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The impact of production interruptions in kitting, an analytical study

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Abstract Efficient transport of materials between the stages of the production process is key to minimizing production costs. Kitting — the collection of the necessary parts for assembly into a specific container prior to arriving at an assembly unit — is an attempt at achieving efficient transport and thus reducing these costs. In this paper we discuss the performance of kitting operations, treating it as a Continuous Time Markov Chain. Specially, the impact of interruptions in the production of parts is investigated. To this end, parts arrive in accordance with an Interrupted Poisson Process, interruptions modelling the downtimes during the production.

Our analysis heavily relies on the use of sparse matrix techniques. Results show that this technique is a valuable queuing theoretic numerical approach for estimating the performance of a kitting process in terms of both solution speed and accuracy.

Keywords Kitting process · Continuous Time Markov Chain · Sparse matrix · Production interruptions · GMRES

1 Introduction

Nowadays manufacturing systems are often composed of multiple in-house fabrication units (Medbo 2003). The semi-finished products stemming from these units are the input materials for other fabrication units or for assembly lines. Hence, efficient transport of materials between the different stages of the production process is a key issue for overall production cost minimization.

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Kitting is a particular strategy for supplying materials to an assembly line. Instead of delivering parts in containers of equal parts, kitting collects the necessary parts into a specific container, referred to as kit, prior to arriving at an assembly unit (Bozer and McGinnis 1992; Som et al 1994; Bryznér and Johansson 1995; Medbo 2003; Ramachandran and Delen 2005; Ramakrishnan and Krishnamurthy 2008).

Kitting mitigates storage space requirements at the assembly station since no part inventories need to be kept there. Moreover, parts are placed in proper positions in the container such that assembly time reductions can be realized. Additional benefits include reduced learning time of the workers at the assembly stations and increased quality of the product. Although kitting is a non-value adding activity, its application can reduce the overall materials handling time (Ramakrishnan and Krishnamurthy 2008). Indeed activities such as selecting and gripping parts are made more efficient. Furthermore, the whole operator walking time is drastically reduced and in some cases eliminated due to the fact that the kits of components are brought as a whole to the assembly station (Johansson and Johansson 1990). The advantages mentioned above do not come for free since the kitting operation itself also incurs additional costs such as the time and effort for planning the allocation of the parts into kits and the kit preparation itself. Moreover, the introduction of a kitting operation in a production process involves a major investment. Therefore it is important to analyse the performance of kitting in a production environment prior to the actual introduction of this operation.

In literature, most authors consider a kitting process as a queuing system with stochastic part arrivals and kit assembly. Hopp and Simon (1989) developed a model for a kitting process with exponentially distributed processing times for kits and Poisson arrivals. They define an accurate boundary for the capacity of the buffer. However, the model is limited to processes with two basic components. Som et al (1994) consider a kitting process as a delivery system for assembly that is mainly based on the model of Hopp and Simon. The latest assumes a finite-buffer-capacity. Of course a buffer has always a finite capacity. However, if the capacity is large enough, we can have a good approximation of a process with a finite capacity on the basis of a model with unlimited capacity. This means that there is always enough space for upcoming parts which simplifies the analysis. Unfortunately, the assumption of an infinite buffer is not valid for kitting processes. If the capacity is assumed to be infinite, then the model will degrade to an unstable stochastic model. This was demonstrated by Latouche (1981) that studied waiting lines with paired customers. We can consider his analysis as an abstraction of a kitting process with two types of parts. Furthermore, in the article "Assembly-like queues", Harrison (1973) confirms that, to ensure stability in the operations of a kitting process, it is necessary to impose a restriction on the size of the buffer. Under this assumption, the probability to have a certain long-term stock position is equal and independent of the current stock position.

In this work, we develop a Markov Modulated Chain to model interruptions of the production in the parts of a kitting process. The parts arrive accord-

ing to an Interrupted Poisson Process and the processing times for kits are exponentially distributed. We use the iterative method GMRES to solve the linear equations. The body of this paper is organized in five sections. Section 2 describes the model of a production interrupted kitting process. In section 3, we define the kitting process as a CTMC satisfying the Chapman-Kolmogorov equation. The sparse matrix techniques is also explained in this section. Section 4 discusses numerical examples. Finally, a conclusion is presented in section 5.

2 Model description

A general kitting process is showed in figure 1. Each of the two types of parts are necessary to compose one kit, such that kitting blocks when one of the buffers is empty. We assume that the capacity of the two part inventories is respectively equivalent to C_1 and C_2. when a part entering the system encounters a full buffer, we speak of loss of parts. We consider also that on average one kit per unit time can be made so that the processing intensity μ_i equals one in every queuing state i. Concerning part arrivals, they arrive in accordance with an Interrupted Poisson Process (abbreviated as IPP). In the queuing analysis, an IPP is a stochastic process in which two states are possible and which one of the two has an intensity equal to zero. This process is divided into two periods, namely the active and inactive period (Heyman and Sobel 1982). We start with an active period and during this interval there are components arriving according to a Poisson process with intensity λ_i^*. The length of this period is exponentially distributed with mean α^{-1}. At the end of an active period begins a period of inactivity in which components do not arrive, the length is exponentially distributed with mean β^{-1}. At the end of this period begins another new active period and so on. All active and inactive periods are i.i.d. The parameter α (β) describes the intensity to go from an active (inactive) to an inactive (active) period in an infinitesimal time interval.

Alternatively, this process can also be characterized by the parameters σ and κ defined as follows:

$$\sigma_i = \frac{\beta_i}{\alpha_i + \beta_i}.$$

The symbol σ is the fraction of the time that the process is in an active state. We call this parameter the *active rate*. The symbol κ, which we call the *switch-over time* is equal to the sum of the average length of the active and the inactive period:

$$\kappa_i = \frac{1}{\alpha_i} + \frac{1}{\beta_i}.$$

Finally, we determine the workload λ_i on the basis of the equation:

$$\lambda_i = \sigma.\lambda_i^*.$$

This means that the product of the arrival intensity in the active period λ_i^* with the active rate σ is equal to the workload λ_i.

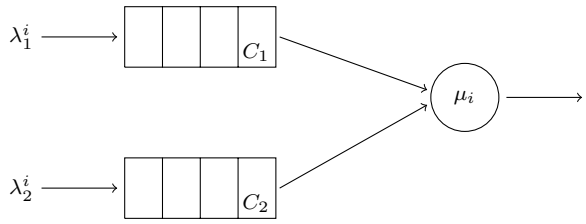


Fig. 1 Kitting process

3 Analysis

First, we describe the transition rate diagram of our kitting model. Then, we define this model satisfying the Chapman-Kolmogorov equation, thence we can calculate the stationary probability vector. Finally, we explain the methodology used in MATLAB to develop numerical results.

3.1 A Continuous Time Markov Chain

Figure 2 shows a fragment of the transition rate diagram of our kitting model in state (i, j, k). The two first values placed in the circles represent respectively the number of parts in buffer 1 and 2 where $0 \leq i \leq C_1$ and $0 \leq j \leq C_2$. Two independent input streams arrive at the buffers at intensity $\lambda_{1,k}$ and $\lambda_{2,k}$ respectively and wait there till they are collected into a kit. A kit is composed of the two parts and is processed at intensity μ_k. The last value k stands for the queuing state. Depending on whether the production of parts is subject to interruptions or not, the arrival intensity λ_k has a different value. If for example part 2 is not produced during a certain period of time, then $\lambda_{2,k} = 0$.

In the following, we model the process as a Continuous Time Markov Chain. We define a stochastic process $X(t)$ as a Markov chain with continuous time parameter where $s, t, u \geq 0$ and all non-negative integer values i, j and r belong to the discrete state space X. It is true that:

$$\mathrm{P[X(t) = j | X(s) = i, X(u) = r, u \leq s < t] = P[X(t) = j | X(s) = i].}$$

This definition is based on the *Markov-property*. A stochastic process has the Markov-property if the conditional probability distribution of future states of the process depend only upon the present state. Suppose now that:

$$\mathrm{p_{ij}(s, t) \equiv P[X(t) = j | X(s) = i],}$$

where $t \geq s$.

We assume that our Markov chain is *homogeneous*. A chain is homogeneous if all *transition functions* $\mathrm{p_{ij}(s, t)}$ depend only on the difference $(t - s)$ and are independent of the absolute epochs s and t. Transition functions give the probability that a situation will occur given a current state. Among

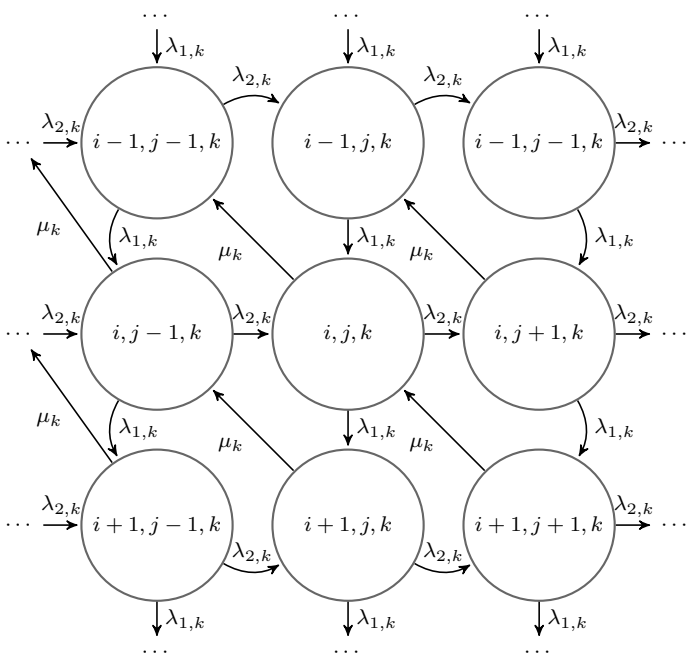


Fig. 2 Fragment of the transition rate diagram for state (i, j, k)

other books, the "Discrete Event Systems" book written by Cassandras and Lafortune (2008) demonstrates on the basis of the law of total probability $P[A] = \Sigma_i P[A|P[B_i].P[B_i]$ that transition functions satisfy the *Chapman-Kolmogorov equation*:

$$p_{ij}(t-s) = p_{ij}(u-s).p_{ij}(t-u),$$

knowing that $s \leq u \leq t$.

The Chapman-Kolmogorov equation is an identity relating the joint probability distributions of different sets of coordinates on a stochastic process. The equation allows us to define the generator matrix $\mathbf{Q}$. This matrix is the continuous case of the transition matrix. It gives us the probability to go from one state to another in an infinitesimal time interval ($t - s = \triangle t \to 0$). The multiplication of the matrix $\mathbf{Q}$ with the stationary probability vector π equals zero. The elements of this vector are steady state probabilities i.e. the probability that a certain state occurs when the chain has reached equilibrium. On the basis of this information we calculate performance measures for the kitting process.

3.2 Methodology: the sparse matrix techniques

Queuing models for kitting processes are rather complicated. Since two queues are involved (one for each part in the kit) and can whether be in a productive or unproductive state of the parts, the state space of the associated Markov chain is inherently multidimensional. Multidimensionality leads to huge state spaces; this is the state space explosion problem. A second complication is more intricate, as mentioned above, the infinite-buffer-capacity assumption is not applicable for kitting processes. If the capacity is assumed infinite, the model degrades to an unstable stochastic model in which some or all of the queues have an unlimited number of parts available all the time with a positive probability.

Consequently, the multidimensionality of the state space and the inapplicability of the infinite-buffer assumption yield Markov chains with a finite but very large state space. However, the number of possible state transitions from any specific state is limited. This means that most of the entries in the generator matrix are zero; the matrix is sparse. In contrast to matrix-analytic methods, sparse matrix techniques have hardly been used in queuing theory. Using sparse matrices and their associated specialized algorithms resulted in less memory consumption and processing times, compared to standard algorithms. The reason is that the complexity is smaller for sparse than for dense matrices. In the model where both parts are subject to production interruptions, the number of elements of the generator matrix for $C_1 = C_2 = 100$ is 40804^2. By considering this matrix as sparse, only $3 * 40804$ elements need to be stored. Indeed, the storage of the matrix requires less memory because only the non-zero elements are kept.

The method used to solve linear equations of sparse matrices is the iterative method GMRES (Generalized Minimum Residual). Direct methods are not applied because they are too slow or even unusable for large sparse-matrices. The GMRES method approximates the exact solution $A.x = b$ by the vector $x_n \epsilon K_n$ in a Krylov subspace K_n that minimizes the norm of the residual $A.x_n - b$. Since every subspace is contained in the next subspace, the residual decreases monotonically. However, the major drawback to GMRES is that the amount of work and storage required per iteration rises linearly with the iteration count. The cost of the iterations grow like $O(n^2)$, where n is the iteration number. The usual way to overcome this limitation is by restarting the iteration. After a chosen number of iterations m, the accumulated data are cleared and the intermediate results are used as the initial data for the next m iterations. This procedure is repeated until convergence is achieved. The difficulty is in choosing an appropriate value for m. If m is too small, GMRES may be slow to converge, or fail to converge entirely. A value of m that is larger than necessary involves excessive work and uses more storage. Saad and Schultz (1986) have proven several useful results. In particular, they show that if the coefficient matrix $\mathbf{A}$ is real and *nearly* positive definite, then a "reasonable" value for m may be selected. The method stagnates and convergence takes place at the m^{th} step. To generate the numerical examples below we used a value for

m equal to 140. Another important parameter to be defined is the initial vector. It is standard programmed as a zero vector. A first improvement is to consider the vector as equiproportional. Even if this assumption is incorrect, it accelerates the calculations. This is because the sum of the state probabilities equals one. When a plot is created where the capacity of the buffers vary, then the previous calculated probability vector could be used. In case the initial vector is adapted, it would be more accurate than an equiproportional vector. The reason is that when the capacity of the buffers is subject to little changes, there is a high chance that the state probabilities almost remain the same. However, the determination of this vector is time consuming because the increase in C_1 has a different effect on the to be calculated vector than a larger C_2. Furthermore, the accuracy of the steady state probability vector was not improved as expected. Further research needs to be done. On the other side, when varying the workload, there is no need to adapt the calculated vector because it is independent of the value given to the workload. As with varying capacity, there is also a high chance that the state probabilities have the same value when the workload is increasingly changing. In terms of speed, the outcome was clearly better than when varying capacities. Indeed, the time required for constructing numerical examples was reduced by a factor of 10.

4 Numerical results

In this section, we present some numerical examples in order to evaluate the effect of production interruptions on the performance of a kitting process. Three models are illustrated. The first model considers both parts arriving according to a Poisson process with an arrival intensity λ^* equal to $0,8$. In the second model part 1 is subject to production downtimes and its arrival is therefore modelled as an Interrupted Poisson Process. In 40 percent of the time, part 1 arrive with intensity λ^* equal to 2. The third model represents a kitting process where both components are subject to production interruptions. The two Interrupted Poisson Processes are independent and equal. The numerical examples showed assume a time length κ equal to ten and a workload λ equal to $0,8$. The workload, i.e. the average arrival intensity over the productive and unproductive period, must be the same for both components. If this is not the case and the buffers are sufficiently large, then the buffer with the highest workload is almost always full. The system can then be considered as a queue with just one buffer, the one that is always full.

Figure 3 represents the loss probability according to different levels of the buffer capacities for each model. Important to mention is that because we assume that the buffers have the same workload, the average loss probability calculated for both buffers together equals that for the buffers separately. A first observation is that the probability decreases as the capacities increase and that for the three models. Less components are lost when the buffers are sufficiently large so that more kits can be processed. Therefore the difference between the models diminishes as the capacity increases. Secondly,the values

for the third model are higher than that for the first model. As expected, the performance of a process subject to production interruptions is worse than a process without. When the arrival process is modelled as a Poisson Process, such as the first model, the probability that the buffer is full and the loss probability are equal. This equality is a consequence of the PASTA-property. Thanks to the memoryless property of the Poisson process, the stochastic properties of parts on the arrival times are the same than that on random times. On the other hand, these two probabilities are not equal for an arrival process modelled as an IPP. Indeed, the average loss probability has greater values than the probability that the buffer is full.

Figure 4 show the probability that buffer 1 and 2 are full for the three models together. We can notice that downtimes in the production of part 1 have a greater impact on buffer 2 than on its own buffer. It also appears that adding production interruptions at part 2 doesn't have a significant impact on its own buffer but does on the other buffer. The lines for the second and third model are almost identical in the second subfigure, which is not the case in the first subfigure.

Now, instead of varying the capacity we assume different workload values for both parts. In figure 6 the mean in buffer 1 for the model where both parts are subject to production downtimes is represented. The mean starts to increase significantly as the workload is greater than $0,8$. Indeed, as the processing intensity μ equals one, we are close to a situation of overload, i.e. $\frac{\lambda}{\mu}$ is equal or greater than one. This effect is amplified as C_1 is increasing. In a model that is not subjected to production interruptions and the workload approximately equals $1,8$, the mean in buffer 1 aims at being equal to its buffer capacity. Here, this equality is not reached yet due to the production downtimes of the parts. This means that when the load is sufficiently high depending on the modelled arrival process, the buffer will be full.

Finally, figure 5 and 6 represent the probability that one of the buffers is empty and the loss probability on a logarithmic scale. These two probabilities are related as the probability loss rate PLR equals:

$$PLR = \frac{\lambda_1 + \lambda_2 - 2 * TP}{\lambda_1 + \lambda_2}$$

where $TP = \mu * (1 - K_1)$ equals the throughput and K_1 the probability that one of the buffers is empty. In figure 5, when the workload is smaller than one, there is no difference in value for different buffer capacity levels. However when the workload is greater than one, the higher the workload and buffer capacity, the higher the value of the probability that one of the buffers is empty. Concerning the loss probability represented in figure 6, it has a higher value when the workload is small and the buffer capacity is high. As the workload increases, the value of the buffer capacities becomes irrelevant.

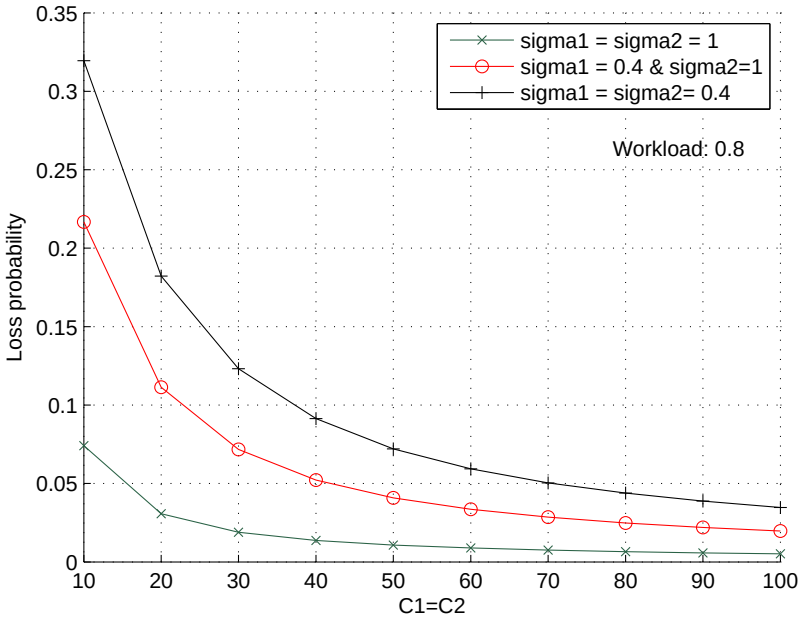
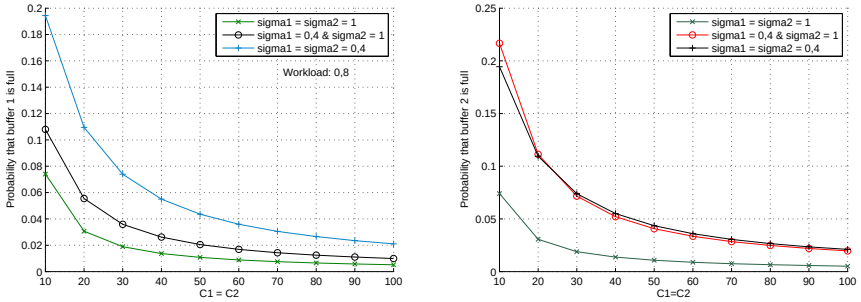


Fig. 3 Loss Probability



(a) Probability that buffer 1 is full

(b) Probability that buffer 2 is full

5 Conclusion

In this paper, we investigate the impact of production inefficiencies of the parts on a kitting process with two queue lines using performance measures. We show that the buffer sizes need to be large enough to catch production inefficiencies. Furthermore, the numerical examples we present lead us to believe that the two part buffers are correlated. When part 1 suffers of inefficiencies, buffer 2 will have a higher probability to be full than buffer 1. Indeed, production downtimes of one component mainly affects the behaviour of the buffer of

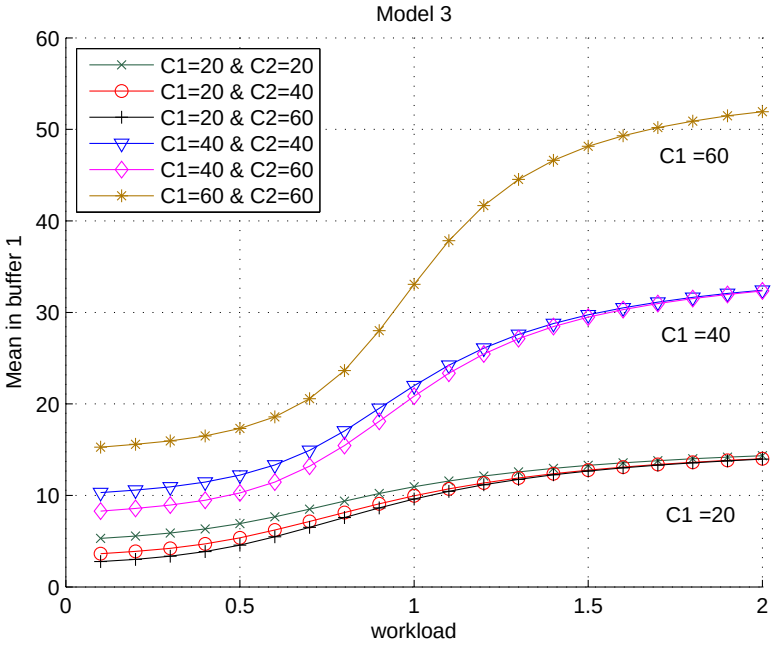


Fig. 4 Mean in Buffer 1

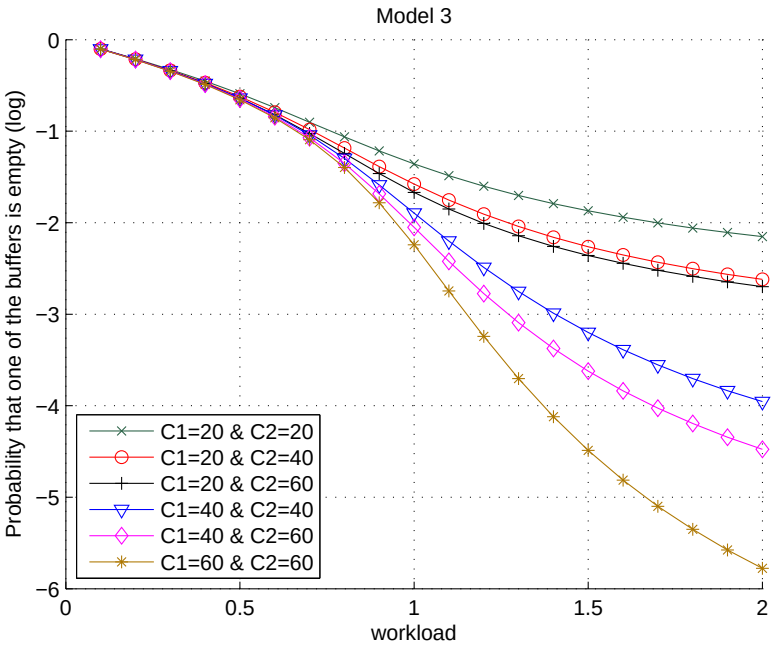


Fig. 5 Probability that one of the buffer is empty

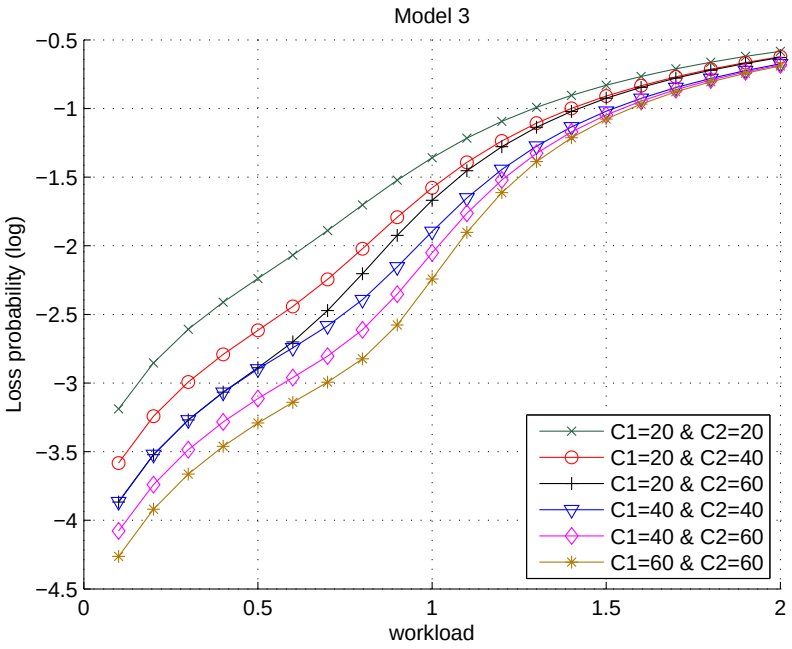


Fig. 6 Loss Probability

the other component. As most of the entries in the generator matrix have a value equal to zero, we apply sparse matrix techniques. To determine the unknowns of the system, we used the method GMRES (Generalized Minimum Residual). The solution is not exact but performs well in terms of solution speed and accuracy. We can establish that the sparse matrix techniques are a valuable queuing theoretic numerical approach to estimate the performance of the kitting process.

Queuing models for determining the performance of kitting processes are currently insufficiently studied. Consequently, there is room for further research. First, the assumptions made could be gradually alleviated or removed. We restrict ourselves to two components, while the process could easily be expanded to multiple components. The selected performance measures are also rather limited and only focused on part buffers. Furthermore, to better approximate the reality, additional factors that affect the performance of the process should be taken into account. When companies start to implement kitting activities in their production process, in addition to the performance, the cost of the kitting process is relevant.

References

Bozer Y, McGinnis L (1992) Kitting versus line stocking: A conceptual framework and a descriptive model. International Journal of Production Economics 28:1–19

Bryznér H, Johansson M (1995) Design and performance of kitting and order picking systems. International Journal of Production Economics 41:115–125

Cassandras CG, Lafortune S (2008) Introduction to Discrete Event Systems, Second Edition. Springer Science and Business Media

Harrison J (1973) Assembly-like queues. Journal Of Applied Probability 10(2):354–367

Heyman DP, Sobel MJ (1982) Stochastic models in Operation Research: Stochastic processes and Operating Characteristics. Mc Graw-Hill Book Company, p.112

Hopp WJ, Simon JT (1989) Bounds and heuristics for assembly-like queues. Queueing Systems 4:137 – 156

Johansson B, Johansson M (1990) High automated Kitting system for small parts: a case study from the Volvo uddevalla plant. In: Proceedings of the 23rd International Symposium on Automotive Technology and Automation, p.75-82, Vienna, Austria

Latouche G (1981) Queues with paired customers. Journal of Applied Probability 18:684–696

Medbo L (2003) Assembly work execution and materials kit functionality in parallel flow assembly systems. International Journal of Industrial Ergonomics 31:263 – 281

Ramachandran S, Delen D (2005) Performance analysis of a Kitting process in stochastic assembly systems. Computers & Operations Research 32(3):449 – 463

Ramakrishnan R, Krishnamurthy A (2008) Analytical approximations for Kitting systems with multiple inputs. Asia-Paific Journal of Operations Research 25(2):187 – 216

Saad Y, Schultz M (1986) Gmres: A generalized minimal residual algorithm for solving nonsymmetric linear systems. SIAM Journal on Scientific and Statistical Computing 7:586–869

Som P, Wilhelm W, Disney R (1994) Kitting process in a stochastic assembly system. Queueing Systems 17:471 – 490