

# Comparison of the sensitivities of MC-CDMA and MC-DS-CDMA to carrier frequency offset

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## Abstract

*In this contribution, we consider the effect of a carrier frequency offset on the performance of MC-CDMA and MC-DS-CDMA, which are both a combination of the multicarrier transmission technique and the CDMA multiple access technique. We show that both systems strongly degrade in the presence of a carrier frequency offset. Furthermore, we show that, for given spreading factor, the MC-DS-CDMA system is less (more) sensitive to carrier frequency offset than MC-CDMA when the number of modulated carriers in the former system is smaller (larger) than the spreading factor.*

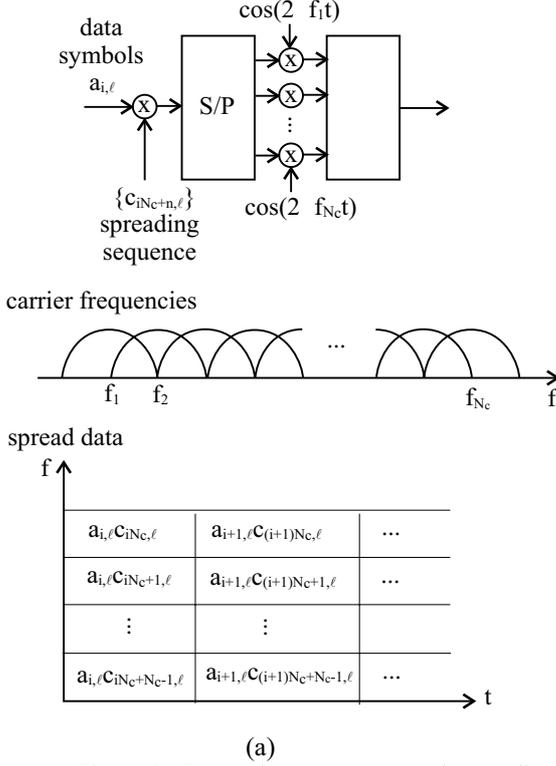
## 1. Introduction

The enormous growth of wireless services (cellular telephones, wireless LAN's, ...) during the last decade gives rise to the need for a bandwidth efficient modulation technique that can reliably transmit high data rates. As multicarrier (MC) techniques combine a good bandwidth efficiency with an immunity to channel dispersion, these techniques have received considerable attention [1]-[5]. To be able to support multiple users, the multicarrier transmission technique can be combined with a code-division multiple access (CDMA) scheme. In the literature, (see [6]-[8] and the references therein), different combinations of the multicarrier transmission technique and CDMA are investigated in the context of high-rate communication over dispersive channels. Two techniques that make use of carriers satisfying the orthogonality condition with minimum frequency separation are multicarrier CDMA (MC-CDMA) and multicarrier direct-sequence CDMA (MC-DS-CDMA).

- In the MC-CDMA technique [9]-[11], the original data stream is first multiplied with the spreading sequence and then modulated on the orthogonal carriers, as shown in figure 1(a): as the chips belonging to the same symbol are modulated on different carriers, the spreading is done in the frequency domain.
  - In the MC-DS-CDMA technique [12]-[13], the serial-to-parallel converted data stream is multiplied with the spreading sequence and then the chips belonging to the same symbol modulate the same carrier, as shown in figure 1(b): the spreading is done in the time domain.
- Both MC-CDMA and MC-DS-CDMA have been considered for mobile radio communication [9]-[13].

The transmitter of a digital bandpass communication system contains a carrier oscillator that upconverts the data-carrying baseband signal to the bandpass signal to be transmitted. At the receiver, the received bandpass signal is downconverted using a local oscillator. The receiver must make an estimation of the carrier frequency and phase used at the transmitter, based upon the received signal. Because of the presence of interference, noise and other disturbances, these estimations are not perfect, which results in carrier phase errors. In the literature [14]-[17], it has been reported that multicarrier systems are very sensitive to some types of carrier phase errors when a large number of carriers is used. Particularly carrier frequency offset is detrimental for multicarrier systems. The effect of a small carrier frequency offset between the carrier oscillators at the transmitter and the receiver (caused e.g. by using a free-running local oscillator or by Doppler shift) has already been investigated for orthogonal frequency-division multiplexing (OFDM) [14]-[15] and MC-CDMA [16]-[17]; the system performance rapidly degraded in the presence of carrier frequency offset when the number of carriers increases.

transmitter structure



transmitter structure

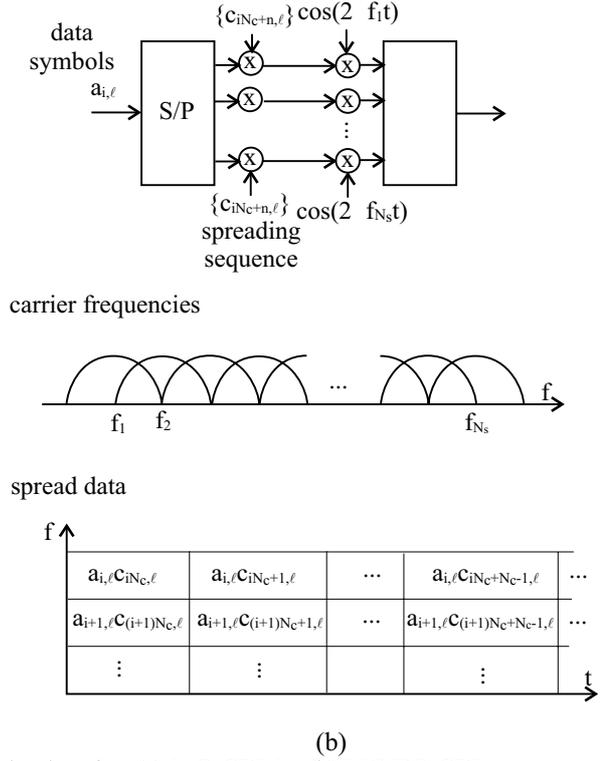


Figure 1: Transmitter structure and spreading of the data for (a) MC-CDMA (b) MC-DS-CDMA

In this paper, we consider the effect of carrier frequency offset on MC-CDMA and MC-DS-CDMA in the case of downlink transmission, and determine for both systems an analytical expression for the performance degradation in terms of the system parameters. With these results, we compare the sensitivities of both systems.

## 2. System Description

### The MC-CDMA System

The conceptual block diagram of a downlink MC-CDMA system is shown in figure 2. The data symbols  $a_{i,\ell}$  are generated at a rate  $R_s$ , where  $a_{i,\ell}$  denotes the  $i^{\text{th}}$  symbol transmitted to user  $\ell$ . Each data symbol is multiplied with a spreading sequence  $\{c_{iN_c+n,\ell}/n=0,\dots,N_c-1\}$  with a spreading factor  $N_c$ , where  $c_{iN_c+n,\ell}$  is the  $n^{\text{th}}$  chip during the  $i^{\text{th}}$  symbol interval of the sequence belonging to user  $\ell$ . The  $N_c$  components of the spread data symbol  $a_{i,\ell}$ , i.e.  $\{a_{i,\ell} c_{iN_c+n,\ell}/n=0,\dots,N_c-1\}$ , are transmitted in parallel on the different carriers of an orthogonal multicarrier system; the  $n^{\text{th}}$  chip is mapped on the  $k_n^{\text{th}}$  carrier, with  $k_n$  belonging to a set  $I_c$  of  $N_c$  carrier indices. Hence, the spreading is accomplished in the frequency domain. Each component  $a_{i,\ell} c_{iN_c+n,\ell}$  has a duration of  $1/R_s$ . To modulate the spread data symbol on the orthogonal carriers, we use an  $N_F$ -point inverse fast Fourier transform (inverse FFT). As in

conventional OFDM, interference between successive transmitted FFT blocks is avoided by cyclically extending each FFT block at the inverse FFT output with a cyclic prefix of  $N_p$  samples. The resulting samples are applied at a rate  $1/T=(N_F+N_p)R_s$  to a square-root raised-cosine transmit filter  $P(f)$  with rolloff factor  $\alpha$ . In the following, it is assumed that the carriers inside the rolloff area are not modulated, i.e. they have zero amplitude. Hence, of the  $N_F$  available carriers, only  $N_c$  carriers are actually used ( $N_c \leq (1-\alpha)N_F$ ). Considering the carrier index corresponds to the carrier frequency  $k/(N_F T)$  and assuming  $N_c$  is even, the set of carriers actually used is given by  $I_c = \{0, \dots, N_F/2\} \cup \{N_F - N_F/2, \dots, N_F - 1\}$ , and the chips are mapped on the carriers as shown in figure 3. The corresponding carrier spacing  $1/(N_F T)$  and system bandwidth  $B$  are given by

$$\begin{aligned} \frac{1}{N_F T} &= R_s \frac{N_F + N_p}{N_F} \cong R_s \\ B &= \frac{N_c}{N_F T} = N_c R_s \frac{N_F + N_p}{N_F} \cong N_c R_s \end{aligned} \quad (1)$$

The above approximations assume that  $N_p \ll N_F$ .

In a multiuser scenario, the basestation broadcasts the sum of  $N_u$  user signals to all  $N_u$  active users. To be able to extract the signal of the reference user at the mobile receiver, each user is assigned a unique spreading

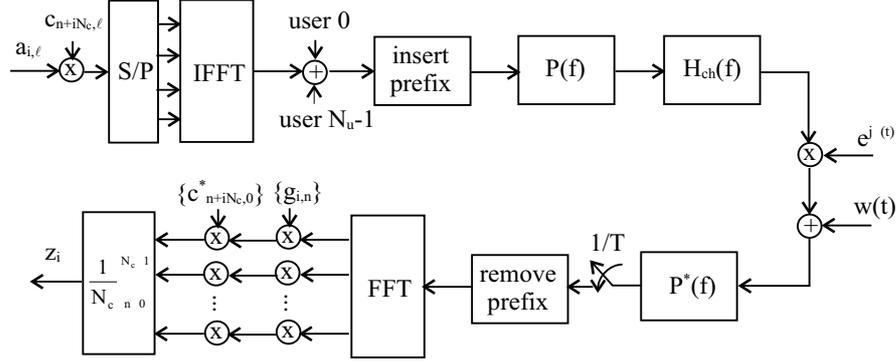


Figure 2: Block diagram for MC-CDMA

sequence. In this contribution, we restrict our attention to orthogonal sequences, consisting of user-dependent Walsh-Hadamard (WH) sequences multiplied with a complex-valued random scrambling sequence that is common to all users. Hence, the maximum number of users equals  $N_c$ , i.e. the number of WH sequences of length  $N_c$ . This indicates that the number of used carriers equals the spreading factor, which in turn equals the maximum number of users.

The broadcast signal reaches the receiver of the reference user ( $\ell=0$ ) through a channel with transfer function  $H_{ch}(f)$ . The output of the channel is affected by a carrier phase error  $\phi(t)=2\pi\Delta Ft+\phi(0)$ , where  $\Delta F$  is a small carrier frequency offset ( $\Delta F T \ll 1$ ). Furthermore, the received signal is disturbed by additive white Gaussian noise (AWGN)  $w(t)$ , with uncorrelated real and imaginary parts, each having a power spectral density  $N_0/2$ .

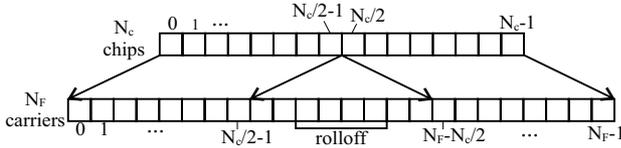


Figure 3: Mapping of the chips on the carriers

The resulting signal is applied to the receiver filter, which is matched to the transmit filter, and sampled at a rate  $1/T$ . For each FFT block of  $N_F+N_p$  samples, the receiver removes the  $N_p$  samples corresponding to the cyclic prefix and keeps the remaining  $N_F$  samples for further processing. The receiver consists of an  $N_F$ -point FFT followed by one-tap equalizers  $g_{i,n}$  that scale and rotate the  $k_n$ <sup>th</sup> FFT output during the  $i$ <sup>th</sup> symbol interval, where  $k_n$  denotes the carrier index that contains the  $n$ <sup>th</sup> chip of the spread data symbol. The equalizer outputs are multiplied with the corresponding chip of the spreading sequence of the reference user, and summed to obtain the sample  $z_i$ , from which a decision is made about the data symbol  $a_{i,\ell}$ . Assuming the duration of the cyclic prefix

exceeds the duration of the impulse response of the cascade of the transmit filter, the channel and the receiver filter (with transfer function  $H(f)=H_{ch}(f)/P(f)^2$ ), the sample  $z_i$  at the input of the decision device can be decomposed as

$$z_i = \sqrt{\frac{N_F}{N_F + N_p}} \sum_{\ell=0}^{N_c-1} a_{i,\ell} I_{i,\ell} + W_i \quad (2)$$

where  $I_{i,\ell}$  denotes the contribution from the symbol  $a_{i,\ell}$  to the sample  $z_i$  at the input of the decision device of the reference user. The quantity  $I_{i,\ell}$  is determined by

$$I_{i,\ell} = \frac{1}{N_c} \sum_{n,n'=0}^{N_c-1} c_{iN_c+n,0}^* c_{iN_c+n',\ell} g_{i,n} H_n A_{i,n,n'} \quad (3)$$

$$A_{i,n,n'} = e^{j\phi(0)} e^{j2\pi\Delta Ft(N_F+N_p)} D\left(\frac{k_n - k_{n'} + \Delta FT}{N_F}\right) \quad (4)$$

$$D(x) = \frac{1}{N_F} \sum_{m=0}^{N_F-1} e^{j2\pi mx} = e^{j\pi(N_F-1)x} \frac{\sin(\pi N_F x)}{N_F \sin(\pi x)} \quad (5)$$

In (3),  $H_n = H(\text{mod}(k_n; N_F)/(N_F T))/T$ , where  $\text{mod}(x; N_F)$  is the modulo- $N_F$  reduction of  $x$ , yielding a result in the interval  $[-N_F/2, N_F/2]$ . The sample  $z_i$  from (2) contains a useful component with coefficient  $I_{i,0}$ . This component can be decomposed into an average useful component  $E[I_{i,0}]$  and a zero-mean fluctuation  $I_{i,0} - E[I_{i,0}]$  about its average, i.e. the self-interference. The power of the average useful component is denoted  $P_U$ . For  $\ell \neq 0$ , the contribution  $I_{i,\ell}$  corresponds to the multiuser interference. The sum of the powers of the self-interference and the multiuser interference is denoted  $P_I$ . The additive noise contribution  $W_i$  has a power  $P_N$ , given by

$$P_N = E[|W_i|^2] = N_0 \frac{1}{N_c} \sum_{n=0}^{N_c-1} |g_{i,n}|^2 \quad (6)$$

We select the equalizer coefficients such that the coefficient  $E[I_{i,0}]$  of the average useful component equals 1. Assuming  $H_n$ ,  $\phi(0)$  and  $\Delta F$  can be estimated accurately, the equalizer coefficients yield

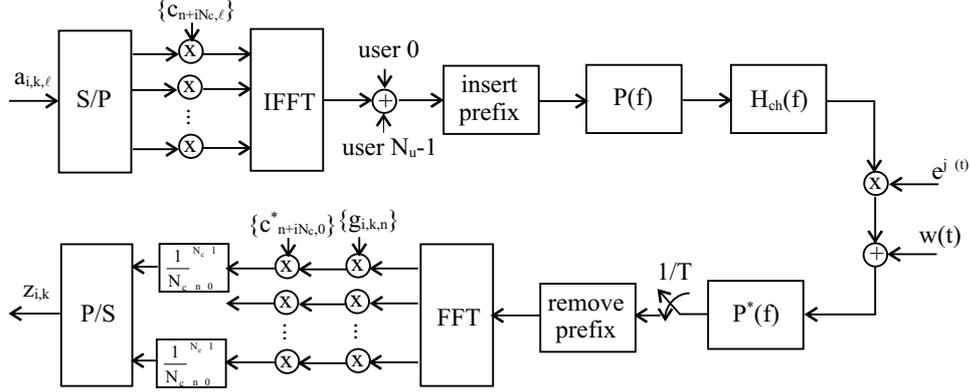


Figure 4: Block diagram for MC-DS-CDMA

$$g_{i,n} = \frac{e^{-j\phi(0)} e^{-j2\pi\Delta Ft(N_F+N_p)}}{H_n D(\Delta Ft)} \quad (7)$$

This equalizer compensates for the phase rotation and the scaling of the average useful component, but is not able to eliminate the interference ( $P_i \neq 0$ ).

### The MC-DS-CDMA System

The conceptual block diagram of a downlink MC-DS-CDMA system is shown in figure 4. In MC-DS-CDMA, the complex data symbols to be transmitted at a rate  $R_s$  to user  $\ell$  are split into  $N_s$  symbol sequences, each having a rate  $R_s/N_s$  and each modulating a different carrier of the multicarrier system. We denote by  $a_{i,k,\ell}$  the  $i^{\text{th}}$  symbol sent on carrier  $k$  to user  $\ell$ ;  $k$  belongs to a set  $I_s$  of  $N_s$  carrier indices. The data symbol  $a_{i,k,\ell}$  is multiplied with a spreading sequence  $\{c_{iN_c+n,\ell}/n=0,\dots,N_c-1\}$  with a spreading factor  $N_c$ , where  $c_{iN_c+n,\ell}$  denotes the  $n^{\text{th}}$  chip of the sequence that spreads the data symbols transmitted to user  $\ell$  during the  $i^{\text{th}}$  symbol interval. The resulting  $N_c$  components of the spread data symbol  $a_{i,k,\ell}$ , i.e.  $\{a_{i,k,\ell}c_{iN_c+n,\ell}/n=0,\dots,N_c-1\}$ , are serially transmitted on the  $k^{\text{th}}$  carrier of an orthogonal multicarrier system. Hence, in contrast with MC-CDMA, the spreading is done in the time domain. Each component  $a_{i,k,\ell}c_{iN_c+n,\ell}$  has a duration of  $(N_s/N_c)/R_s$ . Analogous as in MC-CDMA, the modulation of the spread data symbols on the carriers can be accomplished by means of an  $N_F$ -point inverse FFT, and each FFT block is cyclically extended with a prefix of  $N_p$  samples to avoid interference between successive transmitted FFT blocks. The resulting sequence is applied at a rate  $1/T=(N_F+N_p)/(N_c/N_s)R_s$  to a square-root raised-cosine transmit filter  $P(f)$  with rolloff factor  $\alpha$ . As in the MC-CDMA system, we do not use the carriers inside the rolloff area; only  $N_s$  of the  $N_F$  available carriers are actually modulated ( $N_s \leq (1-\alpha)N_F$ ). For  $N_s$  is even, the set of carriers actually used is  $I_s=\{0,\dots,N_s/2\} \cup \{N_F-$

$N_s/2,\dots,N_F-1\}$ . The corresponding carrier spacing  $1/(N_F T)$  and system bandwidth  $B$  are given by

$$\begin{aligned} \frac{1}{N_F T} &= \frac{N_c}{N_s} R_s \frac{N_F + N_p}{N_F} \cong \frac{N_c}{N_s} R_s \\ B &= \frac{N_s}{N_F T} = N_c R_s \frac{N_F + N_p}{N_F} \cong N_c R_s \end{aligned} \quad (8)$$

The above approximations are valid for  $N_p \ll N_F$ . For given data rate  $R_s$  and spreading factor  $N_c$ , the system bandwidths of MC-CDMA and MC-DS-CDMA are the same. However, the carrier spacing for MC-DS-CDMA is larger (smaller) than for MC-CDMA when the number of carriers  $N_s$  is smaller (larger) than the spreading factor  $N_c$ .

In a multiuser scenario, each user is assigned a different spreading sequence. For MC-DS-CDMA, we consider the same set of orthogonal sequences as in the MC-CDMA system. Note that in MC-DS-CDMA, the number of carriers  $N_s$  can be selected independently of the spreading factor  $N_c$ , which in turn equals the maximum number of users.

As in the MC-CDMA system, the sum of the  $N_u$  user signals, broadcast by the basestation, reaches the receiver of the reference user ( $\ell=0$ ) through a dispersive channel with transfer function  $H_{ch}(f)$ , and the signal is disturbed by a small carrier frequency offset  $\Delta F$  ( $\phi(t)=2\pi\Delta Ft+\phi(0)$ ,  $\Delta Ft \ll 1$ ) and AWGN  $w(t)$  (of which the real and imaginary parts are uncorrelated and have the same power spectral density  $N_d/2$ ).

After applying the received signal to the receiver filter, which is matched to the transmit filter, and sampling the receiver filter output at a rate  $1/T$ , the receiver removes the cyclic prefix from each FFT block of  $N_F+N_p$  samples. The remaining  $N_F$  samples outside the prefix are fed to an FFT. The FFT outputs are applied to one-tap equalizers  $g_{i,k,n}$  that scale and rotate the  $k^{\text{th}}$  FFT output during the  $n^{\text{th}}$  FFT block

in the  $i^{\text{th}}$  symbol interval. The equalizer outputs are multiplied with the corresponding chip of the spreading sequence of the reference user and summed to obtain the sample  $z_{i,k}$ , from which a decision is made about the symbol  $a_{i,k,0}$ . The sample  $z_{i,k}$  at the input of the decision device can be written as

$$z_{i,k} = \sqrt{\frac{N_F}{N_F + N_p}} \sum_{\ell=0}^{N_u-1} \sum_{k' \in I_s} a_{i,k',\ell} I_{i,k,k',\ell} + W_{i,k} \quad (9)$$

where  $I_{i,k,k',\ell}$  denotes the contribution from the symbol  $a_{i,k',\ell}$  to the sample  $z_{i,k}$  at the input of the decision device of the reference user

$$I_{i,k,k',\ell} = \frac{H_{k'}}{N_c} \sum_{n,n'=0}^{N_c-1} c_{iN_c+n,0}^* c_{iN_c+n',\ell} g_{i,k,n} A_{i,k,k',n} \quad (10)$$

$$A_{i,k,k',n} = e^{j\phi(0)} e^{j2\pi\Delta FT(n+iN_c)(N_F+N_p)} D\left(\frac{k'-k}{N_F} + \Delta FT\right) \quad (11)$$

In (10),  $H_k = H(\text{mod}(k; N_F)/(N_F T))/T$ . The sample  $z_{i,k}$  (9) contains a useful component with coefficient  $I_{i,k,k,0}$ . In contrast with MC-CDMA, the useful component contains no fluctuation, i.e. there is no self-interference. For  $k' \neq k$ , the quantity  $I_{i,k,k',0}$  corresponds to intercarrier interference from other symbols transmitted to the reference user in the considered symbol interval, and for  $\ell \neq 0$ , the quantity  $I_{i,k,k',\ell}$  corresponds to multiuser interference. The power of the useful component is denoted  $P_{U_k}$  and the sum of the powers of the intercarrier interference and the multiuser interference is denoted  $P_{I_k}$ . The additive noise contribution  $W_{i,k}$  has a power  $P_{N_k}$ , given by

$$P_{N_k} = E[|W_{i,k}|^2] = N_0 \frac{1}{N_c} \sum_{n=0}^{N_c-1} |g_{i,k,n}|^2 \quad (12)$$

Similarly as in MC-CDMA, the equalizer coefficients are selected such that the coefficient  $I_{i,k,k,0}$  of the useful component equals 1. This yields

$$g_{i,k,n} = \frac{e^{-j\phi(0)} e^{-j2\pi\Delta FT(n+iN_c)(N_F+N_p)}}{H_k D(\Delta FT)} \quad (13)$$

Assuming that accurate estimates of  $H_n$ ,  $\phi(0)$  and  $\Delta F$  are available, the equalizer (13) compensates the phase rotation and the scaling of the average useful component, and eliminates the multiuser interference ( $I_{i,k,k',\ell} = 0$  for  $\ell \neq 0$ ). However, the equalizer can not eliminate the intercarrier interference ( $I_{i,k,k,0}$  for  $k' \neq k$ ).

### 3. System Analysis

#### The MC-CDMA System

The performance measure of the MC-CDMA system is the signal-to-noise ratio (SNR) at the input of the decision device, i.e. the ratio of the power of the average useful component to the sum of the powers of the interference and the noise. This yields

$$SNR(\Delta FT) = \frac{P_U}{P_N + P_I} \quad (14)$$

where

$$P_U = \frac{N_F}{N_F + N_p} E_{s_0}$$

$$P_I = \frac{N_F}{N_F + N_p} \sum_{\ell=0}^{N_u-1} E_{s_\ell} \frac{1}{N_c^2} \sum_{\substack{n,n'=0 \\ n \neq n'}}^{N_c-1} \left| \frac{H_{n'}}{H_n} \right|^2 \frac{\left| D\left(\frac{k_{n'} - k_n}{N_F} + \Delta FT\right) \right|^2}{|D(\Delta FT)|^2}$$

$$P_N = N_0 \frac{1}{|D(\Delta FT)|^2} \frac{1}{N_c} \sum_{n=0}^{N_c-1} \frac{1}{|H_n|^2} \quad (15)$$

In (15),  $E_{s_\ell} = E[|a_{i,\ell}|^2]$  is the energy per symbol transmitted to user  $\ell$ . In the absence of a carrier frequency offset, the SNR (14) reduces to

$$SNR(0) = \frac{N_F}{N_F + N_p} \frac{E_{s_0}}{N_0} \left( \frac{1}{N_c} \sum_{n=0}^{N_c-1} \frac{1}{|H_n|^2} \right)^{-1} \quad (16)$$

The degradation (expressed in dB), caused by the carrier frequency offset, is given by  $Deg = 10 \log(SNR(0)/SNR(\Delta FT))$ .

In the following, we consider the case of an ideal channel ( $H_n = 1$ ,  $n=0, \dots, N_c-1$ ) and all users having the same energy per symbol ( $E_{s_\ell} = E_s$ ,  $\ell=0, \dots, N_u-1$ ), in order to clearly isolate the effect of the carrier frequency offset. In this case, the degradation is given by

$$Deg = -10 \log |D(\Delta FT)|^2 + 10 \log \left( 1 + SNR_0 \frac{N_u}{N_c} X \right) \quad (17)$$

with  $SNR_0 = (E_s/N_0)(N_F/(N_F+N_p))$  and

$$X = \frac{1}{N_c} \sum_{n,n'=0}^{N_c-1} \left| D\left(\frac{k_{n'} - k_n}{N_F} + \Delta FT\right) \right|^2 - |D(\Delta FT)|^2 \quad (18)$$

The summation over the chip indices  $n$  and  $n'$  in (18) corresponds to the summation over the carrier indices belonging to the set  $I_c$  of  $N_c$  modulated carriers. A simple upper bound on the degradation is obtained by extending the summation over all  $N_F$  available carriers. This yields

$$Deg \leq -10 \log |D(\Delta FT)|^2 + 10 \log \left( 1 + SNR_0 \frac{N_u}{N_c} \left( 1 - |D(\Delta FT)|^2 \right) \right) \quad (19)$$

The upper bound is reached when all carriers are modulated ( $N_c=N_f, \alpha=0$ ). For  $\alpha>0$ , this upper bound yields an accurate approximation of the actual degradation when the number of unmodulated carriers is small, i.e.  $N_c \approx N_f$ .

### The MC-DS-CDMA System

The performance measure of the MC-DS-CDMA system is the SNR at the input of the decision device, which is, similarly as in MC-CDMA, defined as the ratio of the power of the average useful component to the sum of the powers of the remaining contributions, i.e.

$$SNR_k(\Delta FT) = \frac{P_{U_k}}{P_{N_k} + P_{I_k}} \quad (20)$$

where

$$P_{U_k} = \frac{N_F}{N_F + N_p} E_{s_{k,0}}$$

$$P_{I_k} = \frac{N_F}{N_F + N_p} \sum_{\substack{k' \in I_s \\ k' \neq k}} E_{s_{k',0}} \left| \frac{H_{k'}}{H_k} \right|^2 \frac{\left| D\left(\frac{k'-k}{N_F} + \Delta FT\right) \right|^2}{|D(\Delta FT)|^2} \quad (21)$$

$$P_{N_k} = N_0 \frac{1}{|D(\Delta FT)|^2} \frac{1}{|H_k|^2}$$

In (21),  $E_{s_{k,\ell}} = E[|a_{i,k,\ell}|^2]$  is the energy per symbol transmitted on carrier  $k$  to user  $\ell$ . In the absence of a carrier frequency offset, the SNR (20) reduces to

$$SNR_k(0) = \frac{N_F}{N_F + N_p} \frac{E_{s_{k,0}}}{N_0} |H_k|^2 \quad (22)$$

The performance degradation (expressed in dB), caused by the carrier frequency offset, can be defined in a similar way as in MC-CDMA as  $Deg_k = 10 \log(SNR_k(0)/SNR_k(\Delta FT))$ . Note that in MC-DS-CDMA, the performance in general depends on the carrier index  $k$ .

To clearly isolate the effect of a carrier frequency offset, we consider the case of an ideal channel ( $H_k=1, k \in I_s$ ), and all users have the same energy per symbol on each carrier ( $E_{s_{k,\ell}}=E_s, k \in I_s, \ell=0, \dots, N_u-1$ ). In this case, the degradation is given by

$$Deg_k = -10 \log |D(\Delta FT)|^2 + 10 \log(1 + SNR_0 Y) \quad (23)$$

where

$$Y = \sum_{k' \in I_s} \left| D\left(\frac{k'-k}{N_F} + \Delta FT\right) \right|^2 - |D(\Delta FT)|^2 \quad (24)$$

For given  $\Delta FT$ , the degradation (23) is independent of the spreading factor  $N_c$  and the number of users  $N_u$ . The summation over  $k'$  in (24) ranges over the set  $I_s$  of modulated carriers. Similarly as in MC-CDMA, a simple upper bound on the degradation is obtained by extending the summation over all  $N_F$  available carriers, i.e.  $k'=0, \dots, N_F-1$ . This yields

$$Deg \leq -10 \log |D(\Delta FT)|^2 + 10 \log \left( 1 + SNR_0 \left( 1 - |D(\Delta FT)|^2 \right) \right) \quad (25)$$

which is independent of the carrier index  $k$ . The upper bound is reached when all carriers are modulated ( $N_s=N_f, \alpha=0$ ). For  $\alpha>0$ , this upper bound yields an accurate approximation of the actual degradation for carriers near the center of the signal band.

## 4. Performance Comparison

In this section, we compare the degradations (19) and (25) of MC-CDMA and MC-DS-CDMA, respectively. First, we notice from (5) that for  $|x| \ll 1$ ,  $|D(x)|$  is essentially a function of  $N_f x$ , as  $\sin \pi x \approx \pi x$ . Hence, for  $|\Delta FT| \ll 1$ , the degradations (19) and (25) are a function of  $N_f \Delta FT$ , which denotes the ratio of the frequency offset  $\Delta F$  to the carrier spacing  $1/(N_f T)$ . Secondly, it follows from (19) and (25) that the degradation for MC-CDMA increases with increasing load  $N_u/N_c$ , whereas the degradation for MC-DS-CDMA does not depend on the load (because in the latter system, MUI is eliminated by the equalizer). For given  $N_f \Delta FT$ , the degradation for full-load MC-CDMA is the same as for MC-DS-CDMA.

Figure 5 shows the degradation of MC-DS-CDMA and full-load MC-CDMA as function of  $N_f \Delta FT = (\Delta F/B) N_s$ , where  $B$  is the system bandwidth and  $N_s$  is the number of modulated carriers ( $N_s=N_c$  for MC-CDMA). For given  $\Delta F/B$ , MC-DS-CDMA yields a larger (smaller) degradation than does MC-CDMA when the number of carriers  $N_s$  is larger (smaller) than the spreading factor  $N_c$ . A small degradation is obtained only when  $|\Delta F/B| \ll 1/N_s$ , in which case the degradation is proportional to  $(\Delta F/B)^2 N_s^2$ . The condition  $|\Delta F/B| \ll 1/N_s$  is much more severe than in the case of single-carrier DS-CDMA, where obtaining a small degradation only requires  $|\Delta F/B| \ll 1$ . This indicates the high sensitivity of MC-DS-CDMA and MC-CDMA as compared to single-carrier DS-CDMA, especially when the number of carriers is large.

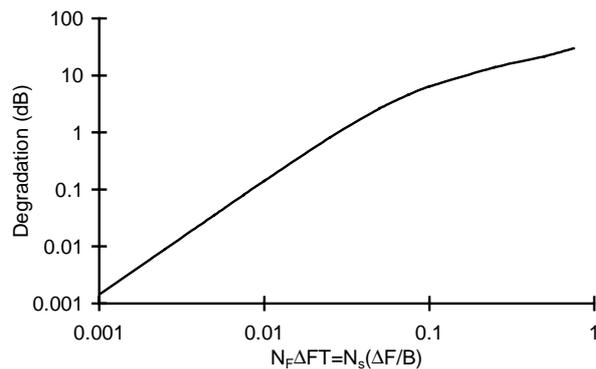


Figure 5: Degradation caused by carrier frequency offset,  $N_u = N_c$ ,  $SNR_0 = 20dB$

## 5. Conclusions

In this contribution, we have determined the effect of carrier frequency offset on the performance of MC-CDMA and MC-DS-CDMA, which are both a combination of the multicarrier transmission technique and the CDMA multiple access technique. We have determined analytical expressions for the SNR at the input of the decision device, and for the degradation of the SNR caused by carrier frequency offset. Under the assumption of full load ( $N_u = N_c$ ), both systems yield the same degradation when their ratio of frequency offset  $\Delta F$  to carrier spacing  $1/(N_f T)$  is the same. Hence, for given values of the data rate and the spreading factor, the degradation of MC-DS-CDMA is larger (smaller) than for MC-CDMA when the number of carriers in the former system is larger (smaller) than the spreading factor.

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