PILOT-SYMBOL ASSISTED ITERATIVE CARRIER SYNCHRONIZATION FOR BURST TRANSMISSION

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Abstract - **This contribution considers the joint estimation of the carrier phase and the frequency offset from a noisy linearly modulated burst signal containing random data symbols (DS) as well as known pilot symbols (PS). The corresponding Cramer-Rao lower bound (CRB) is derived. This bound indicates that it is potentially more accurate to estimate carrier phase and frequency from such a 'hybrid' burst than from a burst without PS or from the limited number of PS only. The new bound is then compared with the performance of new and existing carrier synchronizers. We present the iterative Soft-Decision-Directed estimator with combined Data-Aided/Non-Data-Aided initialization, which performs closely to the CRB, and provides a large improvement over the classical Non-Data-Aided estimator at low and moderate Signal-to-Noise Ratio.**

Carrier Synchronization, Cramer-Rao Lower Bound, Phase Estimation, Frequency Estimation, Pilot Symbols

I. INTRODUCTION

In burst transmission with coherent detection, the recovery of the carrier phase and frequency offset is a key aspect. We assume that phase coherence over successive bursts cannot be maintained, so that the carrier phase and frequency offset have to be recovered on a burst-by-burst basis. Most burst synchronizers belong to one of the following types: *Data-Aided* (DA) synchronizers use known pilot symbols (PS), while *Non-Data-Aided* (NDA) and *Decision-Directed* (DD) estimators operate on modulated data symbols (DS). DD estimators are similar to DA estimators, but use, instead of PS, hard or soft decisions regarding the DS, that are provided by the detector; NDA estimators apply a non-linearity to the received signal to remove the data modulation.

Assuming the parameter estimate is unbiased, the variance of the estimation error is often used as a performance measure. The Cramer-Rao lower bound (CRB) is a fundamental lower bound on the variance of any unbiased estimate [1], and is also known to be asymptotically achievable for a large enough number of observations, under mild regularity conditions. The $CRB_{PS}(N_p)$ for phase and/or frequency estimation from N_p known PS has been derived in [2-3]. The $CRB_{DS}(N_d)$ for joint carrier phase and frequency estimation from N_d random DS has been addressed in [4-7]. In order to avoid the computational complexity related to the true CRB_{DS}, a modified CRB (MCRB) has been derived in [8,9]. The MCRB is much easier to evaluate than the CRB, but is in general looser (i.e. lower) than the true CRB, especially at low Signal-to-Noise Ratio (SNR). In [10], the high-SNR limit of the CRB_{DS} has been obtained analytically, and has been shown to coincide with the MCRB.

Except for very high SNR and/or very large bursts, accurate estimation of large frequency offsets is far from trivial, because of the *threshold phenomenon* [2]. In [11], it has been shown that a frequency estimator that utilizes both PS and DS may provide the combined advantages of DA estimators and NDA estimators, and allow more accurate synchronization at lower SNR. It is obvious that phase estimation may also benefit from the simultaneous use of PS and DS to resolve the *phase ambiguity*, caused by the rotational symmetry of the constellation.

This contribution further examines joint phase and frequency estimation from the observation of a burst that contains N_p pilot symbols as well as N_d data symbols. We derive the corresponding true $CRB_{PS\text{-}DS}(N_p,N_d)$, which can be viewed as a generalization of both CRB_{DS} and CRB_{PS}. Comparing to the true CRB the performance of the estimation algorithm from [11], that uses both the PS and the DS, it is concluded that more efficient algorithms may exist that perform more closely to the CRB. We propose some new algorithms, including an iterative soft-DD estimator with combined DA/NDA initialization that yields a close agreement between the simulated performance and $CRB_{PS\text{-}DS}$.

II. PROBLEM FORMULATION

Consider the following observation model

$$r_k = a_k e^{j\theta_k} + w_k, \qquad k \in I = \{-K, -K+1, \ldots, K\} \qquad (1)$$

In (1), $\{a_k: k \in I\}$ is a sequence of $L = 2K+1$ transmitted M-PSK, M-QAM or M-PAM symbols ($E[|a_k|^2]=1$). The symbol a_k denotes a known PS for k belonging to the set of indices $I_p=\{k_0, k_1, \ldots, k_{Np-1}\} \subseteq I$, where N_p denotes the number of PS. For $k \in I_d=\{I \setminus I_p\}$, a_k denotes an unknown DS. The N_d ($=L-N_p$) DS are assumed to be statistically independent and

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uniformly distributed over the constellation. The sequence $\{w_k: k \in I\}$ consists of zero-mean complex Gaussian noise variables, with independent real and imaginary parts each having a variance of $N_0/2E_s$. The quantities E_s and N_0 denote the symbol energy and the noise power spectral density, respectively. The quantity $\theta_k = \theta + 2\pi kFT$ is the instantaneous carrier phase, where θ represents the carrier phase at k = 0, F is the frequency offset and T is the symbol duration. Both θ and F are unknown but deterministic parameters.

Let us denoted by $p(\mathbf{r};\mathbf{u})$ the probability density function (pdf) of the observation vector **r**, where **u** is an unknown deterministic vector parameter. Suppose one is able to produce from **r** an unbiased estimate $\hat{\mathbf{u}}$ of the parameter **u**. Then the estimation error covariance matrix $R_{\hat{\mathbf{u}}-\mathbf{u}} = E[(\hat{\mathbf{u}}-\mathbf{u})(\hat{\mathbf{u}}-\mathbf{u})^T]$ satisfies

$$R_{\hat{\mathbf{u}}-\mathbf{u}} - J^{-1}(\mathbf{u}) \geq 0 \qquad (\textit{positive semi definite}) \tag{2}$$

where $J(\mathbf{u})$ is the Fisher information matrix (FIM). The (i,j)-th element of $\mathbf{J}(\mathbf{u})$ is given by

$$\mathbf{J}_{i,j}(\mathbf{u}) = E_{\mathbf{r}}\left[\frac{\partial}{\partial u_i}\ln(p(\mathbf{r};\mathbf{u}))\frac{\partial}{\partial u_j}\ln(p(\mathbf{r};\mathbf{u}))\right] \tag{3}$$

Note that $\mathbf{J}(\mathbf{u})$ is a symmetrical matrix. When the element $\mathbf{J}_{ij}(\mathbf{u}) = 0$, the parameters u_i and u_j are said to be *decoupled*. The expectation $E_{\mathbf{r}}[.]$ in (3) is with respect to $p(\mathbf{r};\mathbf{u})$. The pdf $p(\mathbf{r};\mathbf{u})$ of **r**, corresponding to a given value of **u**, is called the *likelihood function* of **u**, $\ln(p(\mathbf{r};\mathbf{u}))$ is the *log-likelihood function* of **u**. When the observation **r** depends not only on the parameter **u** to be estimated but also on a nuisance vector parameter **v**, $p(\mathbf{r};\mathbf{u})$ is obtained by averaging the likelihood function $p(\mathbf{r}|\mathbf{v};\mathbf{u})$ of the vector $(\mathbf{u},\mathbf{v})$ over the a priori distribution of the nuisance parameter: $p(\mathbf{r};\mathbf{u}) = E_{\mathbf{v}}[p(\mathbf{r}\mid\mathbf{v};\mathbf{u})]$.

Considering the joint estimation of the carrier phase θ and frequency offset F from the observation vector $\mathbf{r} = \{r_k\}$ from (1), we take $\mathbf{u} = (\theta,F)$. The nuisance parameter vector $\mathbf{v}=\{a_k: k\in I_d\}$ consists of the unknown DS. Within a factor not depending on F, θ and **a**, $p(\mathbf{r}|\mathbf{a};F,\theta)$ is given by

$$p(\mathbf{r}\mid\mathbf{a};F,\theta) = \prod_{k\in I} F(a_k,\tilde{r}_k) \tag{4}$$

where $F(a_k,\tilde{r}_k) = e^{\frac{E_s}{N_0}\left(2\,\mathrm{Re}(a_k^*\tilde{r}_k)-|a_k|^2\right)}$ and $\tilde{r}_k = r_k e^{-j(2\pi kFT+\theta)}$. Averaging (4) over the DS yields $p(\mathbf{r}|F,\theta)$. For $\ln(p(\mathbf{r}|F,\theta))$ we obtain, within a term that does not depend on (F,θ)

$$\ln p(\mathbf{r}\mid F,\theta) = 2\frac{E_s}{N_0}\mathrm{Re}\left(e^{-j\theta}\sum_{k\in I_p}a_k^*\tilde{r}_k\right) + \sum_{l\in I_d}\ln I(\tilde{r}_l) \tag{5}$$

where $I(\tilde{r}_k) = \sum_{i=0}^{M-1}F(\alpha_i,\tilde{r}_k)$, and $\{\alpha_0, \alpha_1, ..., \alpha_{M-1}\}$ denotes the set of constellation points.

It follows from (2) that the error variance regarding the estimation of θ and F is lower bounded by the Cramer-Rao Bound (CRB):

$$E_r[(\hat{\theta}-\theta)^2] \geq CRB_\theta^{PS-DS} = \left(\mathbf{J}^{-1}_{(\theta,F)}\right)_{11} \tag{6}$$

$$E_r[(\hat{F}-F)^2] \geq CRB_{FT}^{PS-DS} = \left(\mathbf{J}^{-1}_{(\theta,F)}\right)_{22} \tag{7}$$

Similarly, (2) yields a lower bound on the variance of estimation error on the instantaneous phase:

$$\begin{aligned} E_r[(\hat{\theta}_k-\theta_k)^2] &\geq \mathrm{CRB}_{\theta_k}^{PS-DS} \\ &= \left(\mathbf{J}^{-1}_{(\theta,F)}\right)_{11} + 4\pi kT\left(\mathbf{J}^{-1}_{(\theta,F)}\right)_{12} + 4(\pi kT)^2\left(\mathbf{J}^{-1}_{(\theta,F)}\right)_{22} \end{aligned} \tag{8}$$

As the evaluation of the various expectations in $\mathbf{J}(\theta,F)$ and $p(\mathbf{r}|\theta,F)$ is quite tedious, a simpler lower bound called modified CRB (MCRB), has been derived in [8,9], i.e., $E[(\hat{x}-x)^2] \geq CRB_x^{PS-DS} \geq MCRB_x^{PS-DS}$, where $MCRB_x^{PS-DS}$ is defined as CRB_x^{PS-DS} in (6-8) but with the FIM $\mathbf{J}(\theta,F)$ replaced with the modified FIM (MFIM) $\mathbf{J}_M(\theta,F)$ given by

$$\mathbf{J}_M = \frac{2E_s}{N_0}L\begin{bmatrix}1 & 0\\ 0 & \pi^2T^2(L^2-1)/3\end{bmatrix} \tag{9}$$

III. EVALUATION OF THE CRB

Differentiation of (5) with respect to θ and F and substituting the result into (3) yields

$$\begin{aligned}\mathbf{J}^{PS-DS} &= \frac{2E_s}{N_0}\begin{bmatrix}\sum_{k\in I}\beta_k & 2\pi T\sum_{k\in I}k\beta_k\\ 2\pi T\sum_{k\in I}k\beta_k & (2\pi T)^2\sum_{k\in I}k^2\beta_k\end{bmatrix}\\ &= J_{11}\begin{bmatrix}1 & 2\pi Tk_G\\ 2\pi Tk_G & (2\pi T)^2(k_G^2+\sigma_G^2)\end{bmatrix}\end{aligned} \tag{10}$$

where

$$\beta_k = \begin{cases} |a_k|^2, & k\in I_p\\ \dfrac{2E_s}{N_0}E_r\left[\left(\sum_{i=0}^{M-1}F(\alpha_i,r)\,\mathrm{Im}(\alpha_i^*r)/I(r)\right)^2\right], & k\in I_d\end{cases} \tag{11}$$

and

$$k_G = \frac{\sum_I k\beta_k}{\sum_I \beta_k} \qquad \sigma_G^2 = \frac{\sum_I (k-k_G)^2\beta_k}{\sum_I\beta_k} \tag{12}$$

In (11), $E_r[.]$ denotes the average over $r = a+n$, where a is a random variable that takes any value from the constellation with equal probability, and n is complex zero-mean AWGN with variance equal to E_s/N_0. The quantity k_G can be interpreted as the center of gravity of the sequence $\{\beta_k\}$. We obtain $J_{12}^{PS-DS} \neq 0$, unless $k_G = 0$, which is achieved if both PS and DS are each located symmetrically about zero, and the PS satisfy $|a_k|=|a_{-k}|$. For $k_G \neq 0$, the parameters θ and F are coupled, meaning that the inaccuracy in the phase estimate has an impact on the frequency offset estimation and vice versa.

Note that the FIM does not depend on θ or F. Substituting (10) into (6-8) yields

$$E[(\hat{\theta}-\theta)^2] \geq CRB_{\theta}^{PS-DS} = \frac{1}{J_{11}}\left(1+\frac{k_G^2}{\sigma_G^2}\right) \quad (13)$$

$$E[(\hat{F}T - FT)^2] \geq CRB_{FT}^{PS-DS} = \frac{1}{4\pi^2\sigma_G^2 J_{11}} \quad (14)$$

$$E[(\hat{\theta}_k-\theta_k)^2] \geq CRB_{\theta_k}^{PS-DS} = \frac{1}{J_{11}}\left(1+\frac{(k-k_G)^2}{\sigma_G^2}\right) \quad (15)$$

The lower bound on $E[(\hat{\theta}_k-\theta_k)^2]$ is *quadratic* in k. Its *minimum* value is achieved at k = k_G, and is equal to $1/J_{11}^{PS-DS}$, which is the CRB for the estimation of the carrier phase when the frequency offset is a priori known. Note from (10-11) that $1/J_{11}^{PS-DS}$ depends on the number (N_p) of PS and the number (N_d) of DS, but not on the specific position of the PS in the burst. The bound (15) achieves its *maximum* value at k = -sign(k_G).K, i.e. at one of the edges of the burst interval (or at both edges if k_G=0). The difference between the minimum and the maximum value of (15) over the burst amounts to $\Delta^2 4\pi^2 CRB_{FT}$, where Δ = K+|k_G.| represents the distance (in symbols intervals) between the positions of the minimum and maximum value of the CRB (15). For given values of $1/J_{11}^{PS-DS}$ and CRB_{FT}^{PS-DS}, the detection of symbols located near the edge k = -sign(k_G).K suffers from a larger instantaneous phase error variance as Δ increases.

Let us define by $\mathbf{J}_\infty$ and $\mathbf{J}_0$ the high-SNR and low-SNR asymptotic FIM, that are obtained as the limit of the FIM for $E_s/N_0 \to \infty$ and $E_s/N_0 \to 0$, respectively. It can be verified that $\mathbf{J}_0$ equals the FIM for estimation from the PS only, that is given by (10) in which the summation over I is replaced with a summation over I_p only [3]; $\mathbf{J}_\infty$ equals the MFIM from (9), that has been shown to coincide with the high-SNR limit of the FIM for estimation from L DS in [10]. This indicates that at very high (very low) SNR, NDA (DA) estimation techniques may perform close to optimal.

Numerical results are obtained for a QPSK constellation. We assume a burst of 321 symbols. We restrict ourselves to symmetric burst structures yielding k_G = 0, so that carrier phase and frequency estimation are decoupled. We will consider two different burst structures: burst structure #1 = (128 DS, 65 PS, 128 DS), burst structure #2 = (112 DS, 32 PS, 33 DS, 32 PS, 112 DS) [11]. Both burst structures contain approximately 20% PS. Figs. 1-2 show the ratio CRB^{PS-DS}/MCRB for carrier phase and frequency estimation as a function of the SNR. The following observations can be made

- In Fig.1, the curves for burst structures #1 and #2 coincide, as $1/J_{11}$ does not depend on the specific location of the PS in the burst.
- The CRBs become close to their low SNR asymptote only at very low SNR. This low SNR asymptote is a fundamental lower bound on the performance of any DA estimator operating on the known PS only. The FIM related to DA joint phase and frequency estimation from an arbitrary arrangement of PS, derived in [3], shows that spreading the PS over the burst decreases the CRB_{FT}^{PS}. Indeed, our results predict a better DA frequency estimation performance for burst structure #2 than for burst structure #1.
- For values of SNR larger than about 10 dB, the CRBs become close to the MCRB.
- $CRB_{PS-DS}(N_p, N_d)$ is smaller than both $CRB_{PS}(N_p)$ and $CRB_{DS}(N_p+N_d)$. This indicates that it is potentially more accurate to estimate F and θ from a hybrid burst than from a burst without PS, or from a limited number of PS only. The ratios $CRB_{DS}(N_p+N_d)$ / $CRB_{PS-DS}(N_p,N_d)$ and $CRB_{DS}(N_p)$ / $CRB_{PS-DS}(N_p,N_d)$ depend on the operating SNR and on the burst structure, and indicate to what extent synchronizer performance can be improved by making clever use of the knowledge about the PS and of the presence of the DS in the estimation process.

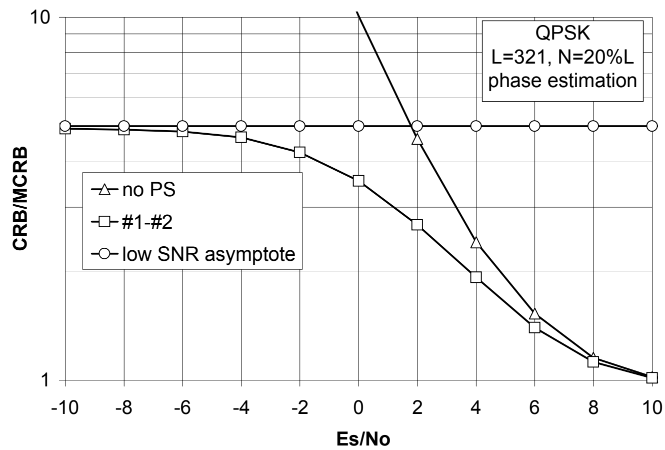

Fig.1: CRB for phase estimation

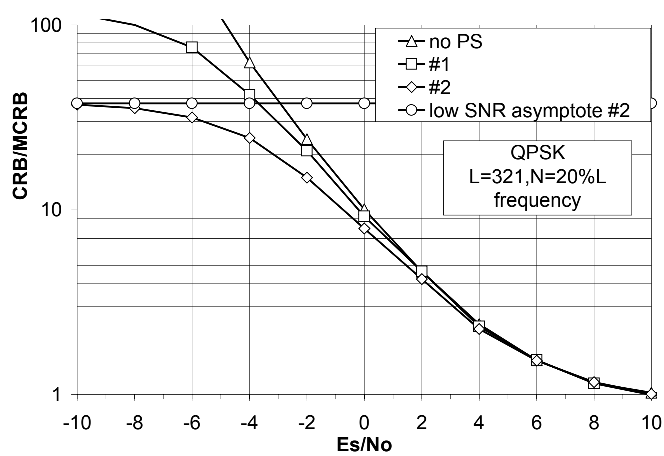

Fig.2: CRB for frequency estimation

IV. PRACTICAL ESTIMATOR PERFORMANCE

The general maximum likelihood (ML) estimator is known to be asymptotically optimal in the sense that it achieves the performance predicted by the CRB for large data records. However, the performance for finite signal durations cannot be determined analytically. In this section the simulated mean square estimation error (MSEE) of practical synchronizers is compared to the CRB.

A. DA synchronization

The DA estimates using only the N_p PS are given by

$$\hat{F} = \arg\max_{\tilde{F}}\left(\left|\sum_{k\in I_p} a_k^* r_k e^{-j2\pi k\tilde{F}T}\right|\right) \quad (16)$$

$$\hat{\theta} = \arg\left\{\sum_{k\in I_p} a_k^* r_k e^{-j2\pi k\hat{F}T}\right\} \quad (17)$$

At high SNR, the $CRB_{PS}(N_p)$ is reached. It is well known that, below a certain SNR threshold, the performance dramatically degrades across a narrow SNR interval, with an MSEE much larger than the CRB. This so-called *threshold phenomenon* results from the occurrence of estimates with large errors, i.e., outlier estimates [2]. The presence of important secondary peaks in the likelihood function results in a large probability of generating outlier frequency estimates at lower SNR, because these secondary peaks can more easily exceed the central peak when noise is added. The SNR threshold decreases with the number of available signal samples N_p. For N_p consecutive PS (as in burst structure #1), the threshold is in general very low (even for small N_p) so that the DA estimator usually operates above threshold. However, the SNR threshold tends to increase as the PS are separated by DS [3,11]. In [11], burst structure #2 has been shown to provide a good compromise between a small value of CRB_{FT}^{PS} and a small SNR threshold.

B. NDA synchronization

Assuming a QPSK constellation, the NDA estimates are given by [2,12]

$$\hat{F} = \frac{1}{M}\arg\max_{\tilde{F}}\left|\sum_{k=-K}^{K}|r_k|^2 e^{j4\arg\{r_k\}} e^{-j2\pi k\tilde{F}T}\right| \quad (18)$$

$$\hat{\theta} = \frac{1}{M}\arg\left\{\sum_{k=-K}^{K}|r_k|^2 e^{j4\arg\{r_k\}} e^{-j8\pi k\hat{F}T}\right\} \quad (19)$$

The resulting MSEE converges to $CRB_{DS}(L)$ at high SNR. Simulation results indicate, however, that the value of SNR at which the MSEE becomes close to the CRB may be quite large. The SNR threshold for the NDA estimator is much higher than for the DA estimator, as the non-linearity increases the noise level. To cope with this problem, a two-stage coarse-fine DA-NDA estimator has been proposed in [11]. A ML DA estimator is used to coarsely locate the frequency offset, and then the more accurate NDA estimator attempts to improve the estimate within the uncertainty of the coarse estimator. In fact, the search range of the NDA estimator is restricted to the neighborhood of the peak of the DA based likelihood function. This excludes a large percentage of secondary peaks from the search range of the NDA estimator, and thus considerably reduces the probability to estimate an outlier frequency. Assuming the MSEE of the initial DA estimate equals the $CRB_{FT}^{PS}(N_p)$, this uncertainty range can be determined as $\pm m\sqrt{CRB_{FT}^{PS}(N_p)}$, where m should be carefully chosen. After frequency and phase correction, the samples for $k\in I_p$ are compared to the original PS and, if necessary, an extra multiple of $\pi/2$ is compensated for. A major disadvantage of this DA-NDA algorithm is that it does not exploit the knowledge of the PS in the NDA fine estimation step. Therefore, its MSEE is lower bounded by the $CRB_{DS}(N_p+N_d) \geq CRB_{PS\text{-}DS}(N_p,N_d)$. This implies that the DA-NDA algorithm is intrinsically suboptimal in the sense that under no circumstances its performance may meet the $CRB_{PS\text{-}DS}$. Some other estimator may yield a MSEE between CRB_{DS} and $CRB_{PS\text{-}DS}$, but it should fully exploit the knowledge of the PS.

C. Iterative DD synchronization

DD estimators extend the sum over I_p in (16-17) with terms over I_d in which the quantities a_k are replaced by hard (hDD) or soft (sDD) decisions, that are based upon a previous estimate of (θ, F). For QPSK, the soft decisions are given by [4]

$$\hat{a}_k = \frac{\sinh\left[\frac{2E_s}{N_0}\mathrm{Re}(\tilde{r}_k^{(n-1)})\right] + j\sinh\left[\frac{2E_s}{N_0}\mathrm{Im}(\tilde{r}_k^{(n-1)})\right]}{\cosh\left[\frac{2E_s}{N_0}\mathrm{Re}(\tilde{r}_k^{(n-1)})\right] + \cosh\left[\frac{2E_s}{N_0}\mathrm{Im}(\tilde{r}_k^{(n-1)})\right]} \quad (20)$$

In (20), $\tilde{r}_k^{(n-1)} = r_k e^{-j(\hat{\theta}^{(n-1)}+2\pi k\hat{F}^{(n-1)}T)}$. The normal operating SNR of the DD estimators is situated above threshold. The required an initial estimate $(\hat{\theta}^{(0)}, \hat{F}^{(0)})$ can be obtained from the NDA method; however, the performance below the NDA threshold rapidly degrades, because of an inaccurate initial estimate. If PS are available, it is better to use DA or combined DA-NDA initialization. We will further refer to these schemes as DA-hDD, DA-sDD, DA-NDA-hDD and DA-NDA-sDD. After phase and frequency correction, the samples for $k\in I_p$ are compared to the original PS and, if necessary, an extra multiple of $\pi/2$ is compensated for.

Numerical results pertaining to the different algorithms are obtained for a QPSK constellation. We assume a burst with 257 DS and 64 PS that are organized as in burst structure #2. In Figs. 3-4, we have plotted the ratio MSEE/MCRB for the estimation of F and θ as a function of the SNR. The phase error of the NDA estimator is measured modulo $\pi/2$. For the DA-NDA estimation we chose m=3. Our results show that:

- The DA estimator achieves optimal $CRB_{PS}(N_p)$ performance, which is considerably worse than the performance of the hybrid estimators. The ratio MSEE/MCRB regarding the DA frequency estimate equals 37.17 (curve not shown) over the whole SNR range from Fig. 4.
- Above its SNR threshold (at about 5dB), the NDA estimator performs very closely to the $CRB_{DS}(L)$.
- At high SNR, the performance of the DA-NDA estimator matches that of the NDA estimator, but the performance below

the SNR threshold degrades less rapidly and is still adequate for reliable receiver operation.

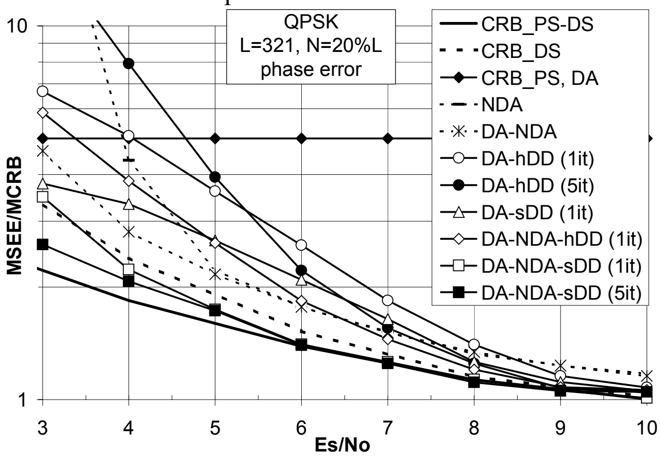

Fig.3: MSEE of the phase estimate

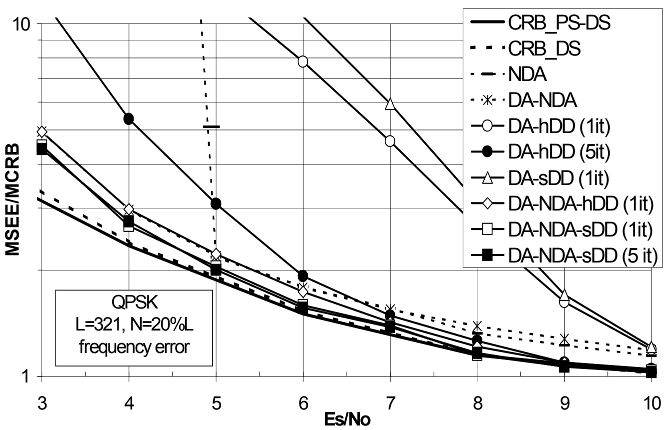

Fig. 4: MSEE of the frequency estimate

- The MSEE resulting from the DA-hDD and DA-NDA-hDD estimators reaches a steady state after about five iterations. Only at high SNR these estimators outperform the DA-NDA estimator. At (very) low SNR, the performance gets worse when the iteration number increases. This can be seen in Fig. 3 when comparing the DA-hDD after 1 and 5 iterations; the same occurs for the DA-NDA-hDD algorithm (curves not shown). This unexpected behavior indicates that at low SNR the initial estimates are more accurate than the steady-state hDD estimates. Hence, hard decisions are not useful at low SNR.
- The MSEE resulting from the DA-sDD estimator reaches a steady state after 10 to 20 iterations. The DA-NDA-sDD estimator yields the same steady state performance as the DA-sDD estimator, but after considerably less (no more than 5) iterations. This indicates the importance of an accurate initial estimate to speed up convergence. Because these estimators fully exploit PS information, their estimation variance must be compared with $CRB_{PS\text{-}DS}$. The steady state phase MSEE of the DA-NDA-sDD estimator is located between CRB_{DS} and $CRB_{PS\text{-}DS}$, and attains $CRB_{PS\text{-}DS}$ for SNR ≥ 6 dB (Fig. 3). The DA(-NDA)-sDD estimators outperform by far the DA(-NDA)-hDD estimators and provide a considerable improvement over the DA-NDA estimator.

V. Conclusions

In this contribution, we have investigated the joint phase and frequency estimation from the observation of a 'hybrid' burst that contains PS as well as DS. We have compared the $CRB_{PS\text{-}DS}$ with the performance of new and existing carrier synchronizers. Numerical evaluation of this CRB shows that it is potentially more accurate to estimate carrier phase and frequency from a hybrid burst than from a burst without PS or from a limited number of PS only. We have pointed out that the hybrid DA-NDA estimator proposed in [11] is suboptimal, because it does not fully exploit the knowledge about the PS. Further, we have proposed a new iterative sDD estimator with combined DA/NDA initialization, that outperforms the DA-NDA estimator and operates closely to the $CRB_{PS\text{-}DS}$.

References

[1] H.L. Van Trees, *Detection, Estimation and Modulation Theory*. New York: Wiley, 1968

[2] D.C, Rife and R.R. Boorstyn, "Single-tone parameter estimation from discrete-time observations," *IEEE Trans. Inf. Theory*, vol. IT-20, No. 5, pp. 591-597, Sept. 1974

[3] J.A. Gansman, J.V. Krogmeier and M.P. Fitz, "Single Frequency Estimation with Non-Uniform Sampling," in *Proc. of the 13th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, pp.878-882, Nov. 1996

[4] W.G. Cowley, "Phase and frequency estimation for PSK packets: Bounds and algorithms," *IEEE Trans. Commun.*, vol. COM-44, pp. 26-28, Jan. 1996

[5] F. Rice, B. Cowley, B. Moran, M. Rice, "Cramer-Rao lower bounds for QAM phase and frequency estimation," *IEEE Trans. Commun.*, vol. 49, pp 1582-1591, Sep. 2001

[6] N. Noels, H. Steendam and M. Moeneclaey, "The true Cramer-Rao bound for phase-independent carrier frequency estimation from a PSK signal," in *Proc. IEEE Globecom 2002*, Taipe, Taiwan, paper CTS-04-2, Nov. 2002

[7] N. Noels, H. Steendam and M. Moeneclaey, "The Impact of the Observation Model on the Cramer-Rao Bound for Carrier and Frequency Synchronization", in *Proc. IEEE Int. Conf. Communications 2003*, Anchorage, Alaska, Paper CT04-1, 11-15 May 2003

[8] A.N. D'Andrea, U. Mengali and R. Reggiannini, "The modified Cramer-Rao bound and its applications to synchronization problems," *IEEE Trans. Comm.*, vol COM-24, pp. 1391-1399, Feb.,Mar.,Apr. 1994.

[9] F. Gini, R. Reggiannini and U. Mengali, "The modified Cramer-Rao bound in vector parameter estimation," IEEE Trans. Commun., vol. CON-46, pp. 52-60, Jan. 1998

[10] M. Moeneclaey, "On the true and the modified Cramer-Rao bounds for the estimation of a scalar parameter in the presence of nuisance parameters," *IEEE Trans. Commun.*, vol. COM-46, pp. 1536-1544, Nov. 1998

[11] B. Beahan, "Frequency Estimation of Partitioned Reference Symbol Sequences," Master Thesis, University of South Australia, April 2001, available at *www.itr.unisa.edu.au/rd/pubs/thesis/theses.html*

[12] A.J. Viterbi and A.M. Viterbi, "Nonlinear estimation of PSK-modulated carrier phase with application to burst digital transmission," *IEEE Trans. Inform. Theory*, vol. IT-29, pp. 543-551, July 1983

