The Effect of Carrier Phase Jitter on MC-CDMA Performance

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***Abstract*— In this paper, we investigate the influence of the carrier phase jitter of a phase-locked local oscillator on the performance of a multicarrier code-division multiple-access (MC-CDMA) system in a nondispersive channel. This carrier phase jitter degrades the signal-to-noise ratio at the input of the decision device. When all users have the same power level and phase jitter spectrum, it is shown that for the highest load, the degradation only depends on the jitter variance but not on the specific shape of the jitter spectrum nor on the number of carriers.**

***Index Terms*—MC-CDMA, carrier-phase jitter, system performance.**

I. Introduction

IN THE LITERATURE [1]–[8], different combinations of orthogonal frequency-division multiplexing (OFDM) and code-division multiple access (CDMA) have been investigated in the context of high-rate communication over dispersive channels. One of these combinations is multicarrier CDMA (MC-CDMA), which has been proposed for downlink communication in mobile radio. In MC-CDMA, the data symbols are multiplied by a higher rate chip sequence; the resulting chips are frequency-division multiplexed on a set of orthogonal carriers.

Receivers need a local oscillator for converting the received RF signal to a baseband signal. The effect of local oscillator imperfections on OFDM system performance has been investigated in [9] and [10]. In [9], it was shown that for a free-running local oscillator exhibiting frequency offset and Wiener phase noise, the OFDM system performance rapidly degrades with an increasing number of tones. In order to avoid this strong degradation, it was proposed in [10] to use a phase-locked local oscillator (which was locked to a pilot tone, for instance) in order to get rid of frequency offset and of phase noise components that fall within the bandwidth of the phase-locked loop (PLL). For a given variance of the residual phase jitter of the phase-locked local carrier, it was shown in [10] that the degradation of the OFDM system performance is independent of the number of tones and of the shape of the jitter spectrum.

In [11], the sensitivity of MC-CDMA to local oscillator imperfections has been investigated, assuming the free-running local oscillator model used in [9]; as in [9], a strong dependence on the number of tones has been observed.

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In this contribution we study the effect of local oscillator imperfections on MC-CDMA performance, assuming the phase-locked oscillator model from [10]. In order to clearly isolate the effect of jitter, and to avoid the issue of orthogonality restoration, transmission over a nondispersive channel is assumed.

II. System Description

The conceptual block diagram of the MC-CDMA transceiver is shown in Fig. 1. A data symbol $a_{n,m}$ with unit energy and at symbol rate $R_s = 1/(NT)$, transmitted by the nth user during the mth symbol interval, is multiplied with a CDMA chip sequence $\{c_{n,\ell}\}$ $\ell = 0, N-1$, with N denoting the number of chips. Sequences belonging to different users are assumed to be orthogonal (e.g., Walsh–Hadamard sequences). The resulting samples, at a rate $1/T$, are modulated on N equidistant orthogonal carriers using an inverse discrete Fourier transform. The resulting time domain samples are fed to $p(t)$, a unit energy square root Nyquist filter with respect to the interval T. The transmitted signal occupies an RF-bandwidth $B_{\mathrm{RF}} = NR_s$. The carrier phase jitter is the phase error between the carrier used for up-converting the baseband MC-CDMA signal at the transmitter and the phase-locked carrier used for down-converting at the receiver. All OFDM carriers belonging to the same user exhibit an identical carrier phase jitter as they are up-converted by the same oscillator. The phase error $\phi_n(t)$ of user n is modeled as a stationary zero mean process having a bandwidth much smaller than $1/T$. For $M \leq N$ users, the complex envelope of the received signal in the presence of additive noise $w(t)$ and carrier phase jitter is given by

$$r(t) = \sum_m \sum_{n=0}^{M-1} \frac{\sqrt{E_{s,n}}}{N} a_{n,m} \sum_{k,\ell=0}^{N-1} c_{n,\ell} e^{j2\pi(k\ell/N)} \cdot p(t-(k+mN)T)\, e^{j\phi_n(t)} + w(t). \tag{1}$$

The receiver consists of a filter $p^*(-t)$ matched to the transmit pulse whose output is sampled at the chip rate at the instants $\{kT\}$. In order to detect the symbol $a_{n,m}$, the matched filter output samples are fed to the discrete Fourier transform, and the resulting frequency domain samples are correlated with the chip sequence corresponding to the nth user.

III. Sensitivity to Carrier Phase Jitter

In this section, we compute the degradation (in decibels) of the signal-to-noise ratio (SNR) at the input of the decision device when carrier phase jitter is present. We consider the detection of the symbol $a_{n,0}$.

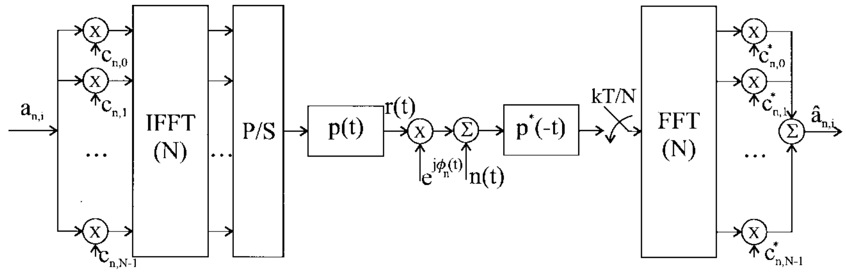

Fig. 1. Conceptual block diagram of an MC/CDMA transceiver for one user.

As the spectrum of the phase jitter is much narrower than $1/T$, the variation of the phase jitter over the impulse response duration of the matched filter, which is in the order of T, can be neglected. The input of the decision device is fed with the sample $\hat{a}_{n,0}$, given by

$$\begin{aligned}\hat{a}_{n,0} &= \sqrt{E_{s,n}}a_{n,0}\underline{c}_n^T E[B_n]\underline{c}_n^* \\ &\quad + \sqrt{E_{s,n}}a_{n,0}\underline{c}_n^T (B_n - E[B_n])\,\underline{c}_n^* \\ &\quad + \sum_{\substack{\ell=0\\ \ell\neq n}}^{M-1} \sqrt{E_{s,\ell}}a_{\ell,0}\underline{c}_\ell^T B_n \underline{c}_n^* + W_{n,0}. \end{aligned} \tag{2}$$

The vector $\underline{c}_n$ contains the chip sequence of user n, where $|c_{n,i}| = 1$, $i = 0, \cdots, N-1$. The elements of the matrix B_n and the additive noise samples $W_{n,0}$ are given by

$$(B_n)_{k,\ell} = \frac{1}{N^2}\sum_{i=0}^{N-1} e^{j2\pi[i(k-\ell)/N]} e^{j\phi_n(iT)} \tag{3}$$

$$W_{n,0} = \frac{1}{N}\sum_{k,\ell=0}^{N-1} c^*_{n,\ell} e^{-j2\pi(k\ell+/N)} \int_{-\infty}^{+\infty} w(t)p^*(t-kT)\,dt. \tag{4}$$

According to (2), the sample at the input of the decision device can be decomposed into four uncorrelated contributions. The first term consists of the mean value, with respect to the carrier phase jitter, of the useful component. The second contribution contains a zero-mean disturbance caused by the fluctuation of the useful component. The third contribution is zero-mean multiuser interference caused by the phase jitter that provokes a loss of orthogonality between the different users. The fourth term is caused by the additive noise.

For small phase jitter $\phi_n(t)$, we can use the approximation $\exp(j\phi n(t)) \cong 1 + j\phi_n(t)$, which reduces the matrix elements $(B_n)_{k,l}$ to

$$(B_n)_{k,\ell} \cong \frac{1}{N}\delta_{k,\ell} + \frac{1}{N^2}\sum_{i=0}^{N-1} j\phi_n(iT)\,e^{j2\pi[i(k-\ell)/N]}. \tag{5}$$

We define the SNR at the input of the decision device as the ratio of the power of the average useful component to the power of the remaining contributions. In the presence of phase jitter, the SNR is reduced as compared to the case of no jitter $E_{s,n}/E[|W_{n,0}|^2]$. This degradation, expressed in decibels, is given by

$$\begin{aligned} D_n = 10\log\Bigg(1 &+ \frac{E_{s,n}}{E\left[|W_{n,0}|^2\right]} E\left[\left|\underline{c}_n^T(B_n - E[B_n])\underline{c}_n^*\right|^2\right] \\ &+ \sum_{\substack{\ell=0\\ \ell\neq n}}^{M-1} \frac{E_{s,\ell}}{E\left[|W_{n,0}|^2\right]} E\left[\left|\underline{c}_\ell^T(B_n - E[B_n])\underline{c}_n^*\right|^2\right]\Bigg). \end{aligned} \tag{6}$$

Assuming that all users have the same jitter spectrum $S_\phi(f)$ and the same energy per symbol E_s, the powers of the fluctuation of the useful component and the multiuser interference yield

$$\begin{aligned} E&\left[\left|\underline{c}_n^T(B_n - E[B_n])\underline{c}_n^*\right|^2\right] \\ &= E_s \int_{-\infty}^{+\infty} S_\phi(f)|H_0(f)|^2 df \end{aligned} \tag{7}$$

$$\begin{aligned} \sum_{\substack{\ell=0\\ \ell\neq n}}^{M-1} & E_{s,\ell} E\left[\left|\underline{c}_\ell^T(B_n - E[B_n])\underline{c}_n^*\right|^2\right] \\ &= \frac{M-1}{N-1} E_s \int_{-\infty}^{+\infty} S_\phi(f)\left(1 - |H_0(f)|^2\right) df \end{aligned} \tag{8}$$

where $M-1$ denotes the number of disturbing users and $|H_0(f)|^2$ is a low-pass filter given by

$$|H_0(f)|^2 = \left|\frac{1}{N}\frac{\sin(\pi f N T)}{\sin(\pi f T)}\right|^2. \tag{9}$$

In obtaining (7) and (8), we have used the approximations $E[c_{n,k}c^*_{n,k'}] \approx \delta_{k,k'}$ and $E[c_{l,k}c^*_{n,k'}c^*_{l,m}c_{n,m'}] \approx N/(N-1)(\delta_{k,m}\delta^{l}_{k,m'} - 1/N\delta_{k,k'}\delta_{m,m'})$, and followed a similar reasoning as in [10]. From (7) and (8) it follows that the fluctuation of the useful component and the multiuser interference mainly exist of the low frequency components $(<1/(NT) = R_s)$ and the high frequency components $(>1/(NT) = R_s)$ of the phase jitter, respectively. The sum of (7) and (8) is linearly increasing with M.

For the highest load, i.e., $M = N$, the sum of the powers of the fluctuation of the useful component (7) and multiuser interference (8) only depends on the jitter variance but not on the specific shape of the jitter spectrum nor on the number of OFDM tones: for N active users, the degradation is given by

$$D_n = 10\log\left(1 + \frac{E_s}{E[|W_{n,0}|^2]}\sigma_\phi^2\right) \tag{10}$$

where the jitter variance σ_ϕ^2 is defined by

$$\sigma_\phi^2 = \int_{-\infty}^{+\infty} S_\phi(f)\, df. \tag{11}$$

When $M < N$, the sum of (7) and (8) does depend on the shape of the jitter spectrum. Let us consider the following cases.

i) When all jitter power is in the interval $|f| < 1/(NT) = R_s$, the multiuser interference power (8) is essentially zero. The degradation of the SNR is independent of the number M of active users and equal to (10).

ii) Decreasing by some amount the jitter power in the interval $|f| < 1/(NT) = R_s$ and simultaneously increasing by the same amount the jitter power in the interval $|f| > 1/(NT) = R_s$, yields a decrease of (7) and an increase of (8). As $M < N$, the sum of (7) and (8) is reduced, so that the degradation of the SNR is smaller that for case i).

iii) The smallest degradation of the SNR is obtained when all jitter power is in the interval $|f| > 1/(NT) = R_s$, in which case the power (7) of the fluctuation of the useful component is essentially zero. The degradation of the SNR is essentially zero for $M = 1$ and equal to (10) for $M \to N$.

IV. Numerical Results and Conclusions

The degradation (6) was computed assuming a phase jitter spectrum given by

$$S_\phi(f) = \begin{cases} \sigma_\phi^2 \dfrac{f_H - |f|}{(f_H - f_L)^2}, & f_L \le |f| \le f_H \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

The spectrum (12) models the jitter spectrum at the output of a PLL with loop bandwidth f_L, which tries to track phase noise having a bandwidth f_H, with $f_H > f_L$. As the PLL is able to track only the phase noise components within its loop bandwidth [12], the residual jitter spectrum (12) is set to zero for $|f| < f_L$.

Fig. 2 shows the degradation (6) as a function of the ratio $(M-1)/(N-1)$ for various values of f_L/R_s and f_H/R_s, assuming $\sigma_\phi^2 = -40$ dB rad^2 and $SNR_{\phi=0} = E_s/E[|W_{n,0}|^2] = 25$ dB.

- For $f_H/R_s = 0.5$, all jitter power is within the interval $|f| < R^s$. The degradation is independent of the number of users and of the PLL bandwidth f_L ($f_L < f_H$).
- For $f_L/R_s = 5$, all jitter power is within the interval $|f| > R_s$. The degradation is independent of the jitter bandwidth f_H ($f_H > f_L$) and assumes its minimum value.
- For $f_L/R_s = 0$ and $f_H/R_s \ge 1$, phase jitter is present in both intervals $|f| < R_s$ and $|f| > R_s$. When f_H/R_s increases, the fluctuation of the useful component decreases and the multiuser interference increases; the net effect is a reduction of the degradation.

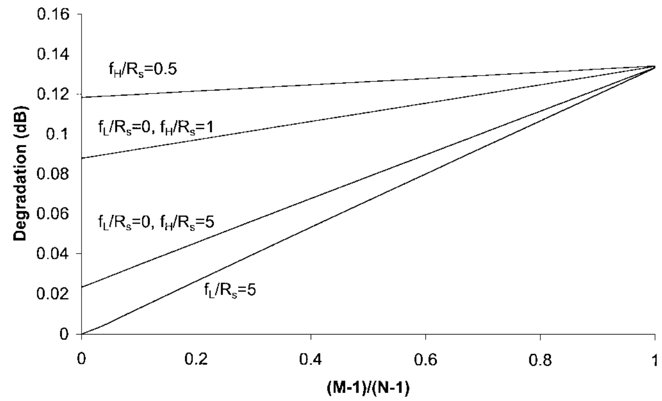

Fig. 2. Degradation as function of M: $\sigma_\phi^2 = -40$ dB, $SNR_{\phi=0} = 25$ dB.

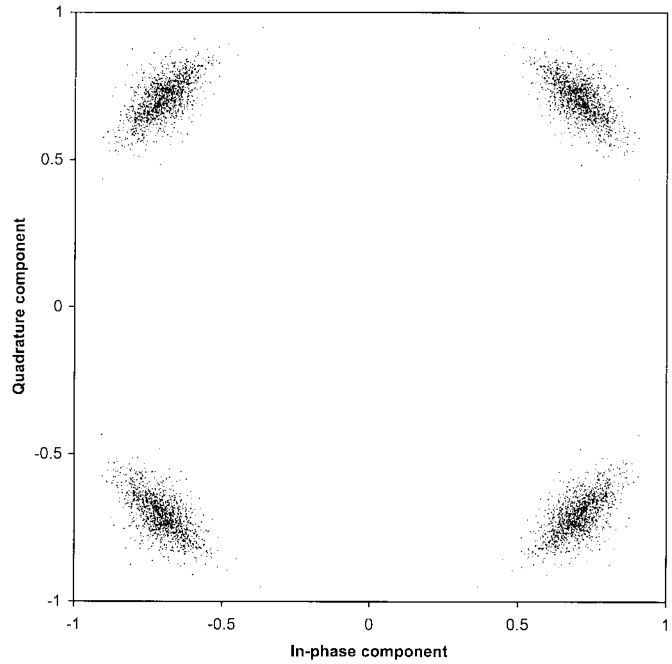

Fig. 3. Scatter diagram for $f_L = 0$, $f_H = 0.02/T$, $N-1$ disturbing users, $N = 128$.

We simulated (2) in the absence of additive noise, assuming $N = 128$ tones with QPSK modulation and Walsh–Hadamard sequences of length 128; for $M = N$ and the jitter spectrum is of (12) with $\sigma_f^2 = -21$ dB rad^2. Figs. 3 and 4 show the scatter diagrams for $f_L = 0$, $f_H = 0.02/T = 2.56R_s$ and $f_L = 0.02/T = 2.56R_s$, $f_H = 0.04/T = 5.12R_s$, respectively. As $M = N$, the mean-square deviation of the samples (2) from the constellation points is the same for both scatter diagrams and equals σ_f^2. However, the scatter diagrams differ considerably.

When $f_L = 0$, $f_H = 2.56R_s$, the jitter in the interval $|f| < R_s$ gives rise to a random rotation of the useful component (i.e., an angular displacement in the scatter diagram), while the jitter in the interval $|f| > R_s$ gives rise to multiuser interference (i.e., a circular cloud in the scatter diagram). The superposition of these effects yields a scatter diagram with an angular displacement that is larger than the radial displacement.

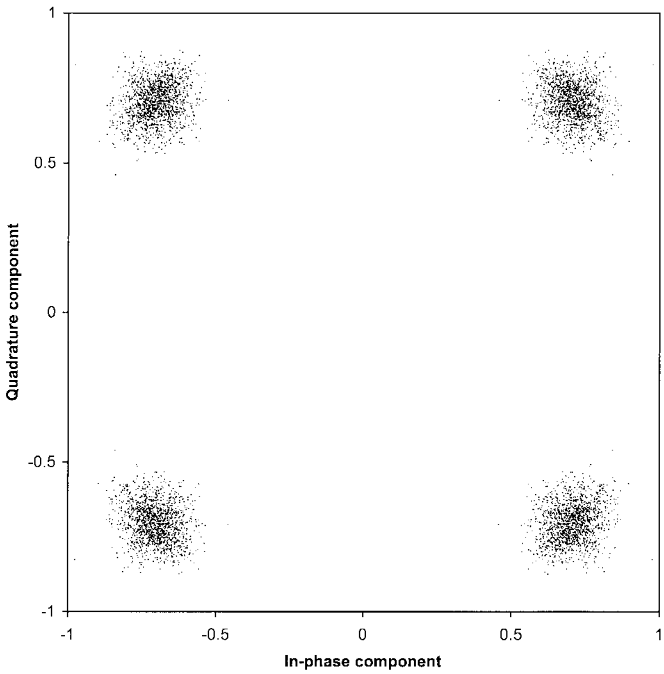

Fig. 4. $f_L = 0.02/T$, $f_H = 0.04/T$, $N-1$ disturbing users, $N = 128$.

When $f_L = 2.56R_s$, $f_H = 5.12R_s$, the random rotation of the useful component is negligible, and the multiuser interference power equals σ_ϕ^2. The scatter diagram only contains the circular clouds caused by multiuser interference.

Similar scatter diagrams have been observed in the case of OFDM [10].

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