PERFORMANCE EVALUATION AND PARAMETER OPTIMIZATION OF MC-CDMA

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Abstract The performance of multicarrier systems depends on the propagation channel behavior. The latter is subject to time and/or frequency selectivity. The designer has to select properly the guard interval and the number of carriers for a given system bandwidth to combat the channel dispersiveness. In this paper we investigate the sensitivity of MultiCarrier Code Division Multiple Access (MC-CDMA) performance to these parameters in different environments. We derive closed form expressions of the useful and the different interference powers after Maximum Ratio Combining (MRC) and despreading. The optimum parameters correspond to the minimum of the Signal to Noise Ratio Degradation (SNRD). It turns out that the derivations in [1] restricted to the Orthogonal Frequency Division Multiplexing (OFDM) system, are a particular case of our results. Numerical evaluation of the analytical expressions reveals that the optimum parameters of the MC-CDMA and its corresponding OFDM system are similar and depend in the same way on the channel characteristics.

1. Introduction

Multicarrier systems have emerged as a powerful candidate to wireless communication systems. The OFDM technique was selected as a transmission standard technique in Digital Audio Broadcasting (DAB) from satellite or fixed terrestrial to mobile users [2] and in Digital Video Broadcasting in Europe [3]. Another important application of MC systems can be found in transmission of high data rate over twisted pair [4]. So far, the conventional CDMA technique that is the core wireless technology in the third generation is limited by channel dispersion, causing Inter Symbol Interference (ISI) which requires advanced detection algorithms to be removed. The combination of the OFDM and CDMA, i.

e. MC-CDMA [5], has drawn a lot of attention due to its robustness to channel dispersion, hence ISI, and its ability to accommodate a higher number of users as compared to CDMA only. Basically we can distinguish three versions namely MC-CDMA, MC-DS-CDMA and MultiTone (MT)-CDMA [5].

The robustness of the MC systems to the channel frequency selectivity that induces Inter Carrier Interference (ICI) (which is caused by other carriers from the same MC symbol) and ISI (which is caused by other MC symbols) is obtained by transmitting the data over subchannels with a very low bandwidth compared to the total system bandwidth. If the subchannel bandwidth is smaller than the channel Coherence Bandwidth (CB), the interference can be removed completely after the FFT at the receiver. Otherwise, the interference can be reduced by adding a guard interval per MC symbol. The subchannel bandwidth is proportional to the inverse of the MC symbol period. Therefore increasing this parameter will reduce the interference level at the input of the decision device. However, there is another constraint that should be fulfilled in order to avoid interference caused by the time selectivity of the channel: the MC symbol duration has to be smaller than the channel Coherence Time (CT) duration related to the time variation of the channel. If this condition is not respected, ICI will be introduced.

Hence the designer has to select the optimum parameters according to a certain criterion. The few contributions found in the literature deal with the OFDM parameter optimization. In [6] and [3] the optimum number of OFDM subchannels is found in the absence of the ISI, i.e. the guard interval is sufficiently high to cope with the ISI. In [1] the authors extend the previous contributions to deal with the effect of the ISI in the OFDM system when the guard interval is smaller than the delay spread.

In this contribution we extend the work done in [1] to MC-CDMA systems equipped with the MRC equalizer and using Walsh Hadamard codes with a random overlay sequence as spreading codes. The results obtained in [1] for the OFDM system can be seen as particular case of our MC-CDMA system with a processing gain of one and will serve as a reference system. We derive analytical expressions for the useful and interference power and the SNRD after MRC equalization and despreading. The SNRD will serve as a cost function to be minimized in order to find the optimum number of carriers and guard interval. In the numerical evaluation, we consider an aeronautical channel with different Dopplers to come up with general optimization rules that are channel independent.

The outline of this paper is as follows: first the system model is described in section 2. In section 3, we derive analytical expressions for the useful power, the interference power and the SNRD. Numerical evaluations can be found in section 4 before we conclude in section 5.

2. System model

The composite signal to be transmitted is obtained as follows: after serial to parallel conversion of P symbols, the p-th data symbol $a_{i,p}$ of the i-th block is spread by the spreading sequence $c_{p,s}$ that repeats from frame to frame with $p = 0, \ldots, P-1$ and $s = 0, \ldots, N_s$ where N_s is the spreading factor. Each chip $a_{i,p}c_{p,s}$ is mapped to a carrier denoted $n_{p,s}$ where $0 \leq n_{p,s} \leq N-1$. In total we have $N = PN_s$ carriers. In order to achieve frequency diversity, the assignment of the carriers to chips is made such that the frequency separation among carriers conveying the chips of the same data symbol is maximized. One possible mapping is as follows: the p-th data stream is transmitted on N_s carriers with frequencies $f_{p+sP} = f_c + (p+sP)/(NT)$ where $1/T$ is the system bandwidth and f_c is the RF carrier frequency of the MC system. After feeding the i-th block of spread data symbols to an Inverse Fast Fourier Transform (IFFT) of length N, the resulting block of N time domain samples is cyclically extended by a prefix of ν samples. The m-th sample of the resulting MC block of $N+\nu$ samples is given by:

$$s_{i,m} = \sqrt{\frac{E_s}{N+\nu}} \sum_{p=0}^{P-1} a_{i,p} \sum_{s=0}^{N_s-1} c_{p,s} \exp\left[j2\pi m \frac{n_{p,s}}{N}\right], \quad m = -\nu, \ldots, N-1. \tag{1}$$

Considering a tapped delay line channel $h(l;k)$ with symbol spacing, the received samples $r(k)$ of the j-th frame (after removal of the cyclic prefix) are:

$$r(k) = \sum_{i=\infty}^{+\infty} \sum_{m=-\nu}^{N-1} s_{i,m} h(k-m-i(N+\nu);k) + n(k), \quad k = j(N+\nu), \ldots, j(N+\nu)+N-1. \tag{2}$$

We concentrate on the MC symbol detection during the frame $j=0$. The output $b_{0,p',s'}$ of the FFT corresponding to the chip $a_{0,p'}c_{p',s'}$ can be written as:

$$b_{0,p',s'} = \sqrt{\frac{1}{N}} \sum_{k=0}^{N-1} r(k) \exp\left[-j2\pi k \frac{n_{p',s'}}{N}\right].$$

After the expansion of the received samples, it can be shown that

$$\begin{aligned} b_{0,p',s'} &= \sqrt{\frac{N}{N+\nu}E_s} \sum_{i=-\infty}^{+\infty} \sum_{p=0}^{P-1} \sum_{s=0}^{N_s-1} a_{i,p} c_{p,s} \gamma\left(n_{p',s'}; n_{p,s}; i\right) \\ &+ \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} n(k) \exp\left[-j2\pi k \frac{n_{p',s'}}{N}\right] \end{aligned} \tag{3}$$

where

$$\begin{aligned} \gamma\left(n_{p',s'}; n_{p,s}; i\right) &= \frac{1}{N} \sum_{m=-\nu}^{N-1} \sum_{k=0}^{N-1} \exp\left[-j2\pi \frac{n_{p',s'}k - n_{p,s}m}{N}\right] \\ & \qquad h\left(k-m-i(N+\nu); k\right). \end{aligned} \tag{4}$$

Note that the useful part in the FFT output $b_{0,p',s'}$ in (3) can be expressed as

$$\left(b_{0,p',s'}\right)_U = \sqrt{\frac{N}{N+\nu}E_s} a_{0,p'} c_{p',s'} \gamma\left(n_{p',s'}; n_{p',s'}; 0\right). \tag{5}$$

Therefore, the MRC equalizer multiplies the $n_{p',s'}$-th FFT output with $d_{0,p',s'} = \gamma^*\left(n_{p',s'}; n_{p',s'}; 0\right)$. After MRC and despreading of the different chips related to the symbol index p', the decision variable $y_{0,p'}$ is as follows:

$$y_{0,p'} = \sum_{s'=0}^{N_s-1} c^*_{p',s'} d_{0,p',s'} b_{0,p',s'} \tag{6}$$

3. Performance analysis

Based on the decision variable in (6), we can identify on top of the useful component and the AWGN component two sources of interference: the Inter-Symbol-Interference (ISI) due to the frequency selectivity of the channel and the Inter-Carrier-Interference (ICI) due to both the time and frequency selectivity of the channel. The power at the output of the equalizer can be decomposed as $E_s \frac{N}{N+\nu}\left(P_U + P_{ICI} + P_{ISI}\right) + P_{AWGN}$where P_X, $X = U,\ ICI,\ ISI,\ AWGN$ corresponds to the useful, ICI, ISI and AWGN powers, respectively:

$$P_U = \mathrm{E}\left\{\left|\Gamma\left(n_{p'}; n_{p'}; 0\right)\right|^2\right\} \tag{7}$$

$$P_{ICI} = \sum_{\substack{p=0 \\ p\neq p'}}^{P-1} \mathrm{E}\left\{\left|\Gamma\left(n_{p'}; n_p; 0\right)\right|^2\right\} \tag{8}$$

$$P_{ISI} = \sum_{\substack{i=-\infty \\ i\neq 0}}^{+\infty} \sum_{p=0}^{P-1} \mathrm{E}\left\{\left|\Gamma\left(n_{p'}; n_p; i\right)\right|^2\right\} \tag{9}$$

$$P_{AWGN} = \mathrm{E}\left\{\left|\sqrt{\frac{1}{N}} \sum_{s'=0}^{N_s-1} c^*_{p',s'} d_{0,p',s'} \sum_{k=0}^{N-1} n(k) \exp\left[-j2\pi n_{p',s'} k/N\right]\right|^2\right\} \tag{10}$$

where $\Gamma\left(n_{p'}; n_p; i\right) = \sum_{s',s=0}^{N_s-1} c^*_{p',s'} c_{p,s} d_{0,p',s'} \gamma\left(n_{p',s'}; n_{p,s}; i\right)$.

The averaging in P_X is with respect to the statistics of the data symbols, the spreading chips and the channel. In the following we derive closed analytical expressions for the useful and interference powers in terms of the Wide Sense Stationary Uncorrelated Scatterer (WSSUS) channel autocorrelation $R(l;k)$ function defined as $\mathrm{E}\left\{h(l,k)h^*(l',k')\right\} = \delta(l-l')R(l;k-k')$. P_T is the total power defined as $P_T = P_U + P_{ICI} + P_{ISI}$, and can be expressed as

$$P_T = \sum_{i=-\infty}^{+\infty} \sum_{p=0}^{P-1} \mathrm{E}\left\{\left|\Gamma\left(n_{p'}; n_p; i\right)\right|^2\right\}. \tag{11}$$

The derivations related to the total and useful powers are straightforward but quite lengthly and therefore are not included in the manuscript.

One important parameter in the system performance is the Signal to Noise Ratio (SNR) which is the ratio of the useful power and the other sources of interference power:

$$SNR = \frac{E_s \frac{N}{N+\nu} P_U}{E_s \frac{N}{N+\nu}\left(P_{ICI} + P_{ISI}\right) + P_{AWGN}}. \tag{12}$$

To tackle how the performance degrades in the fading channels as compared to the ideal case, i.e. the equivalent frequency-time flat channels, we define the SNR degradation as the SNR reduction compared to the ideal case: $SNRD = 10 \log \left(\frac{SNR_{AWGN}}{SNR} \right)$. It can be shown that

$$SNRD = \qquad (13)$$
$$-10\log\left(\frac{\frac{N}{N+\nu}P_U}{2\left(\left[\frac{1}{N}\sum_{q=-\infty}^{+\infty}\sum_{r=-\infty}^{+\infty}w(q;r)R(q;r)\right]+\frac{E_s}{N_0}\frac{N}{N+\nu}\left(P_T-P_U\right)\right)}\right)$$

where the multivariate function $w(q;r)$ has the same definition as the function $w(q;r)$ in [1].

4. Numerical results

In this section we have evaluated the obtained analytical expressions to optimize the system parameters of the MC-CDMA system. The OFDM system will serve as a reference system and the optimum parameters of the MC-CDMA system will be compared to those of the OFDM system. We consider the aeronautical channel model of the parking scenario in [7] with two different mobile speeds $v = 20$ and $v = 100$ km/h respectively. The channel impulse response is a multipath Rayleigh fading channel with a maximum delay spread of $\tau_{max} = 7.10^{-6}$ s and a power delay profile that is exponentially decreasing with slope time $\tau_{slope} = 10^{-6}$ s. Computations are carried out for a MC-CDMA system using Walsh Hadamard spreading codes followed by an overlay random code of spreading factor $N_s = 8$, a system bandwidth $B = 2$ MHz, i.e. $T = 5\,10^{-7}$ s, and a carrier frequency of $f_c = 1$ GHz. To compute channel Coherence Bandwidth (CB) and Coherence Time (CT) we use the following expressions [8] $B_c = \frac{1}{2\pi\tau_{max}}$ and $T_c = \frac{1}{16\pi f_{D_{max}}}$ respectively where $f_{D_{max}} = \frac{v}{c} f_c$ is the maximum Doppler spread. It turns out that the CB is $B_c = 22.74$ kHz and the maximum Doppler frequency is $f_{D_{max}} = 92.6$ Hz (resp. 18.52 Hz) for a mobile speed of $v = 100$ km/h (resp. $v = 20$ km/h) which results in a CT of $T_c = 1.9$ ms (resp. $T_c = 9.7$ ms). We will refer to the scenario with $v = 100$ km/h (resp. $v = 20$ km/h) as scenario A (resp. scenario B).

The determinant factor in the system performance is the BER that should be minimized or equivalently we have to minimize the SNRD defined in (13). In figure 1 we plot the optimum number of carriers N (left) and guard interval ν (right) for the MC-CDMA system with $N_s = 8$ and for the corresponding OFDM system with $N_s = 1$. It can be noticed that the minimization of the SNRD appears to result in different parameters N and ν for the MC-CDMA system and its equivalent

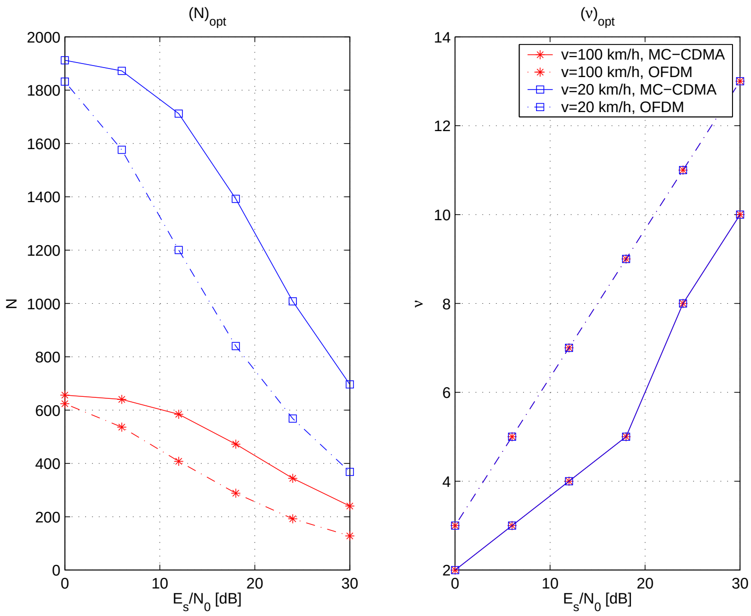

Figure 1. The optimum number of carriers (left) and guard of interval (right) as function of the E_b/N_0 in case of a mobile speed of 20 and 100 km/h.

OFDM system per E_b/N_0. From the figure, it follows that for all E_s/N_0 operating points, the optimum carrier spacing $\left(\frac{1}{NT}\right)_{opt}$ ranges in the interval $\left[\frac{1}{600T}, \frac{1}{200T}\right]$ for mobile speed of $v = 100$ km/h and $\left[\frac{1}{2000T}, \frac{1}{400T}\right]$ for a mobile speed of $v = 20$ km/h. It can easily be verified that these optimum carrier spacings are located within the interval determined by the maximum Doppler spread $f_{D_{max}}$ as a lower bound and the coherence bandwidth, which is inversely proportional to the maximum delay spread τ_{max} as an upper bound: $f_{D_{max}} \ll \left(\frac{1}{NT}\right)_{opt} \ll B_c$.

This can be explained as follows: when the carrier spacing is larger than the Doppler spread, the MC symbol period NT will be smaller than the coherence time of the channel, such that the channel appears almost time flat to the MC system reducing the amount of time selectivity induced ICI. On the other hand if the subchannel bandwidth $\frac{1}{NT}$ is smaller than the coherence bandwidth of the channel, the channel is frequency flat per carrier which is a key factor in MC systems to reduce frequency selectivity induced ICI and ISI. It can be noticed that the scenario B can accommodate more carriers. This can be explained by the fact that the Doppler spread is smaller as compared to scenario A. On the hand, for all situations the optimum guard interval ν_{opt} is not necessarily equal to the maximum delay spread. To eliminate the ISI completely the guard interval should be at least equal to the channel delay spread but this will be translated in a system throughput loss. Generally , the optimum ν is a few samples below the maximum delay spread for both scenarios.

5. Conclusion

In this paper, we derived closed analytical expressions for the useful and interference power of MC-CDMA. Based on these derivations, we carried out computations in order to minimize the SNR degradation and to compute the optimum guard interval and number of carriers. The optimization results of the MC-CDMA were compared to the corresponding OFDM system. Numerical results in different environments confirm the following rule of thumb in selecting the optimum parameters: the carrier spacing should be higher (resp. smaller) as compared to the maximum Doppler (resp. channel coherence bandwidth) while the guard interval has to be few samples smaller than the channel delay spread. Comparing the results of the parameter optimization for OFDM and MC-CDMA, and evaluating the sensitivity of the performance on the choice of the system parameters, it turns out that the results obtained for both multicarrier systems are very similar.

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