1629	1900	1951	1694
184	3	T4	F	T	F	S	35	1709	2318	2315	1819
185	3	T4	F	T	F	S	40	1678	2376	2143	1904
186	3	T4	F	T	F	L	20	1061	1365	1316	1193
187	3	T4	F	T	F	L	25	1316	1773	1642	1354
188	3	T4	F	T	F	L	30	1581	1860	1904	1694
189	3	T4	F	T	F	L	35	1644	2249	2268	1819
190	3	T4	F	T	F	L	40	1753	2419	2240	1904
191	3	T4	F	T	M	S	20	1116	1225	1146	1193
192	3	T4	F	T	M	S	25	1291	1678	1568	1354
193	3	T4	F	T	M	S	30	1566	1931	1893	1694
194	3	T4	F	T	M	S	35	1644	2249	2268	1819
195	3	T4	F	T	M	S	40	1753	2419	2240	1904
196	3	T4	F	T	M	L	20	942	1154	1030	1193
197	3	T4	F	T	M	L	25	1257	1669	1492	1354
198	3	T4	F	T	M	L	30	1566	1885	1888	1694
199	3	T4	F	T	M	L	35	1467	1848	1862	1819
200	3	T4	F	T	M	L	40	1616	2144	2079	1904
201	3	T4	M	LS	F	S	20	1009	1223	1255	1101
202	3	T4	M	LS	F	S	25	1277	2071	1560	1383
203	3	T4	M	LS	F	S	30	1384	2219	1657	1452
204	3	T4	M	LS	F	S	35	1598	2039	1903	1739
205	3	T4	M	LS	F	S	40	1415	2386	1938	1815
206	3	T4	M	LS	F	L	20	921	1119	1106	1101
207	3	T4	M	LS	F	L	25	1277	1888	1573	1383
208	3	T4	M	LS	F	L	30	1280	1772	1550	1452
209	3	T4	M	LS	F	L	35	1556	1874	1833	1739
210	3	T4	M	LS	F	L	40	1394	1970	1804	1815
211	3	T4	M	LS	M	S	20	1018	1130	1260	1101
212	3	T4	M	LS	M	S	25	1155	2003	1423	1383

Table 3 (*continued*)

								H1	H2	H3	H4
NO.	NE	PD	PR	SC	NS	PJ	NJ	TC	TC	TC	TC
213	3	T4	M	LS	M	S	30	1395	2504	1832	1452
214	3	T4	M	LS	M	S	35	1595	2344	2092	1739
215	3	T4	M	LS	M	S	40	1394	1970	1804	1815
216	3	T4	M	LS	M	L	20	878	1068	1099	1101
217	3	T4	M	LS	M	L	25	1059	1680	1362	1383
218	3	T4	M	LS	M	L	30	1268	1875	1585	1452
219	3	T4	M	LS	M	L	35	1600	2085	1970	1739
220	3	T4	M	LS	M	L	40	1299	2191	1965	1815
221	3	T4	M	T	F	S	20	1044	1280	1226	1103
222	3	T4	M	T	F	S	25	1280	2059	1557	1383
223	3	T4	M	T	F	S	30	1434	2302	1672	1456
224	3	T4	M	T	F	S	35	1659	2067	1939	1739
225	3	T4	M	T	F	S	40	1418	2476	1923	1817
226	3	T4	M	T	F	L	20	909	1128	1146	1103
227	3	T4	M	T	F	L	25	1280	1876	1573	1383
228	3	T4	M	T	F	L	30	1351	1807	1560	1456
229	3	T4	M	T	F	L	35	1654	1930	1835	1739
230	3	T4	M	T	F	L	40	1395	1982	1772	1817
231	3	T4	M	T	M	S	20	1053	1167	1253	1103
232	3	T4	M	T	M	S	25	1203	2030	1397	1383
233	3	T4	M	T	M	S	30	1427	2692	1755	1456
234	3	T4	M	T	M	S	35	1653	2331	2092	1739
235	3	T4	M	T	M	S	40	1536	2314	1958	1817
236	3	T4	M	T	M	L	20	884	1105	1097	1103
237	3	T4	M	T	M	L	25	1061	1700	1368	1383
238	3	T4	M	T	M	L	30	1336	2000	1633	1456
239	3	T4	M	T	M	L	35	1657	2114	2013	1739
240	3	T4	M	T	M	L	40	1345	2270	1874	1817
241	3	T5	F	LS	F	S	20	2249	2669	2506	2398
242	3	T5	F	LS	F	S	25	2503	3057	2782	2534
243	3	T5	F	LS	F	S	30	3170	3786	3679	3467
244	3	T5	F	LS	F	S	35	3505	4354	4192	3764
245	3	T5	F	LS	F	S	40	3765	4755	4640	4306
246	3	T5	F	LS	F	L	20	2249	2677	2573	2398
247	3	T5	F	LS	F	L	25	2494	2919	2830	2534
248	3	T5	F	LS	F	L	30	3173	3651	3649	3467
249	3	T5	F	LS	F	L	35	3496	4196	4172	3764
250	3	T5	F	LS	F	L	40	3861	4797	4696	4306
251	3	T5	F	LS	M	S	20	2148	2450	2314	2398
252	3	T5	F	LS	M	S	25	2309	2697	2734	2534
253	3	T5	F	LS	M	S	30	3213	3825	3664	3467
254	3	T5	F	LS	M	S	35	3302	3884	3671	3764
255	3	T5	F	LS	M	S	40	3679	4279	4270	4306
256	3	T5	F	LS	M	L	20	2086	2399	2264	2398
257	3	T5	F	LS	M	L	25	2269	2684	2505	2534
258	3	T5	F	LS	M	L	30	3208	3657	3639	3467
259	3	T5	F	LS	M	L	35	3302	3789	3654	3764
260	3	T5	F	LS	M	L	40	3645	4128	4085	4306
261	3	T5	F	T	F	S	20	2284	2654	2550	2398
262	3	T5	F	T	F	S	25	2583	3114	2951	2538
263	3	T5	F	T	F	S	30	3172	3769	3674	3467
264	3	T5	F	T	F	S	35	3603	4396	4175	3764
265	3	T5	F	T	F	S	40	3924	4867	4728	4374

								H1	H2	H3	H4
NO.	NE	PD	PR	SC	NS	PJ	NJ	TC	TC	TC	TC
266	3	T5	F	T	F	L	20	2255	2656	2553	2398
267	3	T5	F	T	F	L	25	2574	2988	2909	2538
268	3	T5	F	T	F	L	30	3175	3636	3692	3467
269	3	T5	F	T	F	L	35	3550	4193	4148	3764
270	3	T5	F	T	F	L	40	3865	4875	4838	4374
271	3	T5	F	T	M	S	20	2214	2421	2339	2398
272	3	T5	F	T	M	S	25	2403	2733	2772	2538
273	3	T5	F	T	M	S	30	3241	3805	3699	3467
274	3	T5	F	T	M	S	35	3325	3876	3701	3764
275	3	T5	F	T	M	S	40	3759	4304	4261	4374
276	3	T5	F	T	M	L	20	2148	2366	2244	2398
277	3	T5	F	T	M	L	25	2348	2720	2503	2538
278	3	T5	F	T	M	L	30	3246	3634	3677	3467
279	3	T5	F	T	M	L	35	3325	3832	3716	3764
280	3	T5	F	T	M	L	40	3727	4119	4120	4374
281	3	T5	M	LS	F	S	20	2184	2436	2608	2245
282	3	T5	M	LS	F	S	25	2834	3266	3231	2863
283	3	T5	M	LS	F	S	30	2941	4210	3631	3120
284	3	T5	M	LS	F	S	35	3248	3795	2698	3562
285	3	T5	M	LS	F	S	40	3432	4410	3955	3985
286	3	T5	M	LS	F	L	20	2118	2279	2319	2245
287	3	T5	M	LS	F	L	25	2710	3079	3001	2863
288	3	T5	M	LS	F	L	30	2796	3558	3360	3120
289	3	T5	M	LS	F	L	35	3187	3914	3687	3562
290	3	T5	M	LS	F	L	40	3306	4082	3689	3985
291	3	T5	M	LS	F	S	20	2188	2288	2482	2245
292	3	T5	M	LS	F	S	25	2663	3061	3087	2863
293	3	T5	M	LS	F	S	30	3064	4599	3628	3120
294	3	T5	M	LS	F	S	35	3219	3937	3704	3562
295	3	T5	M	LS	F	S	40	3390	4013	4010	3985
296	3	T5	M	LS	F	L	20	2047	2225	2264	2245
297	3	T5	M	LS	F	L	25	2380	2836	2749	2863
298	3	T5	M	LS	F	L	30	3898	3841	3458	3120
299	3	T5	M	KS	F	L	35	3143	3777	3620	3562
300	3	T5	M	LS	F	L	40	3240	3871	3643	3985
301	3	T5	M	LS	F	S	20	2190	2407	2440	2245
302	3	T5	M	LS	F	S	25	2867	3254	3334	2863
303	3	T5	M	LS	F	S	30	3023	4216	3669	3122
304	3	T5	M	LS	F	S	35	3287	3815	3684	3562
305	3	T5	M	LS	F	S	40	3435	4403	3774	3987
306	3	T5	M	LS	F	L	20	2149	2325	2272	2245
307	3	T5	M	LS	F	L	25	2740	3073	3047	2863
308	3	T5	M	LS	F	L	30	2835	3662	3410	3122
309	3	T5	M	LS	F	L	35	3217	3940	3660	3562
310	3	T5	M	LS	F	L	40	3307	4073	3679	3987
311	3	T5	M	LS	F	S	20	2219	2336	2474	2245
312	3	T5	M	LS	F	S	25	2733	3136	3113	2863
313	3	T5	M	LS	F	S	30	3072	4583	3667	3122
314	3	T5	M	LS	F	S	35	3307	3972	3704	3562
315	3	T5	M	LS	F	S	40	3392	4003	3943	3987
316	3	T5	M	LS	F	L	20	2125	2317	2256	2245
317	3	T5	M	LS	F	L	25	2415	2846	2734	2863
318	3	T5	M	LS	F	L	30	2935	3864	3450	3122

Table 3 (*continued*)

								H1	H2	H3	H4
NO.	NE	PD	PR	SC	NS	PJ	NJ	TC	TC	TC	TC
319	3	T5	M	LS	F	L	35	3192	3856	3658	3562
320	3	T5	M	LS	F	L	40	3242	3914	3630	3987

NE: number of echelon
SC: shop capacity [T (tight), LS (loose)]
PD: part delivery time distribution [T4 (Table 4), T5 (Table 5)]
NS: number of substitution pairs [F (few). M (many)]
PR: number of A+ part types required by each subassembly [F (few), M (many)]
PJ: proportion of jobs allowed to use substitutes [S (small), L (large)]
NJ: number of jobs
T5: Table LS: loose T6: Table 6 T: Tight F: Few S: Small M: Many L: Large

Table 3. Solution value comparisons of H1, H2, H3, and H4

Delivery dist.	4 days early	3 days early	2 days early	1 day early	on time	1 day late	2 days late
Probability	0·10	0·15	0·15	0·20	0·20	0·10	0·10

Table 4. Part delivery time distribution ('early').

Delivery dist.	2 days early	1 day early	on time	1 day late	2 days late	3 days late	4 days late
Probability	0·10	0·20	0·30	0·15	0·10	0·10	0·05

Table 5. Part delivery time distribution ('late').

categories (1), (2), and (3) (the proportion of jobs in categories 1, 2, and 3 was 35% [24%], 35% [56%], and 30% [20%], respectively, when PJAS is 'small' ['large']). The eighth column indicates number of jobs.

According to Table 3, H1 obtained better solution values than did H2 and H3 in all test problems, and obtained better solution values than did H4 for 317 out of the 320 test problems. Overall, H2 and H3 did not obtain the solution quality of those obtained by H4, even though H4 did not allow parts to be substituted.

The reason that H2 and H3 did not prescribe solutions of the quality of those prescribed by H4 is that heuristics H2 and H3 violate rules (1) and (2). The reason that H1 did not perform as well as H4 on three problems (problems 182, 262, and 267) may be because parts which were supposed to be assigned to subassemblies of two (or more) jobs were used by subassemblies of a job as substitutes. Using part subsections to kit subassemblies of one job may result in kitting delays for subassemblies of other jobs and result in higher tardiness costs. H1 allows subassemblies of a job to 'steal' parts as substitutes from subassemblies of two or more jobs. If this happens and results in more tardiness costs, the solution value obtained from H1 may not be very good.

Table 6 shows the average performance of the four heuristics. According to Table 6, H1 obtained better average solution values than the other three heuristics in all

		H1	H2	H3	H4	H2 − H1	H3 − H1	H4 − H1
NE	2E	1376·70	1670·53	1601·45	1531·05	293·83	224·75	154·35
	3E	2148·53	2697·01	2536·03	2385·95	548·48	387·51	237·43
PD	Early	1084·42	1469·74	1364·14	1238·20	385·32	279·72	153·78
	Late	2440·81	2897·79	2773·34	2678·80	456·99	332·54	237·99
PR	Few	1820·67	2227·98	2145·26	2005·30	307·31	324·59	184·63
	Many	1704·56	2139·56	1992·22	1911·70	435·00	287·66	207·14
SC	Loose	1750·59	2176·50	2064·75	1956·93	425·91	314·16	206·33
	Tight	1774·63	2191·03	2072·73	1960·08	416·40	298·10	185·44
NS	Few	1788·96	2213·79	2102·78	1958·50	424·83	313·83	169·54
	Many	1736·27	2153·74	2034·70	1958·50	417·48	298·43	222·23
PJ	Small	1794·59	2248·88	2112·41	1958·50	454·28	317·82	163·91
	Large	1730·63	2118·66	2025·07	1958·50	388·03	294·44	227·87
NJ	20, 40	1292·36	1495·50	1461·42	1419·94	203·14	169·06	127·58
	25, 50	1551·41	1906·31	1798·45	1681·19	354·91	247·05	129·78
	30, 60	1790·73	2294·02	2106·70	1964·13	503·28	315·97	173·39
	35, 70	2052·61	2504·33	2419·77	2266·63	451·72	367·16	214·02
	40, 80	2125·95	2718·67	2557·36	2460·63	592·72	431·41	334·67
Overall average		1762·61	2183·77	2068·74	1958·50	421·15	306·13	195·89

Table 6. Mean value comparisons of H1, H2, H3, and H4.

different cases. H1 performed much better than did H2, H3, and H4 with E = 3, while H1 did not perform that much better than H2, H3, and H4 with E = 2. The magnitude of E reflects the assembly structure. The larger E is, the more important the coordination of material flows are. Hence, this experience may indicate that H1 may perform much better than H2, H3, and H4 when problems involve more complicated assembly structures as actually found in most assembly systems.

H1 performed much better than H2, H3, and H4 when the part delivery time distribution is 'late' (i.e., as indicated in Table 5). According to Table 5, part deliveries can be up to four days late (compared to 2 days late in Table 4) Part delivery delays create more competition for available parts. Hence, this experience may indicate that H1 may perform much better than H2, H3, and H4 when problems involve more competition for parts. H1 performed much better than H4 when the number of substitution pairs are 'many'. The larger the number of substitution pairs, the larger the 'chance' that subassemblies will use substitutes. Hence, this experience may indicate that H1 may perform better than H4 when problems involve a larger 'chance' of jobs to use substitutes.

H1 performed much better than H4 when the proportion of jobs in Categories 1, 2, and 3 is 24%, 56%, and 20%, respectively. The higher the proportion of jobs in Category 2 is, the more 'free' jobs can use substitutes. Hence, this experience may indicate that H1 may perform much better than H4 when problems involve more jobs which are allowed to use substitutes. H1 performed much better than H2, H3, and H4 when |J| is large. The magnitude of |J| affects the size and complexity of the problem. Hence, this experience may indicate that H1 may also perform much better than H2, H3, and H4 when problems involve large |J|.

6. Conclusions and suggestions for future research

This paper deals with the kitting problem encountered in multi-echelon, multi-product assembly systems with substitutable parts. A model for this type of kitting problem is presented along with rules for using substitutes and a heuristic solution procedure. The average performance of the heuristic is compared empirically with two heuristics commonly applied in industry, and Chen and Wilhelm's heuristic. A set of 320 test problems were generated to test the computational characteristics of our new heuristic, H1. These problems were randomly generated to study the effects of a set of factors:

(1) The magnitude of $|E|$ (number of echelons),
(2) Part delivery time distributions,
(3) The number of different part types in class A^+ required by subassemblies,
(4) Shop capacities (R_{sd}),
(5) The number of substitution pairs in each echelon,
(6) The proportion of jobs allowed to use substitutes, and
(7) The magnitude of $|J|$ (number of jobs).

Computational results indicate that H1 obtained better solution values than did H2 and H3 in all test problems, and H1 obtained better solution values than did H4 in 317 out of 320 test problems. H1 obtained better average solution values than the other three heuristics in all different factor cases.

The model presented in this paper gives a generic structure of the kitting problem with substitutable parts. The proposed rules for using substitutes, along with our new heuristic, are expected to contribute to the productivity of manufacturing managers in multi-echelon, multi-product assembly systems, particularly those engaged in the electronics industry. This research can be used by production and inventory control managers to better coordinate material flows, perhaps leading to dramatic economic improvements of controlling inventory and material flow through assembly and realizing the potential of automated assembly.

It is proposed that this generic model may be extended to incorporate additional features found in certain applications. An example is the feature that substitution may be linked in some applications (for example, using substitutes in one echelon resulting in the necessity of using substitutes in another echelon). Including this feature complicates the kitting problem, since it increases the difficulty of determining when to use substitutes. However, it does present a significant new challenge for developing heuristic procedures.

Appendix A: List of notation

INDICES

d	$= 1, \ldots, D$ index for days in the kitting horizon
e	$= 1, \ldots, E_j$ index of echelons in the assembly structure of job j
s	$= 1, \ldots, S$ index for shops
i	$\in I(j)$ a subassembly composing job *j*
j	$\in J$ a job (customer order)
p	$\in P$ an A^+ part

SETS

I(j)	set of subassemblies comprising job *j*
J	set of jobs
A(1, j)	set of predecessors of assembly 1*j*
K(i, j)	set of immediate successors of subassembly *ij*

M(i, j)	set of immediate predecessors of subassembly *ij*
P	set of A^+ parts
Π_1	set of all jobs
SE(e, j)	set of subassemblies of job *j* in echelon *e*
SS(s, j)	set of subassemblies of job *j* processed in shop *s*
Θ(p)	set of subassemblies that originally require part *p* or may use part *p* as substitutes
θ(i, j)	set of parts originally was required by subassembly *ij*
θ'(i, j)	set of parts that can be used as substitutes for subassembly *ij*

PARAMETERS

Costs

H_{ij}	holding cost/day for subassembly $ij[ij \in I(j)]$ in lot size $N_j = (I/365)V_{ij}$
I	annual inventory carrying cost rate ($/$in inventory/year)
T_j	tardiness cost/day for job *j*
V_{ij}	total value of completed subassembly $ij = \sum_{mj \in M(i,j)} V_{mj} + v_{ij}$
v_{ij}	contributed value added by assembling subassembly *ij*

Assembly networks

KITDAY(i, j)	the kitting day of subassembly *ij*
KITDUE(i, j)	the kitting due date of subassembly *ij*
Q_j	Order quantity of job j
q_{pij}	quantity of part *p* required by subassembly *ij* to assemble N_j end-products
PH(mj, ij)	the set of nodes in the path from node *mj* to node *ij* (inclusive)
σ_{ij}	sojourn time (integer number of days) to produce subassembly *ij* queuing time + processing time to produce subassembly *ij*
γ_j	due date of job *j*

Resources

1A	total number of part *p* (originally) required to complete customer orders in category (1)
IB	total number of part p' (originally) required to complete customer orders in category (1)
IC	total number of part *p* (originally) required to complete customer orders in category (2)
ID	total number of part p' (originally required to complete customer orders in category (2)
IE	total number of part *p* (originally) required to complete customer orders in category (3)
IF	total number of part p' (originally required to complete customer orders in category (3)
ALLT(p)	the sum of part *p* allocated to the jobs that required part *p* and cannot use part p' as substitutes
R_{sd}	capacity of shop *s* on day *d* (i.e. the number of subassemblies that can be released into shop *s* on day *d*)
N_{pd}	cumulative number of part *p* available through day $d = \sum_{t=0}^{d-1} n_{pt}$
n_{pd}	number of part *p* with delivery promised on day *d* ($d \geq 1$)

DECISION VARIABLES

$$X_{ijd} = \begin{cases} 1 \text{ if subassembly } ij \text{ uses all originally required parts and is kitted on day } d \\ 0 \text{ otherwise} \end{cases}$$

$$X'_{ijd} = \begin{cases} 1 \text{ if subassembly } ij \text{ uses any substitutes and is kitted on day } d \\ 0 \text{ otherwise} \end{cases}$$

$$Y_{cpijd} = \begin{cases} 1 \text{ if subassembly } ij \text{ uses } c \text{ units of originally required part, } p\text{, and is kitted on day } d \\ 0 \text{ otherwise} \end{cases}$$

$$Y'_{cpijd} = \begin{cases} 1 \text{ if subassembly } ij \text{ uses } c \text{ units of substitutes (part } p'\text{) and is kitted on day } d \\ 0 \text{ otherwise.} \end{cases}$$

Appendix B: A numerical example for heuristic H1

A numerical example is presented here to demonstrate heuristic H1. Assume that the sojourn times for subassemblies in echelons 3, 2, and 1 are 4, 3, and 2 days, and that shop capacities in shops 3, 2, and 1, are 4, 5, and 7, respectively. Data related to substitutes are given in Table 7. Jobs 1, 2, and 4 are allowed to use substitutes (bi-directional substitution) and job 3 is not. The order quantities for jobs 1, 2, 3, and 4 are 50, 45, 45, and 30, respectively, and the due dates for jobs 1, 2, 3, and 4 are on days 10, 13, 9, and 11, respectively. A^+ part requirements for each subassembly are given in Table 8, part delivery schedules are listed in Table 9, and the tardiness costs for each job and the holding costs for each subassembly are listed in Table 10.

Step 0. *0-A*. $\Pi_1 = \Pi_2 = \{1,2,3,4\}$, POSITION = 3, FLAG1 = 1, FLAG2 = 0, and ALLT(p) = 0, for every p in P.
0-B. $\text{SLACK}_1 = 1$, $\text{SLACK}_2 = 4$, $\text{SLACK}_3 = 0$, $\text{SLACK}_4 = 2$. KITDUE(1,1) = 9, KITDUE(2,1) = 6, KITDUE(3,1) = 6, KITDUE(4,1) = 2, KITDUE(5,1) = 2, KITDUE(6,1) = 2, KITDUE(7,1) = 2, KITDUE(1,2) = 12, KITDUE(2,2) = 9, KITDUE(3,2) = 9, KITDUE(4,2) = 5, KITDUE(5,2) = 5, KITDUE(6,2) = 5, KITDUE(7,2) = 5, KITDUE(1,3) = 8, KITDUE(2,3) = 5, KITDUE(3,3) = 5, KITDUE(4,3) = 1, KITDUE(5,3) = 1, KITDUE(6,3) = 1, KITDUE(7,3) = 1, KITDUE(1,4) = 10, KITDUE(2,4) = 7, KITDUE(3,4) = 7, KITDUE(4,4) = 3, KITDUE(5,4) = 3, KITDUE(6,4) = 3, KITDUE(4,4) = 3.

Step 1. 1-A. (FLAG1 = 1, FLAG2 = 0) $j^* = 3$.
1-B. KITDAY(7,3) = 1, KITDAY(4,3) = 1, KITDAY(6,3) = 1, KITDAY(5,3) = 1, KITDAY(3,3) = 5, KITDAY(2,3) = 5, KITDAY(1,3) = 8.
1-C. KITDAY(1,3) = 8, KITDAY(2,3) = 5, KITDAY(3,3) = 5, KITDAY (5,3) = 1, KITDAY(4,3) = 1, KITDAY(6,3) = 1, KITDAY(7,3) = 1

Part # (p, inferior parts)	Part # (p', superior parts)
4	1
5	3
6	2
10	9
11	8
14	13

Table 7. Substitution pairs.

Subassembly # (i, j)	Part #	q_{pij}	Subassembly #(i, j)	Part #	q_{pij}	Subassembly # (i, j)	Part #	q_{pij}
71	2	200	52	1	180	33	9	45
71	4	200	52	2	180	33	11	45
71	6	200	52	4	180	23	8	90
61	2	150	42	3	135	23	10	45
61	4	200	42	4	90	13	14	45
61	5	150	42	6	90	74	2	120
51	1	200	32	8	45	74	4	90
51	2	200	32	10	45	74	6	60
51	5	100	22	9	45	64	1	90
41	1	200	22	11	90	64	3	120
41	3	100	12	13	90	64	5	120
41	5	150	73	2	90	54	2	60
31	9	50	73	3	90	54	5	120
31	11	50	73	5	180	54	6	120
21	8	100	63	1	135	44	2	60
21	10	50	63	3	135	44	3	60
11	14	50	63	5	135	44	5	120
72	1	90	53	1	135	34	9	30
72	3	90	53	2	135	34	11	60
72	5	180	53	5	180	24	8	30
62	2	180	43	1	180	24	10	30
62	4	90	43	2	135	14	14	30
62	6	180	43	3	90			

Table 8. A^+ part requirements.

1-D. $\Pi_1 = \{1, 2, 4\}$.
1-A. (FLAG1 = 1, FLAG2 = 0) $j^* = 1$.
1-B. KITDAY(4,1) = 1, KITDAY(5,1) = 2, KITDAY(6,1) = 2, KITDAY(7,1) = 2, KITDAY(3,1) = 6, KITDAY(2,1) = 6, KITDAY(1,1) = 9. (Note: subassembly (4,1) uses 110 part 4 as substitutes for part 1, subassembly (7,1) uses 50 part 1 as substitutes for part 4, and subassembly (2,1) uses 20 part 9 as substitutes for part 10).
1-C. KITDAY(1,1) = 9, KITDAY(2,1) = 6, KITDAY(3,1) = 6, KITDAY(7,1) = 2, KITDAY(6,1) = 2, KITDAY(5,1) = 2, KITDAY(4,1) = 2. (Note: subassembly (2,1) still uses 20 part 9 as substitutes for part 10).
1-D. $\Pi_1 = \{2, 4\}$.
1-A. (FLAG1 = 1, FLAG2 = 0) $j^* = 4$.
1-B. KITDAY(4,4) = 1, KITDAY(7,4) = 2, KITDAY(5,4) = 3, KITDAY(6,4) = 3, KITDAY(2,4) = 8, KITDAY(3,4) = 8, KITDAY(1,4) = 11. (Note: subassembly (4,4) uses 60 part 5 as substitutes for part 3, and subassembly (6,4) uses 60 part 3 as substitutes for part 5).
1-C. KITDAY(1,4) = 11, KITDAY(3,4) = 8, KITDAY(2,4) = 8, KITDAY(6,4) = 4, KITDAY(5,4) = 4, KITDAY(7,4) = 4, KITDAY(4,4) = 4.
1-D. $\Pi_1 = \{2\}$.
1-A. (FLAG1 = 1,FLAG2 = 0) $j^* = 2$.
1-B. KITDAY(4,2) = 5, KITDAY(7,2) = 5, KITDAY(6,2) = 1, KITDAY(5,2) = 5, KITDAY(3,2) = 9, KITDAY(2,2) = 9, KITDAY(1,2) = 12.

p	d	W_{pd}	p	d	W_{pd}	p	d	W_{pd}	p	d	W_{pd}	p	d	W_{pd}
1	1	540	3	2	415	5	3	1255	8	5	220	10	9	170
1	2	940	3	3	640	5	4	1255	8	6	220	10	10	170
1	3	1210	3	4	640	5	5	1435	8	7	220	10	11	170
1	4	1210	3	5	820	5	6	1435	8	8	220	10	12	170
1	5	1210	3	6	820	5	7	1435	8	9	265	10	13	170
1	6	1210	3	7	820	5	8	1435	8	10	265	11	5	60
1	7	1210	3	8	820	5	9	1435	8	11	265	11	6	105
1	8	1210	3	9	820	5	10	1435	8	12	265	11	7	155
1	9	1210	3	10	820	5	11	1435	8	13	265	11	8	245
1	10	1210	3	11	820	5	12	1435	9	2	45	11	9	245
1	11	1210	3	12	820	5	13	1435	9	3	45	11	10	245
1	12	1210	3	13	820	6	1	410	9	4	45	11	11	245
1	13	1210	4	1	400	6	2	410	9	5	95	11	12	245
2	1	960	4	2	460	6	3	410	9	6	125	11	13	245
2	2	1510	4	3	460	6	4	410	9	7	125	13	12	90
2	3	1510	4	4	460	6	5	680	9	8	170	13	13	90
2	4	1510	4	5	820	6	6	680	9	9	170	14	7	30
2	5	1510	4	6	820	6	7	680	9	10	170	14	8	75
2	6	1510	4	7	820	6	8	680	9	11	170	14	9	125
2	7	1510	4	8	820	6	9	680	9	12	170	14	10	125
2	8	1510	4	9	820	6	10	680	9	13	170	14	11	125
2	9	1510	4	10	820	6	11	680	10	3	30	14	12	125
2	10	1510	4	11	820	6	12	680	10	4	75	14	13	125
2	11	1510	4	12	820	6	13	680	10	5	75			
2	12	1510	4	13	820	8	2	90	10	6	75			
2	13	1510	5	1	895	8	3	90	10	7	75			
3	1	415	5	2	1255	8	4	220	10	8	170			

Table 9. Part delivery schedule.

$T_1 = 44{\cdot}00$	$T_2 = 39{\cdot}60$	$T_3 = 39{\cdot}60$	$T_4 = 26{\cdot}40$
$H_{11} = 22{\cdot}00$	$H_{12} = 19{\cdot}80$	$H_{13} = 19{\cdot}80$	$H_{14} = 13{\cdot}20$
$H_{21} = 8{\cdot}00$	$H_{22} = 7{\cdot}20$	$H_{23} = 7{\cdot}20$	$H_{24} = 4{\cdot}80$
$H_{31} = 8{\cdot}00$	$H_{32} = 7{\cdot}20$	$H_{33} = 7{\cdot}20$	$H_{34} = 4{\cdot}80$
$H_{51} = 2{\cdot}75$	$H_{42} = 2{\cdot}48$	$H_{43} = 2{\cdot}48$	$H_{44} = 1{\cdot}65$
$H_{61} = 2{\cdot}75$	$H_{52} = 2{\cdot}48$	$H_{53} = 2{\cdot}48$	$H_{54} = 1{\cdot}65$
$H_{71} = 2{\cdot}75$	$H_{62} = 2{\cdot}48$	$H_{63} = 2{\cdot}48$	$H_{64} = 1{\cdot}65$
$H_{11} = 2{\cdot}75$	$H_{72} = 2{\cdot}48$	$H_{73} = 2{\cdot}48$	$H_{74} = 1{\cdot}65$

Table 10. Subassembly holding costs and job tardiness costs.

(Note: subassembly (6,2) uses 140 part 1 as substitutes for part 4 and 180 part 2 as substitutes for part 6, subassembly (5,2) uses 200 part 4 as substitutes for part 1 and 180 part 6 as substitutes for part 2 and subassembly (2,2) uses 20 part 10 as substitutes for part 9).

1-C. KITDAY(1,2) = 12, KITDAY(2,2) = 9, KITDAY(3,2) = 9, KITDAY(5,2) = 5, KITDAY(7,2) = 5, KITDAY(4,2) = 5, KITDAY(6,2) = 5. (Note: subassembly (2,2) still uses 20 part 10 as substitutes for part 9).

1-D. $\Pi_1 = \phi$. D = 12 and go to Step 1-E.

1-E. Since FLAG1 = 1, record the kitting schedule and calculate TCOST (TCOST1 = 26·4). Go to step 2 (FLAG2 = 0).

Step 2. *2-A*. $TC_1 = \max\{0, 44(12 + 2 - 1 - 10\} = 132$, $TC_2 = 0$, $TC_3 = 158{\cdot}4$, $TC_4 = 52{\cdot}8$.
2-B. j+ = 2, ORDER(4) = 2, and POSITION = 3.
2-C. $\Pi_1 = \Pi_2 = \{1, 3, 4\}$. Go to Step 1.

Step 1. *1-A*. (FLAG1 = 0, FLAG2 = 0) $j^* = 3$.
1-B. KITDAY(7,3) = 1, KITDAY(4,3) = 1, KITDAY(6,3) = 1, KITDAY(5,3) = 1, KITDAY(3,3) = 5, KITDAY(2,3) = 5, KITDAY(1,3) = 8
1-C. KITDAY(1,3) = 8, KITDAY(2,3) = 5, KITDAY(3,3) = 6, KITDAY(5,3) = 1, KITDAY(4,3) = 1, KITDAY(6,3) = 1, KITDAY(7,3) = 1.
1-D. $\Pi_1 = \{1, 4\}$.
1-A. (FLAG1 = 0, FLAG2 = 0) $j^* = 1$.
1-B. KITDAY(4,1) = 1, KITDAY(5,1) = 2, KITDAY(6,1) = 2, KITDAY(7,1) = 2, KITDAY(3,1) = 6, KITDAY(2,1) = 6, KITDAY(1,1) = 9 (Note: subassembly (4,1) uses 110 part 4 as substitutes for part 1, subassembly (7,1) uses 50 part 1 as substitutes for part 4, and subassembly (2,1) uses 20 part 9 as substitutes for part 10).
1-C. KITDAY(1,1) = 9, KITDAY(2,1) = 6, KITDAY(3,1) = 6, KITDAY(7,1) = 2, KITDAY(6,1) = 2, KITDAY(5,1) = 2, KITDAY(4,1) = 2. (Note: subassembly (2,1) still uses 20 part 9 as substitutes for part 10).
1-D. $\Pi_1 = \{4\}$.
1-A. (FLAG1 = 0, FLAG2 = 0) $j^* = 4$.
1-B. KITDAY(4,4) = 1, KITDAY(7,4) = 2, KITDAY(5,4) = 3, KITDAY(6,4) = 3, KITDAY(2,4) = 8, KITDAY(3,4) = 8, KITDAY(1,4) = 11. (Note: subassembly (4,4) uses 60 part 5 as substitutes for part 3, and subassembly (6,4) uses 60 part 3 as substitutes for part 5).
1-C. KITDAY(1,4) = 11, KITDAY(3,4) = 8, KITDAY(2,4) = 8, KITDAY(6,4) = 4, KITDAY(5,4) = 4, KITDAY(7,4) = 4, KITDAY(4,4) = 4.
1-D. $\Pi_1 = \phi$. D = 11 and go to Step 1-E.
1-E. Got to Step 2 (FLAG2 = 0).

Step 2. *2-A*. $TC_1 = \max\{0, 44(11 + 2 - 1 - 10\} = 88$, $TC_3 = 118{\cdot}8$, $TC_4 = 26{\cdot}4$.
2-B. j+ = 4, ORDER(3) = 4, and POSITION = 2.
2-C. $\Pi_1 = \Pi_2 = \{1, 3\}$. Go to Step 1.

Step 1. *1-A*. (FLAG1 = 0, FLAG2 = 0) $j^* = 3$.
1-B. KITDAY(7,3) = 1, KITDAY(4,3) = 1, KITDAY(6,3) = 1, KITDAY(5,3) = 1, KITDAY(3,3) = 5, KITDAY(2,3) = 5, KITDAY(1,3) = 8.
1-C. KITDAY(1,3) = 8, KITDAY(2,3) = 5, KITDAY(3,3) = 5, KITDAY(5,3) = 1, KITDAY(4,3) = 1, KITDAY(6,3) = 1, KITDAY(7,3) = 1.
1-D. $\Pi_1 = \{1\}$.
1-A. (FLAG1 = 0, FLAG2 = 0) $j^* = 1$.
1-B. KITDAY(4,1) = 1, KITDAY(5,1) = 2, KITDAY(6,1) = 2, KITDAY(7,1) = 2, KITDAY(3,1) = 6, KITDAY(2,1) = 6, KITDAY(1,1) = 9. (Note: subassembly (4,1) uses 110 part 4 as substitutes for part 1, subassembly (7,1) uses 50 part 1 as substitutes for part 4, and subassembly (2,1) uses 20 part 9 as substitutes for part 10).
1-C. KITDAY(1,1) = 9, KITDAY(2,1) = 6, KITDAY(3,1) = 6, KITDAY(7,1) = 2, KITDAY(6,1) = 2, KITDAY(5,1) = 2, KITDAY(4,1) = 2. (Note: subassembly (2,1) still uses 20 part 9 as substitutes for part 10).
1-D. $\Pi_1 = \phi$. D = 9 and got to Step 1-E.
1-E. Go to Step 2 (FLAG2 = 0).

Step 2. *2-A*. $TC_1 = \max\{0, 44(9+2-1-10)\} = 0$, $TC_3 = 26{\cdot}4$.
2-B. j+ = 1, ORDER(2) = 1, and POSITION = 1.
2-C. $\Pi_2 = \{3\}$. Go to Step 2-D.
2-D. ORDER(1) = 3, $\Pi_1 = \{1,2,3,4\}$, FLAG1 = 1, FLAG2 = 1. Go to Step 1.

Step 1. *1-A*. (FLAG1 = 1, FLAG2 = 1) $j^* = 3$.
1-B. KITDAY(7,3) = 1, KITDAY(4,3) = 1, KITDAY(6,3) = 1, KITDAY(5,3) = 1, KITDAY(3,3) = 5, KITDAY(2,3) = 5, KITDAY(1,3) = 8.
1-C. KITDAY(1,3) = 8, KITDAY(2,3) = 5, KITDAY(3,3) = 5, KITDAY(5,3) = 1, KITDAY(4,3) = 1, KITDAY(6,3) = 1, KITDAY(7,3) = 1.
1-D. $\Pi_1 = \{1,2,4\}$.
1-A. (FLAG1 = 1, FLAG2 = 1) $j^* = 1$.
1-B. KITDAY(4,1) = 1, KITDAY(5,1) = 2, KITDAY(6,1) = 2, KITDAY(7,1) = 2, KITDAY(3,1) = 6, KITDAY(2,1) = 6, KITDAY(1,1) = 9.
(Note: subassembly (4,1) uses 110 part 4 as substitutes for part 1, subassembly (7,1) uses 50 part 1 as substitutes for part 4, and subassembly (2,1) uses 20 part 9 as substitutes for part 10).
1-C. KITDAY(1,1) = 9, KITDAY(2,1) = 6, KITDAY(3,1) = 6, KITDAY(7,1) = 2, KITDAY(6,1) = 2, KITDAY(5,1) = 2, KITDAY(4,1) = 2.
(Note: subassembly (2,1) still uses 20 part 9 as substitutes for part 10).
1-D. $\Pi_1 = \{2,4\}$.
1-A. (FLAG1 = 1, FLAG2-1) $j^* = 4$.
1-B. KITDAY(4,4) = 1, KITDAY(7,4) = 2, KITDAY(5,4) = 3, KITDAY(6,4) = 3, KITDAY(2,4) = 8, KITDAY(3,4) = 8, KITDAY(1,4) = 11.
(Note: subassembly (4,4) uses 60 part 5 as substitutes for part 3, and subassembly (6,4) uses 60 part 3 as substitutes for part 5).
1-C. KITDAY(1,4) = 11, KITDAY(3,4) = 8, KITDAY(2,4) = 8, KITDAY(6,4) = 4, KITDAY(5,4) = 4, KITDAY (7,4) = 4, KITDAY(4,4) = 4.
1-D. $\Pi_1 = \{2\}$.
1-A. (FLAG1 = 1, FLAG2 = 1) $j^* = 2$.
1-B. KITDAY(4,2) = 5, KITDAY(7,2) = 5, KITDAY(6,2) = 1, KITDAY(5,2) = 5, KITDAY(3,2) = 9, KITDAY(2,2) = 9, KITDAY(1,2) = 12.
(Note: subassembly (6,2) uses 140 part 1 as substitutes for part 4 and 180 part 2 as substitutes for part 6, subassembly (5,2) uses 200 part 4 as substitues for part 1 and 180 part 6 as substitutes for part 2, and subassembly (2,2) uses 20 part 10 as substitutes for part 9.)
1-C. KITDAY(1,2) = 12, KITDAY(2,2) = 9, KITDAY(3,2) = 9, KITDAY(5,2) = 5, KITDAY(7,2) = 5, KITDAY(4,2) = 5, KITDAY(6,2) = 5.
(Note: subassembly (2,2) still uses 20 part 10 as substitutes for part 9).
1-D. $\Pi_1 = \phi$. D = 12 and go to Step 1-E.
1-E. Since FLAG1 = 1, record the kitting schedule and calculate TCOST (TCOST2 = 26·4). Got to Step 3 (FLAG2 = 1).

Step 3. The resulting total costs based on earliest-due-date-first, and ORDER are compared to determine the final kitting schedule. In this small example, they are the same (26·4), and the final kitting schedule is determined.

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