# Cramer-Rao Bound for AOA-Based VLP with an Aperture-Based Receiver

Heidi Steendam*, Thomas Q. Wang†, Jean Armstrong‡
* Ghent University, Ghent, Belgium
† University of Technology Sydney, Sydney, Australia
‡ Monash University, Melbourne, Australia

***Abstract*—Visible light positioning (VLP) for indoor applications has recently received considerable attention. In this paper, an aperture based receiver is considered and the position is estimated based on the angle-of-arrival (AOA), because the AOA can be determined by comparing the relative differences in the received signal strengths (RSS) in the different photodiodes (PDs) of the receiver. Hence, in contrast to RSS based positioning solutions, knowledge of the transmitted signal strengths is not required. The performance of the proposed approach depends on the transmitted signal strength only through the dependency on the signal-to-noise ratio. To evaluate the accuracy of the estimates of the AOA, we derive the Cramer-Rao lower bound, and show that the expected error on the AOA is very small. Applying triangulation to determine the position of the receiver, it follows that centimetre accuracy can be obtained with the aperture-based receiver.**

***Index Terms*—VLP, positioning, angle-of-arrival, Cramer-Rao bound**

## I. Introduction

An increasingly number of applications require knowledge of a user's position. GPS technology offers a rough position estimate which is sufficient for many outdoor applications, but because the reception of the GPS signal in indoor areas is reduced, researchers are searching for an alternative for indoor use. Since the beginning of this century, many techniques for indoor positioning have been investigated. However, none of the approaches provide a low-cost, low-power, accurate indoor positioning system. Recently, there has been growing interest in visible light positioning (VLP) [1]-[14]. White LEDs, which are gradually replacing traditional light sources for lighting, can be modulated at frequencies up to several MHz. Consequently, white LEDs can be used for wireless communications and positioning. Compared to RF solutions, visible light can safely be used in all environments, including hospitals and planes, suffers less from multipath interference, and is blocked by opaque walls.

Recent work on VLP focusses on receivers consisting of one or more photodiodes [3]-[14], as they are more energy efficient compared to CCD cameras. The positioning accuracy of these approaches depends on the amount and the type of position-related information that can be extracted from the light. Regarding the type of position related information, we can distinguish solutions based on the received signal strength (RSS) [3]-[4], [7]-[9], [11], time-of-arrival (TOA) [5], [10] and AOA [6], [12]-[14]. However, firstly, approaches based on TOA require accurate synchronization between the different LEDs, which is not straightforward. Secondly, to extract distance-related information from the RSS, knowledge of the transmitted signal strengths is required. In this paper, AOA positioning with the receiver from [15] is considered, which combines PDs with apertures. Because of the placement of the PDs and the apertures, the receiver offers angular diversity, which means that accurate positioning is achievable even with non-directional LEDs. With this receiver, the AOA can be estimated by comparing the relative values of the received signal strengths at different PDs. Hence, the transmitted signal strengths do not need to be known to extract the AOA. Compared to the directional receiver used in [6] and [12], consisting of tilted PDs mounted on the corners of a cube, the receiver structure of [15] is more compact.

In this paper, we consider VLP using a two-step approach based on the AOA: first the AOA is estimated based on the received optical signals, and in the second step, the position of the receiver is determined using triangulation. It is obvious that errors in the estimation of the AOA will degrade the positioning accuracy. Although some papers deal with VLP based on AOA, the optimality of the algorithms considered has not been investigated in detail. A standard technique to evaluate the optimality of an estimate is to compare it with the Cramer-Rao lower bound (CRB). In the literature, only a few papers consider the CRB for VLP using AOA or an aperture-based receiver. In [16], the CRB is derived for direct estimation of the position using an aperture-based receiver, while [17] considers the CRB for hybrid AOA/RSS positioning using directional LEDs and a non-directional receiver. In this paper, the CRB on the AOA is derived for the receiver from [15] where the light comes from a single, non-directional LED. It is shown that the AOA can be estimated very accurately, and that the proposed receiver structure enables three-dimensional (3D) positioning with high accuracy in the order of centimetres.

## II. System Description

### A. Received Signals

Let us consider a positioning system where $K$ white LEDs are attached to the ceiling. We assume that the signals transmitted by the different LEDs can be separated by the receiver. This can for example be achieved by selecting the

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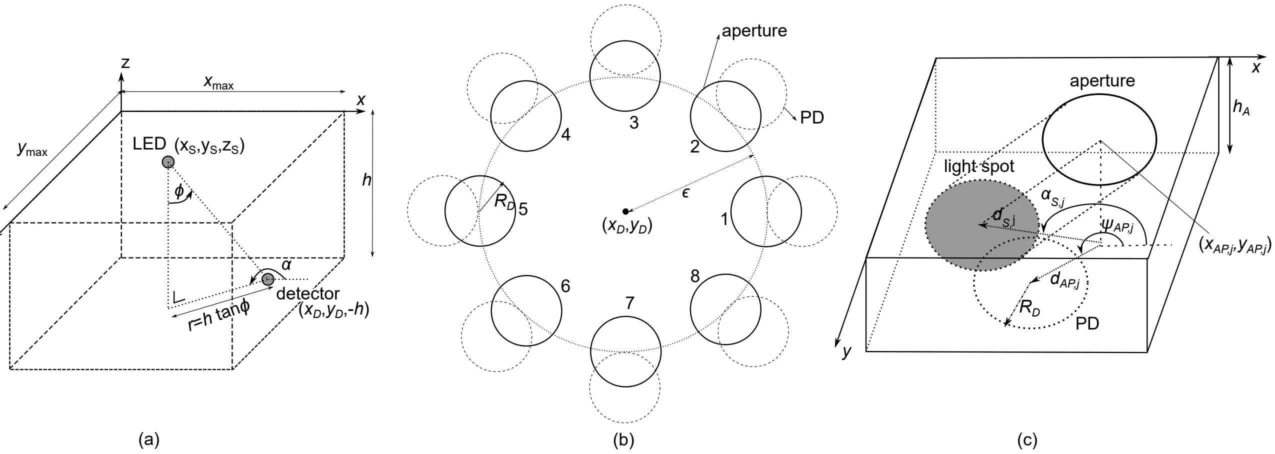



Fig. 1. a) Definition of the incident and polar angles $(\phi, \alpha)$, b) top view of the receiver with 8 REs, and c) top view of one RE.

optical signals corresponding to the different LEDs as dc-biased windowed sinusoids $s_i(t) = A_i w(t)(1 + \cos 2\pi f_{c,i} t)$, $i = 1, \ldots, K$. The incoming signal at the receiver contains the superposition of the different transmitted signals. When the received signal is correlated with the reference signals $\eta_i(t) = w(t) \cos 2\pi f_{c,i} t$ over the interval $T$, the contributions from the different LEDs can easily be separated, provided that the frequencies $f_{c,i}$ are orthogonal over the interval $T$, i.e. $f_{c,i}T$ and $(f_{c,i} - f_{c,i'})T$ must be integer. We assume $w(t)$ is a rectangular window, i.e.

$$w(t) = \begin{cases} 1 & t \in [0, T] \\ 0 & \text{otherwise} \end{cases}. \quad (1)$$

In that case, $\int_0^T s_i(t)dt = A_i$, implying $A_i$ is the optical power transmitted by LED $i$. This selection of the transmitted signals is equivalent to orthogonal frequency division multiplexing (OFDM) [18]. The AOA between a LED and the receiver only depends on the position of that LED and the receiver, but not on the positions of the other LEDs. As a result, because the contributions of the different LEDs can be separated, the presence of other LEDs will not influence the determination of the AOA for a given LED, implying we can restrict our analysis to the case of a single LED. Hence, in the remainder of this section, we drop the index $i$.

We assume the LED is modelled as a generalized Lambertian LED with order $m$, points downwards and has coordinates $(x, y, z) = (x_S, y_S, z_S)$, as shown in Figure 1(a). The receiver contains $M$ receiving elements (RE), where each RE consists of a bare, circular photodiode (PD) combined with an aperture, as shown in Figure 1(b). The apertures are modelled as circular holes with radius $R_A$ in an opaque screen that is placed parallel to the plane of the PDs, at a height $h_A$ above this plane. The radius of the PDs equals $R_D$. In this paper, it is assumed that $R_D = R_A$ and that the only light that reaches a PD is the light that passes through its aperture. Assuming the radius $R_D$ is large compared to the wavelength of the light, the diffraction effects can be ignored, so that the incident light will introduce a circular light spot on the plane of the PD, as shown in Figure 1(c).

We assume the receiver is placed in a plane parallel to the ceiling, at a vertical distance $h = z_S - z_D$ below the LED. To determine the position of the receiver, a reference point on the receiver is selected, and the coordinates $(x_D, y_D, z_D)$ of this reference point are determined. The estimation of the coordinates is based on the light captured by the PDs, which depends on the relative positions of the apertures and the PDs with respect to this reference point. The centre of aperture $j$ is placed at a relative position $(\delta x_j, \delta y_j)$ from the reference point, i.e. $(x_{AP,j}, y_{AP,j}) = (x_D + \delta x_j, y_D + \delta y_j)$. Further, the PD of RE $j$ is shifted relative to its aperture, so that the receiver provides angular diversity. The relative position of the PD with respect to its aperture is defined by the distance $d_{AP,j}$ and angle $\psi_{AP,j}$ (see Figure 1(c)). The parameters $(\delta x_j, \delta y_j)$ and $(d_{AP,j}, \psi_{AP,j})$ determine the receiver layout.

Our aim is to estimate the angles-of-arrival $\phi$ and $\alpha$ between the reference point on the receiver and the LED. We will show at the end of this section that the relative differences between the received signal strengths are essentially independent of $h$, implying that the knowledge of the distance $h$ is not necessary to be able to estimate the AOAs. Taking into account Figure 1(a), the relationship between the coordinates $(x_D, y_D, z_D)$, $(x_S, y_S, z_S)$, and the angles $(\phi, \alpha)$ is given by

$$\begin{aligned} x_D - x_S &= h \tan\phi \cos\alpha \\ y_D - y_S &= h \tan\phi \sin\alpha. \end{aligned} \quad (2)$$

The estimation of the angles $(\phi, \alpha)$ is based on the measured differences in the RSS values in the different PDs of the receiver. It will be shown later in this paper that the RSS value at PD $j$ is a function of the incident and polar angle $\phi_j$ and $\alpha_j$ between the LED and the centre of the aperture of PD $j$. The angles $\phi_j$ and $\alpha_j$ are defined in a similar way as $\phi$ and $\alpha$ (2):

$$\begin{aligned} x_D - x_S + \delta x_j &= h \tan\phi_j \cos\alpha_j \\ y_D - y_S + \delta y_j &= h \tan\phi_j \sin\alpha_j. \end{aligned} \quad (3)$$

Combining (2) and (3), the relationship between $(\phi, \alpha)$ and $(\phi_j, \alpha_j)$ is found:

$$\tan\phi_j = \frac{1}{h}\left[(\delta x_j + h\tan\phi\cos\alpha)^2 + (\delta y_j + h\tan\phi\sin\alpha)\right]^{\frac{1}{2}}$$
$$\tan\alpha_j = \frac{\delta y_j + h\tan\phi\sin\alpha}{\delta x_j + h\tan\phi\cos\alpha} \quad (4)$$

Note that, when $\phi = 0$, the polar angle can not be defined.

To extract the AOA at the receiver, we compare the relative differences between the signal strengths at the different REs. The vector of observations $\mathbf{r} = (r_1 \ldots r_M)^T$, corresponding to the different REs, can be written as

$$\mathbf{r} = R_p \mathbf{h}_c s + \mathbf{n}, \quad (5)$$

where $r_j = \int_0^T r_j(t)\eta(t)dt$ with $r_j(t)$ the signal contribution received by LED $j$, $\eta(t) = w(t)\cos(2\pi f_c t)$, $R_p$ is the responsivity of the PD, $\mathbf{h}_c$ is the $M \times 1$ vector of channel gains, $s = \int_0^T s(t)\eta(t)dt = \frac{AT}{2}$, and $\mathbf{n} = (n_1 \ldots n_M)^T$ is the shot noise. Following [19], the shot noise is modelled as i.i.d. additive white Gaussian noise. Assuming $N_0$ is the one-sided noise spectral density and $\eta(t) = 0$ for $t \notin [0, T]$, the noise variance equals $\frac{N_0 T}{2}$. The noise level $N_0 = 2qR_p p_n A_D \Delta\lambda$ is a function of the charge $q$ of an electron, the responsivity $R_p$ of the photodiode, the background spectral irradiance $p_n$, the area $A_D$ of the PD and the bandwidth $\Delta\lambda$ of the optical filter placed in front of the PD. Further, the channel gain at RE $j$ is given by

$$h_c^{(j)} = \frac{m+1}{2\pi h^2} A_0^{(j)} \cos^{m+3}\phi_j. \quad (6)$$

where $A_0^{(j)}$ is the area of overlap between the light spot and PD $j$

$$A_0^{(j)} = \begin{cases} 2R_D^2 \arccos\left(\frac{d_j}{2R_D}\right) & 0 \le d_j \le 2R_D \\ \quad -\frac{d_j}{2}\sqrt{4R_D^2 - d_j^2} & \\ 0 & d_j > 2R_D \end{cases}, \quad (7)$$

which is determined by the distance $d_j$ between the centre of the light spot and the centre of the PD:

$$\begin{aligned} d_j &= \Big[(d_{S,j}\cos\alpha_{S,j} - d_{AP,j}\cos\psi_{AP,j})^2 \\ &+ (d_{S,j}\sin\alpha_{S,j} - d_{AP,j}\sin\psi_{AP,j})^2\Big]^{\frac{1}{2}} \\ &= \left[\left(h_A\tan\phi\cos\alpha + \frac{\delta x_j}{h}h_A + d_{AP,j}\cos\psi_{AP,j}\right)^2\right. \\ &+ \left.\left(h_A\tan\phi\sin\alpha + \frac{\delta y_j}{h}h_A + d_{AP,j}\sin\psi_{AP,j}\right)^2\right]^{\frac{1}{2}} \quad (8) \end{aligned}$$

where the coordinates of the centre of the light spot are given by $d_{S,j} = h_A \tan\phi_j$ and $\alpha_{S,j} = \pi + \alpha_j$, and (4) is used to rewrite $d_j$ as function of $(\phi, \alpha, h)$.

Let us look more closely at the dependency of the channel gain (6) on the vertical distance $h$ between the LED and the receiver. The channel gain contains a factor $1/h^2$, and also $\phi_j$ and $A_0^{(j)}$ are functions of $h$. However, in general $\delta x_j$ and $\delta y_j$ will be small as compared to $h$. As a result, it follows from (4) that $\phi_j \approx \phi$ and $\alpha_j \approx \alpha$, and from (8) that $d_j$ is essentially independent of $h$. Hence, as $\cos\phi_j$ is essentially independent of the index $j$ and $A_0^{(j)}$ is independent of $h$, the channel gain depends on the distance $h$ through the factor $\cos^{m+3}\phi/h^2$ only, when $\delta x_j, \delta y_j \ll h$. As a result, if we compare the relative differences between the received signal strength values in the different REs, e.g. by evaluating the ratios $r_j/\max_j r_j$, the ratios will be essentially equal to $A_0^{(j)}/\max_j A_0^{(j)}$, which is independent of $h$, implying the angles $\phi$ and $\alpha$ can be determined without knowledge of $h$. In section III, we will show that the height $z_D = z_S - h$ of the receiver can be determined using triangulation.

### B. Cramer-Rao Bound

The Cramer-Rao bound [20] is commonly used to evaluate the performance of an estimator, as it lower bounds the mean-squared error (MSE) of an unbiased estimate. For an unbiased estimate $\hat{\boldsymbol{\theta}}$ of a parameter vector $\boldsymbol{\theta}$, the error covariance matrix is lower bounded by

$$E[(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})^T(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})] \ge \mathbf{F}^{-1}. \quad (9)$$

The Fisher information matrix (FIM) $\mathbf{F}$ is defined as

$$\mathbf{F} = E\left[(\boldsymbol{\nabla}_\theta \ln p(\mathbf{r}|\boldsymbol{\theta}))(\boldsymbol{\nabla}_\theta \ln p(\mathbf{r}|\boldsymbol{\theta}))^T\right], \quad (10)$$

where $\mathbf{r}$ is the vector of observations and $\boldsymbol{\theta} = (\phi, \alpha)$.

Taking into account that $\mathbf{r}|\boldsymbol{\theta}$ is a Gaussian random variable with mean $R_p\mathbf{h}_c s$ and covariance matrix $\frac{N_0 T}{4}\mathbf{I}_K$, where $\mathbf{I}_K$ is the $K \times K$ identity matrix, the FIM can be rewritten as

$$\mathbf{F} = \gamma(\boldsymbol{\nabla}_\theta \mathbf{h}_c^T)^T(\boldsymbol{\nabla}_\theta \mathbf{h}_c^T) \quad (11)$$

where $\gamma = \frac{R_p^2 A^2 T}{N_0}$ and $\boldsymbol{\nabla}_\theta \mathbf{h}_c^T$ is a $2 \times M$ matrix with elements

$$(\boldsymbol{\nabla}_\theta \mathbf{h}_c^T)_{\theta_i, j} = \frac{\partial}{\partial\theta_i} h_c^{(j)} \quad (12)$$

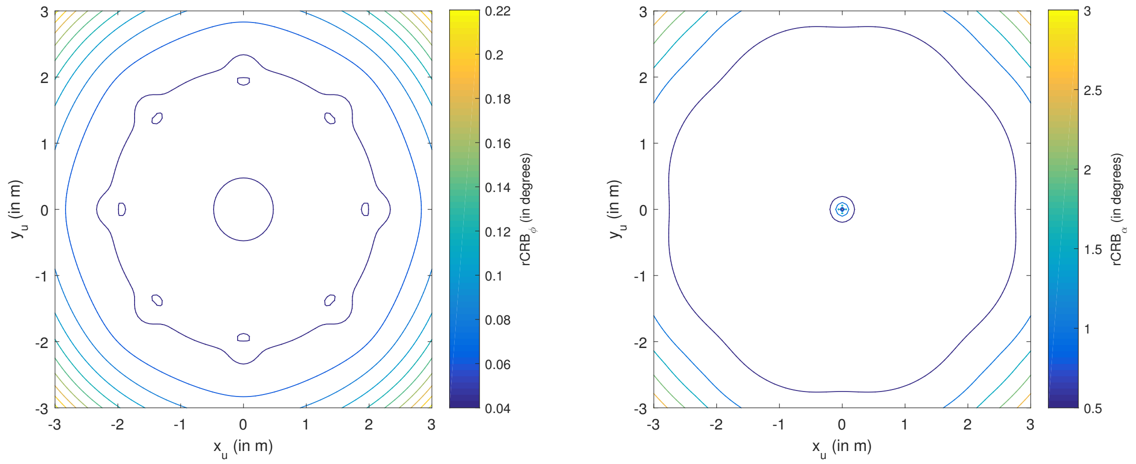
with $i = 1, 2$ and $j = 1, \ldots, M$. The derivation of the elements $\boldsymbol{\nabla}_\theta \mathbf{h}_c^T$ is straightforward.

When $\phi = 0$, the angle $\alpha$ is not defined. In that case, the CRB on $\phi$ can be computed with $F_{\phi=0} = \mathbf{F}_{1,1}$.

## III. Triangulation

Triangulation is a technique to measure the distance of an object to a reference point based on the angles of a triangle. The technique was already known in the ancient Greek period and is commonly used in navigation and topography. We apply this measuring technique to our VLP system to extract the position $\boldsymbol{\rho}_D = (x_D, y_D, z_D)$ of the receiver from the AOA between the receiver and different LEDs. Let us assume the receiver detects the light from $K$ LEDs that have positions $(x_{S,i}, y_{S,i}, z_{S,i})$, $i = 1, \ldots, K$ and are pointing downwards, where we assume $z_{S,i} > z_D$. The incident and polar angle between the reference point $(x_D, y_D, z_D)$ of the receiver and the position $(x_{S,i}, y_{S,i}, z_{S,i})$ of LED $i$ are defined as $\phi_i$ and $\alpha_i$ according to (2). The least squares (LS) estimate of $\boldsymbol{\rho}_D$ is obtained as:

$$\hat{\boldsymbol{\rho}}_{D,LS} = (\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{u}, \quad (13)$$




(a) (b)

Fig. 2. Square root of (a) the CRB of the incident angle $\phi$ and (b) the CRB of the polar angle $\alpha$ as function of the position $(x_D, y_D)$ of the receiver in the room, for $\gamma = 192.8$ dB, $M = 8$, $x_{\max} = y_{\max} = 6$ m, $T = 1$ ms, $h = 2$ m, $\epsilon = 5$, $\zeta = 1$, and $R_D = 1$ mm.

where $\mathbf{u} = (\mathbf{u}_1^T \ldots \mathbf{u}_K^T)$, $\mathbf{u}_i = (x_{S,i} + z_{S,i} \tan\phi_i \cos\alpha_i, y_{S,i} + z_{S,i} \tan\phi_i \sin\alpha_i)^T$, $\mathbf{Z} = (\mathbf{Z}_1^T \ldots \mathbf{Z}_K^T)^T$, with

$$\mathbf{Z}_i = \begin{pmatrix} 1 & 0 & \tan\phi_i \cos\alpha_i \\ 0 & 1 & -\tan\phi_i \sin\alpha_i \end{pmatrix}. \tag{14}$$

Taking into account that the position of the receiver is determined by three parameters, i.e., $\boldsymbol{\rho}_D = (x_D, y_D, z_D)$, and we have two angles per LED, this implies that at least two LEDs are required to allow 3D positioning.

## IV. Numerical Results

To evaluate the CRB on the AOA and the resulting position uncertainty, a receiver with $M$ receiving elements that are placed in a circle around the reference point $(x_D, y_D)$ on the receiver is considered, i.e. $(\delta x_j, \delta y_j) = (\epsilon R_D \cos j\frac{2\pi}{M}, \epsilon R_D \sin j\frac{2\pi}{M})$. Further, the relative positions of the PDs with respect to their apertures are given by $d_{AP,j} = \zeta R_D$ and $\psi_{AP,j} = j\frac{2\pi}{M}$. By slightly displacing the PD with respect to its aperture and by selecting the angles $\psi_{AP,j}$ uniformly over the interval $[0, 2\pi]$, the receiver will have good angular diversity. As a result, it will be shown that accurate estimates of the AOA are possible. We assume the LED has Lambertian order $m = 1$ and restrict our attention to the case where the receiver is placed in a plane parallel to the ceiling, and RE 1 lies on the $x$-axis[1]. Further, it is assumed that the plane of the apertures is at height $h_A = R_D$ above the plane of the PDs. To determine the level $N_0$ of the shot noise, a background spectral irradiance $p_n = 5.8 \times 10^{-6}$ W/cm$^2$·nm [19] is considered. The PD has a responsitivity $R_p = 0.4$ mA/mW [21] and the optical filter placed in front of the PD passes only visible light frequencies in the range 380 to 740 nm, implying $\Delta\lambda = 360$ nm. The noise spectral density corresponding to a PD with radius $R_D = 1$ mm equals $N_0 = 8.4 \times 10^{-24}$ A$^2$/Hz. Further, it is assumed that the optical power of the LED equals $A = 1$ W and the time interval $T = 1$ ms. This corresponds to a ratio $\gamma = 192.8$ dB.

First, we consider the CRB on the AOA for a single LED. The square root of the CRB (rCRB) is evaluated as a function of the position $(x_D, y_D)$ of the receiver compared to the LED. To obtain deeper insight into the results, the angles in the figure are shown in degrees, although the expressions for the CRB use radians. Figure 2 shows the rCRB for the incident angle $\phi$ and the polar angle $\alpha$ in a 6 m×6 m area around the LED, which is placed at position $(x_S, y_S) = (0, 0)$. The rCRB for the angles $\phi$ and $\alpha$ is roughly independent of the polar angle, but mainly depends on the incident angle only. This is due to the angular symmetry of the receiver. It can be observed that the rCRB for the angles is less than three degrees for the whole area, implying very accurate estimates for the AOA can be obtained. The spatial average of the rCRB, rCRB$_{\text{av}}$, i.e., the average of the rCRB over all positions in the considered area, equals rCRB$_{\text{av},\phi} = 0.058$ degrees and rCRB$_{\text{av},\alpha} = 0.479$ degrees. We further investigate the influence of the incident angle on the rCRB in Figure 3. As mentioned in section II-A, the rCRB for the polar angle $\alpha$ is not defined just below the LED. When $\phi \approx 0$, the variation of the polar angle as function of the position of the receiver increases. However, because of the small incident angle $\phi$, the larger variance on $\alpha$ is expected to have only a small effect on the position uncertainty. The rCRB for both $\phi$ and $\alpha$ increases when the incident angle between the LED and the receiver is large. This can be explained by the directionality of the receiver and the radiation pattern of the LED, i.e. the receiver will capture less light when the incident angle is larger. When the incident angle increases above a threshold $\phi_{\max}$, i.e., the

[1] A tilt of the receiver will only influence the incident angle $\phi$ of the light on the receiver structure, and a rotation of the receiver will only result in a transformation of the $(x, y)$ coordinates. Although a rotation and a tilt will affect the ability to determine the position of the receiver if the rotation and tilt are unknown, this rotation and tilt can accurately be determined using inertial measurement units (IMUs), such as a gyroscope, an accelerometer and a compass, which are commonly available in current wireless devices. Hence, a tilt and a rotation of the receiver can easily be compensated for.

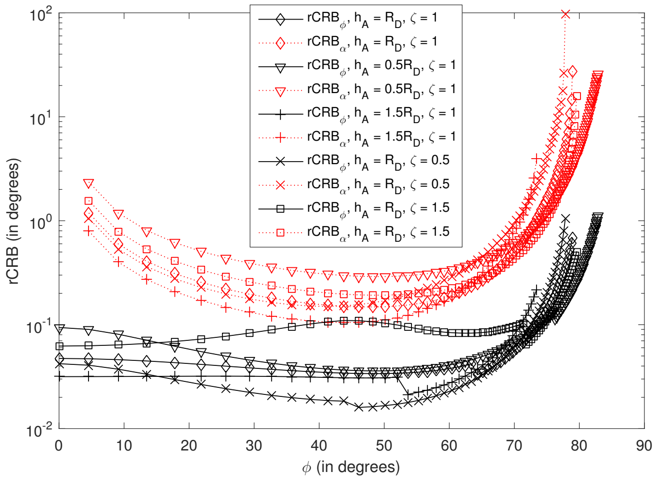



Fig. 3. The rCRB as a function of the incident angle $\phi$ for $y_D = 0$ m, $\gamma = 192.8$ dB, $M = 8$, $T = 1$ ms, $h = 2$ m, $\epsilon = 5$ and $R_D = 1$ mm.

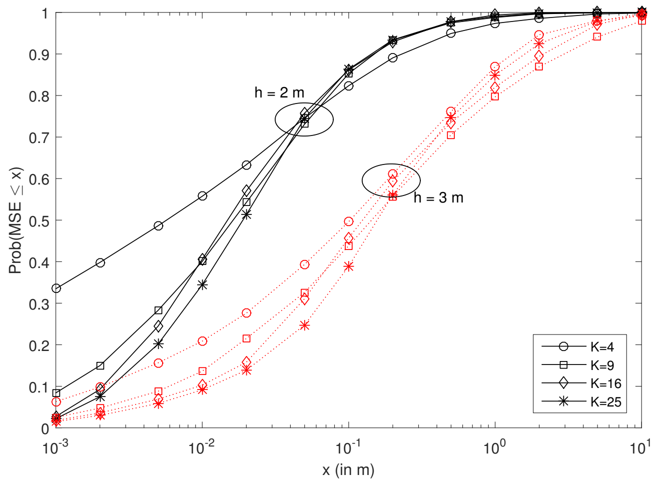



Fig. 4. Probability that the position uncertainty $MSE \geq x$ for $\gamma = 192.8$ dB, $M = 8$, $T = 1$ ms, $h_A = R_D$, $\epsilon = 5$, $\zeta = 1$ and $R_D = 1$ mm, $L = 100$.

smallest incident angle above which none of the PDs will receive light, the FIM will be singular, implying the AOA between the receiver and the LED can not be estimated. As in the triangulation step, the AOA between the receiver and at least two LEDs must be available, the upper bound $\phi_{\max}$ will limit the spacing between the LEDs. For example, assuming $h = 2$ m and defining $r_{\max} = h \tan \phi_{\max}$, for the receiver with $h_A = R_D$ and $\zeta = 1$, the LED can be detected by the receiver within a radius $r_{\max} = 6$ m around this LED, and with $h_A = 0.5R_D$ and $\zeta = 1$, this radius increases to 7.5 m. Note that the rCRB on $\phi$ shows a discontinuity at an intermediate incident angle $\phi_\diamond$. This angle $\phi_\diamond$ corresponds to the smallest incident angle for which one of the PDs will no longer capture the light from the LED.

Errors in the estimation of the AOA will result in a degradation of the positioning accuracy in the triangulation phase. To evaluate the effect of errors in the estimation of $\phi$ and $\alpha$ on the position uncertainty, we consider a positioning system in a 10 m $\times$ 10 m area where $K = P^2$ LEDs are attached to the ceiling in a uniform grid. In the considered area, the receiver is placed at positions selected from a uniform grid with spacing 10 cm, at a distance $h$ below the ceiling. We assume the estimates $\hat{\phi}_i$ and $\hat{\alpha}_i$ can be modelled as Gaussian random variables with averages $E[\hat{\phi}_i] = \phi_i$ and $E[\hat{\alpha}_i] = \alpha_i$ and variances $\sigma_\phi^2 = \text{CRB}_\phi$ (rad$^2$) and $\sigma_\alpha^2 = \text{CRB}_\alpha$ (rad$^2$), i.e. we have an unbiased and efficient estimator. As the variance of an unbiased estimator is lower bounded by its CRB, this would correspond to the best case scenario, and the resulting position uncertainty can serve as a lower bound on the position uncertainty for practical estimators. In Figure 4, we show the probability that the position uncertainty is larger than $x$ (in m), for different values of $K$ and $h$. Due to the attenuation of the channel, the amount of light that reaches the receiver will be smaller if the distance $h$ is larger. Hence, the positioning accuracy for $h = 3$ m will be worse than for $h = 2$ m. Further, we observe that for small values of $x$, the configuration with $K = 4$ LEDs outperforms the configurations with larger $K$. This can be explained as follows. When $K = 4$, the positions with the best position accuracy are located in the middle of the considered area, where the receiver is surrounded by the LEDs, and the incident angle for the LEDs is around 60 degrees, implying accurate estimates of the AOA for all LEDs are present. When $K$ is large, the receiver will capture light from the nearest LEDs, but also from LEDs that are located further away. For large $\phi$, the uncertainty on $\alpha$ can become very large, as can be observed in Figure 3. For the LEDs that are far away from the receiver[2], the reliability of the estimates of $\phi$ and $\alpha$ will therefore be low. As in the triangulation phase, the contributions from all LEDs are equally taken into account, i.e. there is no weighting according to the reliability of the estimates, the uncertainty on the estimates for these far-away LEDs will degrade the position accuracy. Note that this effect can be reduced by excluding the contributions from the LEDs with an incident angle $\phi_i$ larger than a threshold. When $x = 10$ cm or larger, increasing the number of LEDs will result in a marginal improvement of the positioning accuracy. When $h = 2$ m, more than 85% of the positions can be determined with an accuracy of 10 cm or better, whereas for $h = 3$ m, this reduces to 50%. This positioning performance can be improved by increasing the ratio $\gamma$ (11), e.g. by increasing the time interval $T$ or by increasing the optical power. For example, when $T = 10$ ms, the probability that the positioning error is smaller than 10 cm increases to 97% for $h = 2$ m and to 88% for $h = 3$ m. Hence, centimetre accuracy can be obtained with the proposed compact receiver.

## V. Conclusions

In this paper, we studied the Cramer-Rao bound on the AOA for 3D visible light indoor positioning using an aperture-based receiver and non-directional LEDs. Because of the structure of the receiver, the receiver is able to extract the direction of

[2]For $K = 9$, the incident angle between the receiver placed in the middle of the area and the farthest LEDs equals $\phi = 67$ degrees, for $K = 16$, $\phi = 69$ degrees and for $K = 25$, $\phi = 70.5$ degrees.

arrival of the light by comparing the received signal strengths in the different photodiodes. As a result, knowledge of the transmitted signal strength is not required to estimate the AOA, in contrast to RSS-based approaches. The performance only depends on the transmitted signal strength through the signal-to-noise ratio, which is proportional to the ratio $\gamma$ defined in this paper. Based on the derived CRB, the position uncertainty obtained in the triangulation phase is evaluated. The results show that real-time centimetre-accuracy positioning is possible with this compact receiver. Hence, the proposed approach is an excellent candidate for indoor positioning. The centimetre accuracy obtained with the proposed approach is in line with the accuracy obtained in other VLP papers. The advantages of the proposed approach with respect to the state-of-the-art in VLP are the ability to achieve this performance using non-directional LEDs, in contrast to solutions using a non-directional receiver requiring many directional LEDs, i.e. with small half-angles, the compact receiver structure compared to the tilted directional receiver from [6] and [12], and the independency of the knowledge of the transmitted signal strength compared to RSS-based approaches.

Although the theoretical derivations presented in this paper promise very accurate positioning, in practice, a degradation of the position accuracy will occur. This degradation originates from physical effects not included in the considered theoretical model, such as the resolution of the analog-to-digital converter, irregularities in the shapes of the PDs and the apertures, diffuse light components caused by reflections of the light on objects or variations in the distance $h_A$ between the plane of the apertures and the PDs. However, bearing in mind the experiments on VLP with other receivers, it is expected that the position uncertainty including these effects will still be an order of magnitude better than obtained with e.g. wifi or BLE.

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## References

[1] J. Armstrong, Y. A. Sekercioglu and A. Neild, "Visible Light Positioning: A Roadmap for International Standardisation," *IEEE Communications Magazine*, Vol 51, No 12, pp. 68-73, Dec 2013.

[2] M. Biagi, S. Pergolini, A.-M. Vegni, "LAST: A Framework to Localize, Access, Schedule and Transmit in Indoor VLC Systems," *Journal of Lightwave Technology*, Vol 33, No 9, May 2015, pp. 1872-1887.

[3] S. Hann, J.-H. Kim, S.-Y. Jung, C.-S. Park, "White LED Ceiling Lights Positioning Systems for Optical Wireless Indoor Applications," in *Proc. ECOC 2010*, Torino, Italy, Sep 2010.

[4] S.-Y. Jung, S. Hann, C.-S. Park, "TDOA-Based Optical Wireless Indoor Localization Using LED Ceiling Lamps," *IEEE Transactions on Consumer Electronics*, Vol 57, No 4, pp. 1592-1597, Nov 2011.

[5] K. Panta, J. Armstrong, "Indoor Localization Using White LEDs," *Electronics Letters*, Vol 48, No 4, pp. 228-230, Feb 2012.

[6] A. Arafa, X. Jin, R. Klukas, "Wireless Indoor Optical Positioning With a Differential Photosensor," *IEEE Photonics Technology Letters*, Vol 32, No 14, pp. 2480-2485, Jul 2012.

[7] S.-Y. Jung, S. Hann, S. Park, C.-S. Park, "Optical Wireless Indoor Positioning System Using Light Emitting Diode Ceiling Lights," *Microwave and Optical Technology Letters*, Vol 54, No 7, pp. 1622-1626, Jul 2012.

[8] M. Rahaim, G.B. Prince, T.D.C. Little, "State Estimations and Motion Tracking for Spatially Diverse VLC Networks," in *Proc. Globecom Workshops*, Anaheim, CA, USA, Dec 2012, pp. 1249-1253.

[9] H.-S. Kim, D.-R. Kim, S.-H. Yang, Y.-H. Son, S.-K. Han, "An Indoor Visible Light Communication Positioning System Using a RF Allocation Technique," *Journal of Lightwave Technology*, Vol 31, No 1, pp. 134-144, Jan 2013.

[10] U. Nadeem, N.U. Hassan, M.A. Pasha, C. Yuen, "Highly Accurate 3D Wireless Indoor Positioning System Using White LED Lights," *Electronics Letters*, Vol 50, No 11, pp. 828-830, May 2014.

[11] G. Kail, P. Maechler, N. Preyss, A. Burg, "Robust Asynchronous Indoor Localization Using LED Lighting," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP2014)*, Florence, Italy, May 2014.

[12] S.-H. Yang, H.-S. Kim, Y.-H. Son, and S.-K. Han, "Three-Dimensional Visible Light Indoor Localization Using AOA and RSS With Multiple Optical Receivers," *Journal of Lightwave Technology*, Vol 32, No 14, pp. 2480-2485, Jul 2014.

[13] M. Yasir, S.-W. Ho, B.N. Vellambi, "Indoor Positioning Using Visible Light and Accelerometer," *Journal of Lightwave Technology*, Vol 32, No 19, Oct 2014, pp. 3306-3316.

[14] Y.U. Lee, S.-M. Lee, "Random Distributed Angle-of-Arrival Parameter Estimation Technique for Visible Light Positioning," in *Proc. Int. Conf. on Telecom. and Sign. Proc., TSP2015*, Prague, Czech Rep., Jul 2015, pp. 467-471.

[15] T.Q. Wang, C. He, J. Armstrong, "Angular Diversity for Indoor MIMO Optical Wireless Communications," in *Proc. IEEE International Conference on Communications (ICC2015)*, London, UK, Jun 2015.

[16] H. Steendam, T.Q. Wang, J. Armstrong, "Cramer-Rao Bound for Indoor Visible Light Positioning Using an Aperture-Based Angular-Diversity Receiver," in *Proc. IEEE International Conference on Communications (ICC2016)*, Kuala Lumpur, Malaysia, May 2016.

[17] A. Sahin, Y.S. Eroglu, I. Guvenc, N. Pala, M. Yuksel, "Hybrid 3D Localization for Visible Light Communication Systems," *Journal of Lightwave Technology*, Vol 33, No 22, pp. 4589-4599, Nov 2015.

[18] A. J. Lowery and J. Armstrong, "Orthogonal frequency division multiplexing for dispersion compensation of long-haul optical systems", Optics Express. Vol. 14, 2006, pp. 2079-2084

[19] J.M. Kahn, J.R. Barry, "Wireless Infrared Communications," *Proceedings of the IEEE*, Vol 85, No 2, pp. 265-298, Feb 1997.

[20] H.L. Van Trees, *Detection, Estimation, and Modulation Theory*, New York, NY, USA, Wiley, 1968.

[21] L. Zeng, D.C. O'Brien, H.L. Minh, G.E. Faulkner, K. Lee, D. Jung, Y. Oh, E.T. Wong, "High Data Rate Multiple Input Multiple Output (MIMO) Optical Communications Using White LED Lighting," *IEEE Journal on Selected Areas in Communications*, Vol 27, No 9, pp. 1654-1662, Dec 2009.