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A Theoretical Framework for Soft-Information-Based Synchronization in Iterative (Turbo) Receivers

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This contribution considers turbo synchronization, that is to say, the use of soft data information to estimate parameters like carrier phase, frequency, or timing offsets of a modulated signal within an iterative data demodulator. In turbo synchronization, the receiver exploits the soft decisions computed at each turbo decoding iteration to provide a reliable estimate of some signal parameters. The aim of our paper is to show that such "turbo-estimation" approach can be regarded as a special case of the expectation-maximization (EM) algorithm. This leads to a general theoretical framework for turbo synchronization that allows to derive parameter estimation procedures for carrier phase and frequency offset, as well as for timing offset and signal amplitude. The proposed mathematical framework is illustrated by simulation results reported for the particular case of carrier phase and frequency offsets estimation of a turbo-coded 16-QAM signal.

Keywords and phrases: turbo synchronization, iterative detection, turbo codes, parameter estimation.

1. INTRODUCTION

The impressive performance of turbo codes [1] has triggered in the last decade a lot of research addressing the application of this powerful coding technique to digital communications [2]. More recently, the associated idea of iterative decoding has been extended to other receiver functions. This led to the so-called *turbo principle* which enables to perform (sub)optimal joint detection and decoding through the iterative exchange of soft information between soft-input/soft-output (SISO) stages. See [3, 4] for a review of some existing turbo receivers.

In addition to detection/decoding a receiver has also to perform signal synchronization, that is, to estimate a number of parameters like carrier phase offset, frequency offset, timing offset, and so forth. Synchronization for turbo-encoded systems is a challenging task since the receiver usually operates at low SNR values (which can be defined as the ratio between the mean bit energy and the noise spectral density). In the technical literature a great effort is thus being devoted to the development of efficient estimation techniques to perform the above-mentioned synchronization functions within turbo receivers. We outline here at least two categories of algorithms.

(i) The first category consists of algorithms that try to modify classical SISO iterative detection/decoding in order to embed parameter estimation. In [5, 6], for instance, combined iterative decoding and estimation is performed with modified forward and backward recursions in the SISO decoders using a sort of per-survivor parameter estimation technique. In [7], the conventional turbo decoder structure is modified through the use of a simple phase estimation error model. A different approach is pursued in [8] wherein a method (having only polynomial complexity in the sequence length) of generating soft-decision metrics is illustrated and specifically applied to the problem of adaptive iterative detection of LDPC codes in the presence of time-varying unknown carrier phase offset. Further, simpler approximate receivers are proposed in [9] based on the insertion into each transmitted coded block of a number of pilot symbols with the aim of helping the joint phase estimation and decoding process.

(ii) The second category consists of algorithms that try to use the soft information provided at each iteration by a conventional turbo decoder. This approach will be referred to as *turbo synchronization* in the sequel. In [10], a carrier phase recovery algorithm operating in conjunction with the SISO decoders and exploiting the extrinsic information generated at each iteration is proposed. Furthermore, in [11, 12], for instance, it is proposed to combine soft-decision-directed carrier phase estimation with turbo decoding. Tentative decision-aided synchronization within a turbo decoder is reported in [13, 14].

Algorithms in the latter category seem to be promising but they often do not rely on any theoretical basis. The purpose of this paper is therefore to give a mathematical interpretation of such turbo synchronization algorithms and to generalize them. This can be done by means of the expectation-maximization (EM) algorithm. Such an algorithm has been applied to various problems, as in [15], for instance, wherein it is used for channel and noise variance estimation in combination with optimal BCJR-based detection. The same is done in [16] in combination with a suboptimal filter-based equalizer and in [17] for a coded CPM system. In [18], channel gain, and delay estimation is performed in an uncoded CDMA system with a hard-output iterative serial interference canceller. These ideas have been extended to turbo receivers in [19] (see also references therein) and [20] for channel and noise variance estimation in turbo-CDMA and turbo-MIMO contexts, respectively.

In the present paper, we will focus on the specific problem of synchronization. Section 2 will give a general formulation of iterative ML estimation of unknown parameters in the presence of nuisance parameters by means of the EM algorithm. The particular issue of synchronization (i.e., carrier phase, frequency offset, channel gain, and timing estimation) for a digital data-modulated passband signal will then be addressed in Section 3. This implementation will then be extended to the turbo context by showing that the EM algorithm iterations (for parameter estimation) can be combined with those of a turbo receiver (for symbol detection/decoding). This will lead to a general theoretical framework for turbo synchronization. In particular, it will turn out that algorithms introduced in an ad hoc fashion, such as the blind soft-decision-directed carrier phase turbo synchronizer recently proposed in [11], actually correspond to a particular instance of the general scheme proposed here. In order to illustrate the mathematical considerations, in Section 4 we consider as a case study the practical problem of carrier phase and frequency offsets estimation for a turbo-coded 16-QAM system. The relevant simulation results show that the proposed scheme enables to perform blind reliable synchronization and almost ideal coherent detection at very low SNR as required in a turbo receiver. Section 5 considers the computational complexity of the proposed algorithm, whereas a concluding section eventually ends up the paper.

2. ML ESTIMATION IN THE PRESENCE OF A NUISANCE VECTOR

We denote with $\mathbf{r}$ a random vector obtained by expanding the received modulated signal $r(t)$ onto a suitable basis, and we indicate with $\mathbf{b}$ a deterministic vector of parameters to be estimated from the observation of the received vector $\mathbf{r}$. Assume that $\mathbf{r}$ also depends on a random nuisance parameter vector $\mathbf{a}$ independent of $\mathbf{b}$ and with a priori probability density function (pdf) $p(\mathbf{a})$. The problem addressed in this section is to find the ML estimate $\hat{\mathbf{b}}$ of $\mathbf{b}$, that is to say, the solution of

$$\hat{\mathbf{b}} = \underset{\tilde{\mathbf{b}}}{\operatorname{argmax}} \{ \ln p(\mathbf{r}|\tilde{\mathbf{b}}) \}. \tag{1}$$

The likelihood function to be maximized with respect to the trial value $\tilde{\mathbf{b}}$ of $\mathbf{b}$ is obtained after elimination of the nuisance parameter vector $\mathbf{a}$ as follows:

$$p(\mathbf{r}|\tilde{\mathbf{b}}) = \int_{\mathbf{a}} p(\mathbf{a}) p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}}) d\mathbf{a}. \tag{2}$$

In order to solve (1), we take the derivative of $\ln p(\mathbf{r}|\tilde{\mathbf{b}})$ with respect to $\tilde{\mathbf{b}}$ and we equate it to zero, that is,

$$\begin{aligned} \frac{\partial}{\partial \tilde{\mathbf{b}}} \ln p(\mathbf{r}|\tilde{\mathbf{b}}) &= \frac{\int_{\mathbf{a}} p(\mathbf{a}) p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}}) (\partial/\partial\tilde{\mathbf{b}}) \ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}}) d\mathbf{a}}{\int_{\mathbf{a}} p(\mathbf{a}) p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}}) d\mathbf{a}} \\ &= \int_{\mathbf{a}} \frac{p(\mathbf{a}) p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})}{p(\mathbf{r}|\tilde{\mathbf{b}})} \frac{\partial}{\partial \tilde{\mathbf{b}}} \ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}}) d\mathbf{a} = \mathbf{0}. \end{aligned} \tag{3}$$

Now, it is easily seen using Bayes' rule that the first factor in the integrand into (3) is nothing but the a posteriori conditional pdf $p(\mathbf{a}|\mathbf{r},\tilde{\mathbf{b}})$ of the nuisance vector

$$\frac{p(\mathbf{a}) p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})}{p(\mathbf{r}|\tilde{\mathbf{b}})} = p(\mathbf{a}|\mathbf{r},\tilde{\mathbf{b}}). \tag{4}$$

Therefore, the ML estimation problem given by (1), (2), and (3) is turned into

$$\begin{aligned} \frac{\partial}{\partial \tilde{\mathbf{b}}} \ln p(\mathbf{r}|\tilde{\mathbf{b}}) &= \int_{\mathbf{a}} p(\mathbf{a}|\mathbf{r},\tilde{\mathbf{b}}) \frac{\partial}{\partial \tilde{\mathbf{b}}} \ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}}) d\mathbf{a} \\ &= E_{\mathbf{a}} \left\{ \frac{\partial}{\partial \tilde{\mathbf{b}}} \ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}}) \,|\, \mathbf{r}, \tilde{\mathbf{b}} \right\} = \mathbf{0}. \end{aligned} \tag{5}$$

In other words, the ML estimate $\hat{\mathbf{b}}$ of $\mathbf{b}$ is that value that nulls the conditional a posteriori expectation of the derivative with respect to $\tilde{\mathbf{b}}$ of the conditional log-likelihood function (LLF) $\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})$.

Finding the solution of (5) is not trivial, since $\tilde{\mathbf{b}}$ appears in *both* factors of the integrand. Thus, we try an iterative method that produces a sequence of values $\hat{\mathbf{b}}^{(n)}$ hopefully converging to the desired solution. In particular, we use the previous sequence value $\hat{\mathbf{b}}^{(n-1)}$ to resolve the conditioning on the first factor of the integrand, and we find the current solution $\hat{\mathbf{b}}^{(n)}$ by solving the resulting simplified equation that follows:

$$\int_{\mathbf{a}} p(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})\left[\frac{\partial}{\partial\tilde{\mathbf{b}}}\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})\,|_{\tilde{\mathbf{b}}=\hat{\mathbf{b}}^{(n)}}\right]d\mathbf{a}=\mathbf{0}. \quad (6)$$

If the sequence of estimates $\hat{\mathbf{b}}^{(n)}$ yielded by (6) converges to a finite value, that value is a solution of ML equation (5) [21].

Observe now that the first factor of the integrand in (6) does not depend on $\hat{\mathbf{b}}^{(n)}$. Therefore, we can bring the derivative back out of the integral and obtain the equivalent equation

$$\hat{\mathbf{b}}^{(n)}:\frac{\partial}{\partial\tilde{\mathbf{b}}}\left\{\int_{\mathbf{a}} p\left(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)}\right)\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})d\mathbf{a}\right\}\Bigg|_{\tilde{\mathbf{b}}=\hat{\mathbf{b}}^{(n)}}=\mathbf{0}, \quad (7)$$

that is, the estimate $\hat{\mathbf{b}}^{(n)}$ maximizes the conditional a posteriori expectation of the conditional LLF $\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})$:

$$\hat{\mathbf{b}}^{(n)}=\underset{\tilde{\mathbf{b}}}{\operatorname{argmax}}\{\Lambda(\tilde{\mathbf{b}},\hat{\mathbf{b}}^{(n-1)})\}, \quad (8a)$$

$$\begin{aligned}\Lambda(\tilde{\mathbf{b}},\hat{\mathbf{b}}^{(n-1)})&=E_{\mathbf{a}}\{\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})|\mathbf{r},\hat{\mathbf{b}}^{(n-1)}\}\\&=\int_{\mathbf{a}} p(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})d\mathbf{a}.\end{aligned} \quad (8b)$$

Formulation (8a)-(8b) of our iterative solution can also be derived by means of the EM algorithm [21, 22, 23]. Consider $\mathbf{r}$ as the "incomplete" observation and $\mathbf{z}\triangleq(\mathbf{r}^T,\mathbf{a}^T)^T$ as the "complete" observation. The EM algorithm states that the sequence $\hat{\mathbf{b}}^{(n)}$ defined by

(i) expectation step (E-step):

$$Q(\tilde{\mathbf{b}},\hat{\mathbf{b}}^{(n-1)})=E_{\mathbf{a}}\{\ln p(\mathbf{z}|\tilde{\mathbf{b}})|\mathbf{r},\hat{\mathbf{b}}^{(n-1)}\}, \quad (9a)$$

(ii) maximization step (M-step):

$$\hat{\mathbf{b}}^{(n)}=\underset{\tilde{\mathbf{b}}}{\operatorname{argmax}}\{Q(\tilde{\mathbf{b}},\hat{\mathbf{b}}^{(n-1)})\} \quad (9b)$$

converges to the ML estimate under mild conditions [21, 22]. To make (9a)-(9b) equivalent to (8a)-(8b), we observe that, by using the Bayes rule and considering that the distribution of $\mathbf{a}$ does not depend on the parameter vector to be estimated,

$$\begin{aligned}p(\mathbf{z}|\tilde{\mathbf{b}})&=p(\mathbf{r},\mathbf{a}|\tilde{\mathbf{b}})=p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})p(\mathbf{a}|\tilde{\mathbf{b}})\\&=p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})p(\mathbf{a}).\end{aligned} \quad (10)$$

Therefore, substituting (10) in (9a), we get

$$\begin{aligned}Q(\tilde{\mathbf{b}},\hat{\mathbf{b}}^{(n-1)})=&\int_{\mathbf{a}} p(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})d\mathbf{a}\\&+\underbrace{\int_{\mathbf{a}} p(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})\ln p(\mathbf{a})d\mathbf{a}}_{\zeta}.\end{aligned} \quad (11)$$

The second term ζ in (11) does not depend on $\tilde{\mathbf{b}}$, and as far as the M-step is concerned, it can be dropped. Consequently, the estimation procedure given by (8a)-(8b) and the EM algorithm, defined by (9b) and (11), yield the same sequence of estimates. We explicitly observe that the solution of (1) can be found iteratively by only using a posteriori probabilities $p(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ and the LLF $\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})$.

3. APPLICATION TO SYNCHRONIZATION FOR SOFT-INFORMATION-BASED RECEIVERS

3.1. EM-based synchronization

In this section, we will show how to apply the general framework of the previous section to the estimation of the synchronization parameters for a digital data-modulated bandpass signal. In this context, the nuisance parameter vector $\mathbf{a}$ contains the values of the N unknown (hence random) transmitted symbols, that is, $\mathbf{a}^T=(a_0,\ldots,a_{N-1})$. Those symbols take values in an M-point constellation $\mathcal{A}$ (such as M-PSK, M-QAM, etc.) according to some rule. Thus, the vector $\mathbf{a}$ has a probability mass function (pmf) $P(\mathbf{a}=\boldsymbol{\mu})$, with $\boldsymbol{\mu}^T=(\mu_0,\ldots,\mu_{N-1})$ and $\boldsymbol{\mu}\in\mathcal{A}^N$. The vector $\mathbf{b}$ contains the synchronization parameters to be estimated, that is, $\mathbf{b}^T=(A,\tau,\nu,\vartheta)$ where A, τ, ν, ϑ are the channel gain, symbol timing, carrier frequency, and phase offsets, respectively. Here, the synchronization parameters are assumed as constant within the received code block. This has the advantage of simplifying notably the processing required by the estimation algorithm while inherently is the main limitation of the approach itself. However, a possible yet reasonable solution to handle a time-varying phase offset (due, e.g., to phase noise) is shown in [24]. The idea is quite simple and consists in subdividing the entire block in a number of subblocks within which the phase can be considered approximately as constant, and then in applying to each of them the soft-information-based estimation procedure proposed above. Further, yet again for the sake of simplicity, we will consider in the sequel an AWGN channel as well. Hence, putting all these facts together, the baseband received signal $r(t)$ can be written as

$$r(t)=A\sum_{k=0}^{N-1}a_k g(t-kT-\tau)e^{j(2\pi\nu t+\vartheta)}+w(t), \quad (12)$$

where T is the symbol period, $g(t)$ is a unit-energy (e.g., square-root raised-cosine) pulse, and $w(t)$ is complex-valued AWGN with power spectral density $2N_0$ (assumed to be known).

Neglecting irrelevant terms independent of $\mathbf{a}$ and $\mathbf{b}$, the conditional LLF of (12) is

$$\begin{aligned}\ln p(\mathbf{r}|\mathbf{a},\tilde{\mathbf{b}})=&-2\tilde{A}\,\mathrm{Re}\left\{\sum_{k=0}^{N-1}a_k^* z_k(\tilde{\nu},\tilde{\tau})e^{-j\tilde{\vartheta}}\right\}\\&+\tilde{A}^2\sum_{k=0}^{N-1}|a_k|^2,\end{aligned} \quad (13)$$

where

$$z_k(\tilde{\nu},\tilde{\tau}) \triangleq \int_{-\infty}^{\infty} r(t)e^{-j2\pi\tilde{\nu}t}g(t-kT-\tilde{\tau})\,dt = \left[r(t)e^{-j2\pi\tilde{\nu}t}\right]\otimes g(-t)|_{t=kT+\tilde{\tau}} \tag{14}$$

is obtained by frequency precompensating the received signal by the "trial" value $-\tilde{\nu}$, then applying the result to the matched filter $g(-t)$, and finally sampling the matched filter output at the "trial" instant $kT+\tilde{\tau}$. Substituting (13) into (8b) and dropping the terms which do not depend on $\tilde{\mathbf{b}}$, we get

$$\begin{aligned}\Lambda(\tilde{\mathbf{b}},\hat{\mathbf{b}}^{(n-1)}) &= -2\tilde{A}\,\mathrm{Re}\left\{\sum_{k=0}^{N-1}\left[\int_{\mathbf{a}} a_k p\left(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)}\right)d\mathbf{a}\right]^{*} z_k(\tilde{\nu},\tilde{\tau})e^{-j\tilde{\vartheta}}\right\} \\ &\quad + \tilde{A}^2\sum_{k=0}^{N-1}\left[\int_{\mathbf{a}} |a_k|^2 p\left(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)}\right)d\mathbf{a}\right].\end{aligned} \tag{15}$$

We now define $\eta_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ and $\rho_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$, the a posteriori mean and a posteriori mean square value of the channel symbol a_k, respectively, as follows:

$$\eta_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)}) \triangleq \int_{\mathbf{a}} a_k p(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})\,d\mathbf{a} = \sum_{\alpha_m\in A}\alpha_m P(a_k=\alpha_m|\mathbf{r},\hat{\mathbf{b}}^{(n-1)}), \tag{16a}$$

$$\rho_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)}) \triangleq \int_{\mathbf{a}} |a_k|^2 p(\mathbf{a}|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})\,d\mathbf{a} = \sum_{\alpha_m\in A}|\alpha_m|^2 P(a_k=\alpha_m|\mathbf{r},\hat{\mathbf{b}}^{(n-1)}). \tag{16b}$$

$P(a_k=\alpha_m|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ denotes the marginal a posteriori probability (APP) of the kth channel symbol a_k conditioned on the observation $\mathbf{r}$ and on the estimate $\hat{\mathbf{b}}^{(n-1)}$ at the previous $(n-1)$th step, and α_m the M possible values taken in the constellation A. Equation (15) can then be rearranged as

$$\Lambda(\tilde{\mathbf{b}},\hat{\mathbf{b}}^{(n-1)}) = -2\tilde{A}\,\mathrm{Re}\left\{\sum_{k=0}^{N-1}\eta_k^{*}(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})z_k(\tilde{\nu},\tilde{\tau})e^{-j\tilde{\vartheta}}\right\} + \tilde{A}^2\sum_{k=0}^{N-1}\rho_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)}). \tag{17}$$

We emphasize the similarity between (13) and (17): the latter is formally obtained from the former by simply replacing the terms a_k and $|a_k|^2$ by their respective a posteriori expected values $\eta_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ and $\rho_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$.

The new estimate $\hat{\mathbf{b}}^{(n)}$ at the nth step is then determined by applying (8a) and therefore by maximizing $\Lambda(\tilde{\mathbf{b}},\hat{\mathbf{b}}^{(n-1)})$, given by (17), with respect to $\tilde{\mathbf{b}}$. The corresponding result is

$$[\hat{\nu}^{(n)},\hat{\tau}^{(n)}] = \underset{\tilde{\nu},\tilde{\tau}}{\operatorname{argmax}}\left\{\left|\sum_{k=0}^{N-1}\eta_k^{*}(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})z_k(\tilde{\nu},\tilde{\tau})\right|\right\}, \tag{18a}$$

$$\hat{\vartheta}^{(n)} = \angle\left\{\sum_{k=0}^{N-1}\eta_k^{*}(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})z_k[\hat{\nu}^{(n)},\hat{\tau}^{(n)}]\right\}, \tag{18b}$$

$$\hat{A}^{(n)} = \frac{\left|\sum_{k=0}^{N-1}\eta_k^{*}(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})z_k[\hat{\nu}^{(n)},\hat{\tau}^{(n)}]\right|}{\sum_{k=0}^{N-1}\rho_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})}. \tag{18c}$$

The obtained solution can be interpreted as an *iterative synchronization* procedure, which can be referred to as *soft-decision-directed* (SDD) synchronization. What we call here *soft decisions* are the a posteriori average values $\eta_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ and $\rho_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ of each channel symbol. They are a sort of "weighted average" over all the constellation points according to the respective symbol APPs. Note that, thanks to (16a) and (16b), these a posteriori average values $\eta_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ and $\rho_k(\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ can be computed from the marginals $P(a_k=\alpha_m|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$ only. In other words, due to the particular structure of the digital data-modulated signal, the implementation of the iterative ML estimation algorithm only requires the evaluation of the marginal a posteriori symbol probabilities $P(a_k=\alpha_m|\mathbf{r},\hat{\mathbf{b}}^{(n-1)})$.

We now concentrate on the evaluation of the marginal a posteriori symbol probabilities. Whereas for uncoded transmission the usual assumption is that data symbols are independent and equally likely (yielding $P(\mathbf{a}=\boldsymbol{\mu})=M^{-N}$ for all $\boldsymbol{\mu}\in\mathcal{A}^N$), for a coded transmission with code rate λ, we only have a subset $\mathcal{B}\subset A^N$ of all possible sequences corresponding to $M^{\lambda N}$ legitimate encoder output sequences. Therefore, taking into account that the APP of the symbol sequence $\mathbf{a}$ is given by

$$P(\mathbf{a}=\boldsymbol{\mu}|\mathbf{r},\tilde{\mathbf{b}}) = \frac{P(\mathbf{a}=\boldsymbol{\mu})p(\mathbf{r}|\mathbf{a}=\boldsymbol{\mu},\tilde{\mathbf{b}})}{\sum_{\nu\in\mathcal{B}}P(\mathbf{a}=\nu)p(\mathbf{r}|\mathbf{a}=\nu,\tilde{\mathbf{b}})}, \tag{19}$$

and assuming that

$$P(\mathbf{a}=\boldsymbol{\mu}) = \begin{cases} M^{-\lambda N}, & \boldsymbol{\mu}\in\mathcal{B},\\ 0, & \boldsymbol{\mu}\notin\mathcal{B},\end{cases} \tag{20}$$

we get

$$P(\mathbf{a}=\boldsymbol{\mu}|\mathbf{r},\tilde{\mathbf{b}}) = \begin{cases}\dfrac{p(\mathbf{r}|\mathbf{a}=\boldsymbol{\mu},\tilde{\mathbf{b}})}{\sum_{\nu\in\mathcal{B}}p(\mathbf{r}|\mathbf{a}=\nu,\tilde{\mathbf{b}})}, & \boldsymbol{\mu}\in\mathcal{B},\\ 0, & \boldsymbol{\mu}\notin\mathcal{B},\end{cases} \tag{21}$$

which relates the APP of the symbol sequence to the conditional likelihood function. Note that the result for uncoded transmission is obtained from (21) by taking $B=A^N$. Finally, the marginal APP related to a symbol a_k is obtained by summing the symbol sequence APPs (21) over all symbols a_i with $i\neq k$.

Evaluation of the APPs according to (21) yields a computational complexity that increases exponentially with the sequence length N, as all possible data sequences must be enumerated. However, in systems where the received signal can be modeled as a Markov process, (i.e., transmission over a frequency selective channel, coded systems, MIMO or CDMA systems, etc.), the marginal symbol APPs $P(a_k = \alpha_m | \mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$ can be efficiently obtained using the BCJR algorithm [25], with a complexity that grows only linearly with the sequence length N. Note however that the computations related to the BCJR algorithm must then be carried out once per iteration of the synchronizer.

3.2. Turbo synchronization

The EM-based synchronization procedure proposed in the previous subsection is intrinsically well suited to iterative (turbo) receivers that perform detection/decoding through extrinsic information exchange between SISO stages. Indeed, one usually assumes that such receivers provide, after convergence of the iterative process, soft information that equals channel symbol APPs. This makes synchronization via the EM algorithm and turbo receivers complementary since the symbol APPs needed by the first one can be provided by the second one.

As shown in the previous subsection, the estimation of the synchronization parameters needs at each EM iteration the knowledge of the marginal APPs $P(a_k = \alpha_m | \mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$ in order to compute the a posteriori expected values $\eta_k(\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$ and $\rho_k(\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$ required for the evaluation of (18a), (18b), and (18c). In a strict implementation, this means that at *each EM iteration* the turbo receiver has to reinitialize the extrinsic information, and then has to iterate until the soft information reaches a steady-state value, in order to yield good approximations of the required symbol APPs. It is clear that the main drawback of this approach is the considerable increase in complexity and latency in comparison with the corresponding ideal synchronized turbo receiver, since the turbo system is required to converge at each EM iteration. To deal with such a trouble, an approximate implementation can be used: the turbo decoder is no longer reinitialized and at each EM iteration only one detection/decoding iteration is performed. In other words, the synchronization iterations (EM algorithm) are merged with the detection/decoding ones (turbo decoder). Note that this approximate "merged" procedure strictly differs from the EM algorithm in that performing only one detection/decoding iteration at each EM iteration (especially in the first ones) leads to poorer estimations of the required symbol APPs. To investigate the potential performance degradation that the proposed simplified algorithm may imply, in [26] the BER performance of both the EM-based synchronizer and its approximate version are evaluated in the context of a BICM (bit-interleaved-coded modulation) 8-PSK transmission scheme. The difference between the two different synchronization methods is that at each EM iteration in the former we make additionally 5 detection/decoding iterations whereas in the latter only 1 detection/decoding iteration is performed. In spite of this rough simplification, the simulation results surprisingly indicate a negligible performance degradation at EM iteration 10, even though the EM-based method exhibits a faster convergence due to a more reliable symbol APPs estimates in the first iterations.

When applied to the specific case of carrier phase estimation for turbo-coded QAM transmission, the proposed approximate implementation leads to the algorithm introduced earlier in an ad hoc fashion in [11, 27], wherein the symbol APPs computed at each turbo decoding iteration are properly combined with the received samples in order to provide a reliable estimate necessary for coherent demodulation. This leads in this case to a sort of "bootstrapping effect," wherein decoding helps synchronization that in turn aids decoding and so forth. Therefore, more generally it can be concluded that the proposed mathematical framework provides a theoretical justification to the category of ad hoc algorithms which make use of the available soft decisions in a turbo receiver for the purpose of iteratively estimating the synchronization parameters. Furthermore, if one has to deal with a parameter vector $\mathbf{b}$ for which more than one or two parameters have to be estimated at the same time, it may happen that the turbo receiver must be allowed to proceed for more iterations between the synchronization steps. In this more demanding context, the number of needed detection/decoding iterations has to be selected considering the trade-off between the requirement on providing an accurate estimation of the APPs and the corresponding increase in complexity and latency.

As far as the initial parameter estimate $\hat{\mathbf{b}}^{(0)}$ is concerned, we have to point out that convergence of our iterative, EM-like, synchronization algorithm to the true ML estimate is not unconditional. Due to the highly nonlinear properties shown by the turbo decoding process, a good choice of $\hat{\mathbf{b}}^{(0)}$ certainly affects the system performance and is mandatory in order to enable the convergence of the joint detection and decoding scheme. However, finding a "good" initial value and then refining it through an iterative procedure looks like the acquisition/tracking approach. In our context, the issue of the initial acquisition may be solved in general by making a data-aided preliminary estimate based upon a preamble of pilot symbols. With respect to conventional methods, it is clear that by additionally exploiting the APP information, the length of the pilot sequence may be properly reduced, thereby increasing the spectral efficiency of the transmission system. We will also show in the next section that in some cases (e.g., phase estimation considering turbo-coded QAM transmission) no preamble is required, and acquisition (within a multiple of $\pi/2$) is accomplished as well, provided that the estimate is refined block after block. We will call this approach "time-recursive," and we will reserve the term "iterative" to successive estimation of a parameter on a *single* data block as described above.

4. SIMULATION RESULTS

Theoretical analysis of the proposed algorithms proved to be extremely difficult. We resorted therefore to simulation to derive performance results of the different iterative SDD turbo synchronization algorithms. As a case study, we consider a turbo-coded QAM-modulated transmission scheme. We focus here on the simple case where the channel gain A and the timing offset τ are known to the receiver, so that only the carrier frequency offset ν and phase offset ϑ, assumed to be constant within the received block, have to be estimated. To be more specific, the corresponding joint SDD phase-frequency recovery procedure is based on (18a)-(18b), that is, assuming the estimates of A and τ replaced by their a priori known values can be written as

$$\widehat{\nu}^{(n)} = \underset{\widetilde{\nu}}{\operatorname{argmax}} \left\{ \left| \sum_{k=0}^{N-1} \eta_k^*(\mathbf{r}, \widehat{\mathbf{b}}^{(n-1)}) z_k(\widetilde{\nu}, \tau) \right| \right\}, \tag{22a}$$

$$\widehat{\vartheta}^{(n)} = \angle \left\{ \sum_{k=0}^{N-1} \eta_k^*(\mathbf{r}, \widehat{\mathbf{b}}^{(n-1)}) z_k[\widehat{\nu}^{(n)}, \tau] \right\}. \tag{22b}$$

The required a posteriori average values $\eta_k(\mathbf{r}, \widehat{\mathbf{b}}^{(n-1)})$ given by (16a) are evaluated on the basis of the symbol APPs computed at the output of the turbo decoder (see Section 5 for more details). In the sequel, according to the discussion in Section 3.2, only one decoding iteration is performed at each synchronization iteration, in order to limit the overall complexity and latency. Therefore, at the nth iteration the estimate $\widehat{\nu}^{(n)}$ is found according to (22a) and used to reevaluate the samples $z_k(\widetilde{\nu}, \tau)$ by *frequency* compensating the received signal by $-\widehat{\nu}^{(n)}$ and sampling the matched filter output at the "exact" instant $kT + \tau$. Then, the phase estimate $\widehat{\vartheta}^{(n)}$ is computed by applying (22b) and eventually employed for *phase* compensating the matched filter output samples for the next decoding iteration. As initial estimates for the iterative synchronization procedure, we took $(\widehat{\nu}^{(0)}, \widehat{\vartheta}^{(0)}) = (0, 0)$ in (22a)-(22b).

We consider the simple rate $-\lambda = 3/4$ turbo encoder that encompasses parallel concatenation of two identical binary 16-state rate $-1/2$ recursive systematic convolutional (RSC) encoders with generators $g_1 = (31)_8$ and $g_2 = (33)_8$ [28], via a pseudorandom interleaver with block length $L = 1500$ information bits, and an appropriate puncturing pattern so that the block at the turbo-encoder output comprises 2000 coded bits. This binary turbo code is combined with conventional gray-mapped 16-QAM modulation (giving rise to a transmitted block of 500 symbols) adhering to the so-called suboptimum "pragmatic approach" wherein coding and modulation are performed separately, as illustrated in [29]. Simulation results are provided assuming that the carrier frequency and phase offsets are time-invariant on the transmitted data block. In addition, the above offsets change from one block to the next only in the case of the single-block joint SDD carrier recovery approach, whereas they are considered invariant if the time-recursive algorithm is applied.

The baseband-equivalent architecture of such a turbo-coded transmission system and the encoder schematic are depicted in Figures 1a and 1b, respectively. Note that, in contrast with (14), frequency correction is applied *after* matched filtering. Indeed, in the case of $|\nu T| \ll 1$, this modification causes a negligible performance degradation and, more notably, enables a remarkable reduction in the receiver complexity. At the receiver, consistently to the encoding process, pragmatic disjoint demodulation and binary turbo decoding is performed. As for the latter, to decrease its computational complexity we resort to a suboptimal solution given by the Max-Log-MAP algorithm [30]. Further, the symbol APPs required by the turbo synchronization algorithm can be obtained from the coded bits log-likelihood ratios (LLRs) made available at the output of the binary turbo decoder (see Section 5 for more details).

The proposed synchronization algorithm's performance will be assessed through evaluation of the mean estimated value (MEV) and the root-mean squared estimation error (RMSEE). We will also investigate the overall BER performance of the coded system with carrier recovery as compared to ideal synchronization, taking as main design parameters the number of decoder iterations I and the energy per bit-to-noise spectral density ratio E_b/N_0.

4.1. *MEV curves*

Figure 2 depicts the MEV curves (i.e., the average estimated value $E\{\widehat{\vartheta}\}$ as a function of the true phase offset ϑ) for the SDD phase recovery algorithm based on (22b) for different numbers of decoder iterations $I = 8, 10, 12$, assuming a null frequency offset and with $E_b/N_0 = 6\,\text{dB}$ (roughly corresponding to BER $= 10^{-4}$ with ideal carrier recovery). The difference between the MEV curves is not significant for phase errors $|\vartheta| \leq 20°$, whatever the number of iterations, whereas with larger phase errors the bias of the algorithm is negligible only for $I = 10, 12$. For the particular transmission scheme of Figure 1a, the rotational invariance is not destroyed and the usual $\pi/2$ estimation ambiguity due to the four-fold symmetry of the QAM constellation is apparent, as can be found in [11]. Note that, if one can afford an increase in complexity, the above problem can be easily handled by evaluating the average value of the absolute soft output of the decoder for different multiples of $\pi/2$, and choosing the phase offset that provides the highest reliability according to the approach illustrated in [12].

The MEV curves illustrated in Figure 2 suggest using this estimator as a sort of phase error detector in a *time-recursive* recovery scheme. This can be done on a block-by-block recursive basis as follows. We denote with $\widehat{\vartheta}_m$ the time-recursive phase estimate related to the mth data block and with $\widehat{\varphi}_m^{(I)}$ the phase error estimate after I decoding iterations as described above. After a prerotation of the received samples in the $(m+1)$th data block by $-\widehat{\vartheta}_m$ and a new phase error estimate $\widehat{\varphi}_{m+1}^{(I)}$, the phase estimate for the $(m+1)$th data block is computed as

$$\widehat{\vartheta}_{m+1} = \widehat{\vartheta}_m + \widehat{\varphi}_{m+1}^{(I)} \tag{23}$$

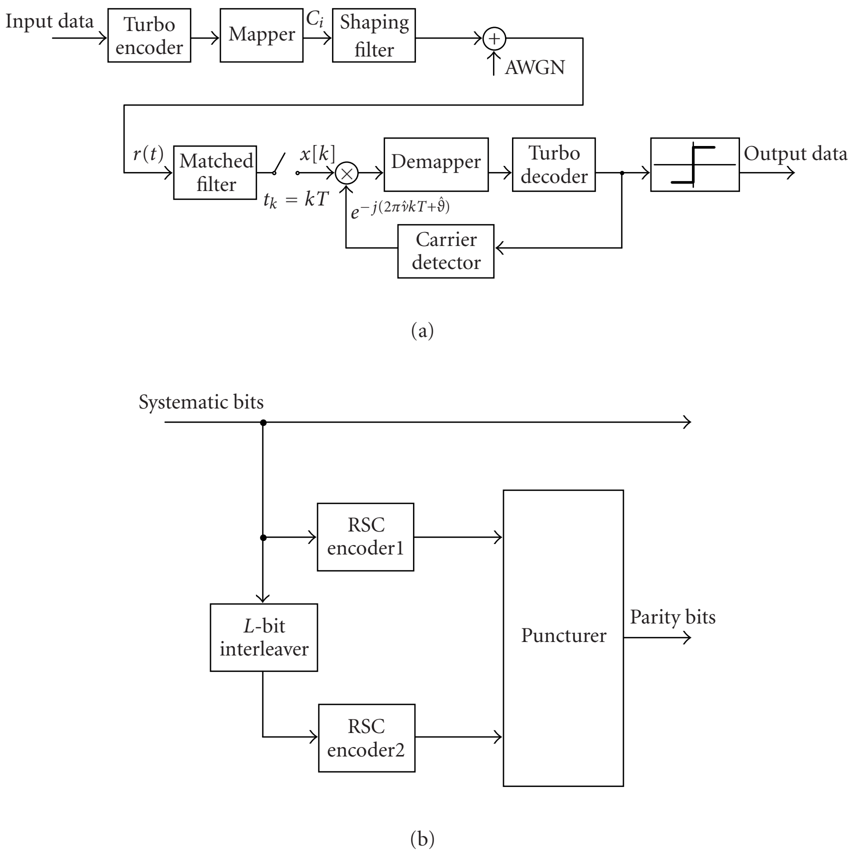

Figure 1: (a) Turbo-coded transmission system and (b) turbo-encoder schematic.

assuming as starting condition $\hat{\vartheta}_0 = \hat{\varphi}_0^{(I)}$. We found by simulation that to accomplish an adequate acquisition only 3 blocks are sufficient (i.e., just 3 updates on m in (23)). In doing so, the operating point of the phase error estimator is progressively brought back to the vicinity of the origin, that is, in a negligible-bias zone. Indeed, the results in Figure 3 obtained for $I = 10$ iterations show the improvement of the recursive algorithm with respect to the one based on a single block.

We now tackle the additional issue of carrier frequency recovery. We have to jointly solve (22a) (where the timing offset is considered perfectly known) and (22b). Figure 4 shows the MEV curves for the single-block estimation of the phase offset, for the true values ($\vartheta = 0°, 10°, 20°, 30°$), as a function of the true normalized frequency offset νT. Results are provided for $I = 12$ decoder iterations and $E_b/N_0 = 6$ dB. The joint estimator works fine up to $|\vartheta| \leq 20°$, but the operating interval for frequency recovery is quite narrow, that is, $|\nu T| < 10^{-4}$, if compared with a conventional data-aided method [31]. This can be easily explained if we consider the following fact. For a given block length, the residual frequency offset causes a phase rotation on the received signal samples leading to a considerable performance degradation for the constituent SISO decoders. Clearly, the larger the frequency offset, the larger will also be the phase rotation on the block samples. Consequently, there exists a threshold value for the frequency offset, such that the overall phase accumulated on a block will be around π, above which the reliability of the decoded bits, even after a few decoding iterations, will stay small. This hinders joint convergence of the (blind) frequency estimator and data decoder.

The time-recursive approach can be used to improve the performance of joint phase-frequency recovery as well. To be more specific, the frequency and phase estimates are used to precorrect the received signal samples in the subsequent block both in frequency and in phase prior to a new iterative estimation. Unfortunately, the improvement for frequency is not as dramatic as for phase estimation, as can be seen from Figure 5. The operating range for the carrier frequency estimator is now $|\nu T| < 3 \cdot 10^{-4}$ for $I = 12$ decoder iterations and $E_b/N_0 = 6$ dB. The conclusion is that some form of "frequency sweeping" is required in order to perform initial frequency acquisition when the offset is larger than the value above. Further enlargement of this range can be alternatively obtained by partitioning the code block into shorter estimation windows, over which we can apply (time-recursive) joint estimation. With shorter windows, a larger frequency operating range is obtained, but the phase estimation accuracy decreases, so that an optimum length will exist for a given E_b/N_0.

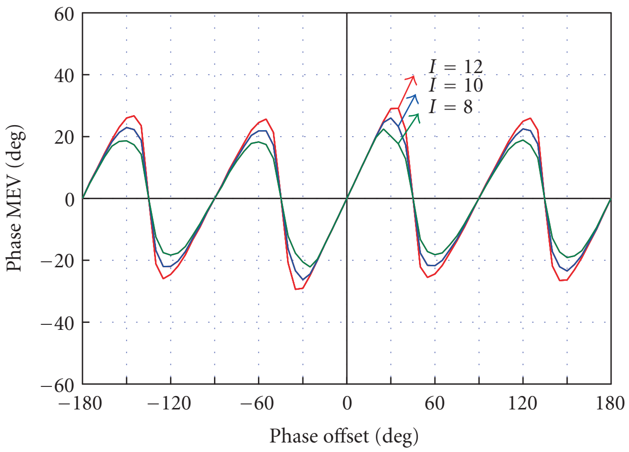

Figure 2: MEV curves for single-block phase SDD recovery with different iteration numbers, 16-QAM, $\lambda = 3/4$, $L = 1500$, $E_b/N_0 = 6$ dB.

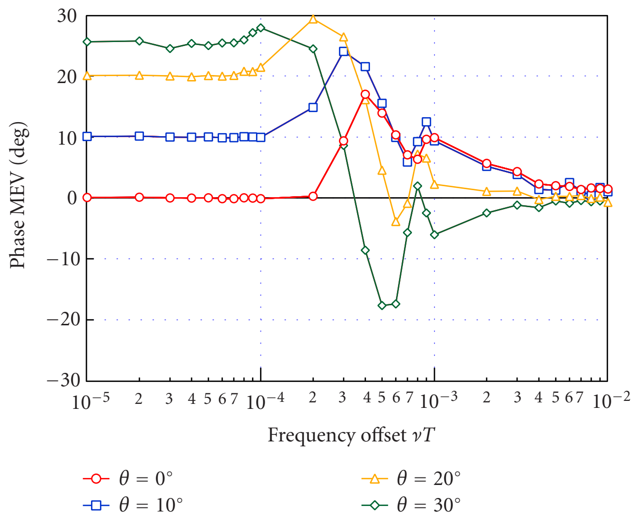

Figure 4: MEV phase curves for single-block joint SDD phase-frequency recovery with different true phase, 16-QAM, $\lambda = 3/4$, $L = 1500$, $E_b/N_0 = 6$ dB.

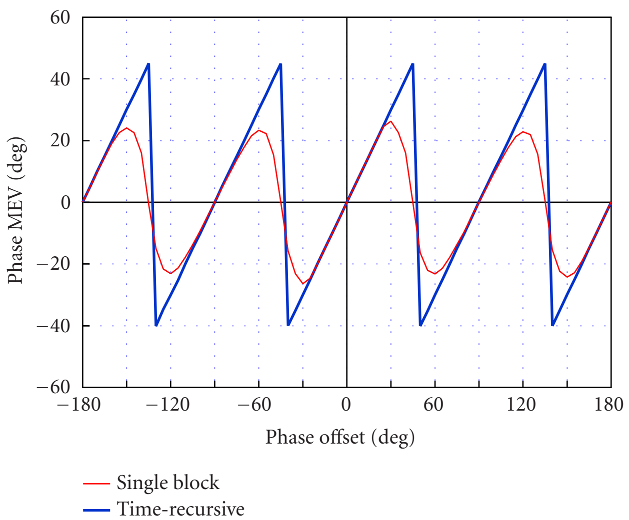

Figure 3: MEV curves for time-recursive and single-block SDD phase recovery, 16-QAM, $\lambda = 3/4$, $L = 1500$, $E_b/N_0 = 6$ dB.

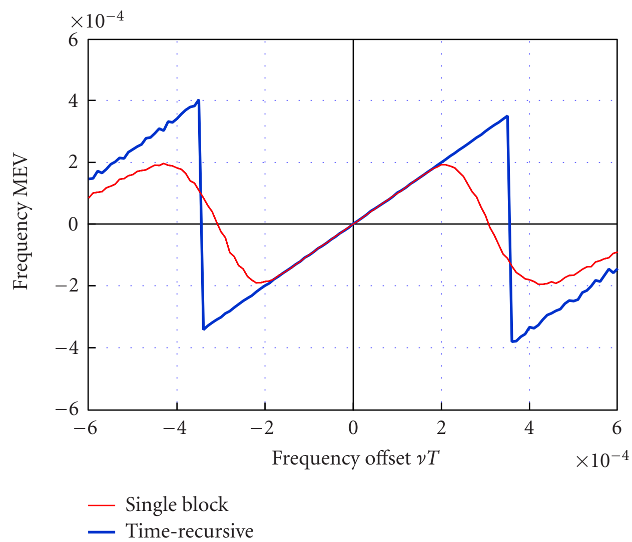

Figure 5: MEV curves for time-recursive and single-block joint SDD phase-frequency recovery, 16-QAM, $\lambda = 3/4$, $L = 1500$, $E_b/N_0 = 6$ dB.

4.2. RMSEE curves

Figure 6 shows the curves of RMSEE σ_θ (i.e., $\sqrt{E\{(\hat{\vartheta} - \vartheta)^2\}}$) of the phase SDD recovery algorithm as a function of E_b/N_0 for various values of the true offset ϑ. The curves are compared to the modified Cramér-Rao bound (MCRB) [31], and with ideal DA estimation that lies exactly on the MCRB. Conversely, the RMSEE performance of SDD gets approximately close to the bound for $E_b/N_0 \geq 6$ dB only, that is, in the interval where soft-data decisions are reliable enough (as will be illustrated in the sequel). It is also noted that the RMSEE curve for conventional hard-decision-directed (HDD) phase estimation, that is, based on the decisions taken at the decoder input, is catastrophic. This is easily explained by noting that the BER of hard-detected 16-QAM in our SNR range is definitely poor, leading to an inaccurate phase estimate. On the other side, a different solution is based on applying the proposed iterative estimation algorithm (22b) using the hard-detected QAM symbols taken from the decoder output at each decoding iteration. This kind of scheme can be referred to as iterative hard-decision-directed (IHDD). As illustrated in Figure 6, the performance degradation with respect to SDD of IHDD is small as long as the phase error is $|\vartheta| \leq 10°$, but gets more important for larger values of initial phase offset.

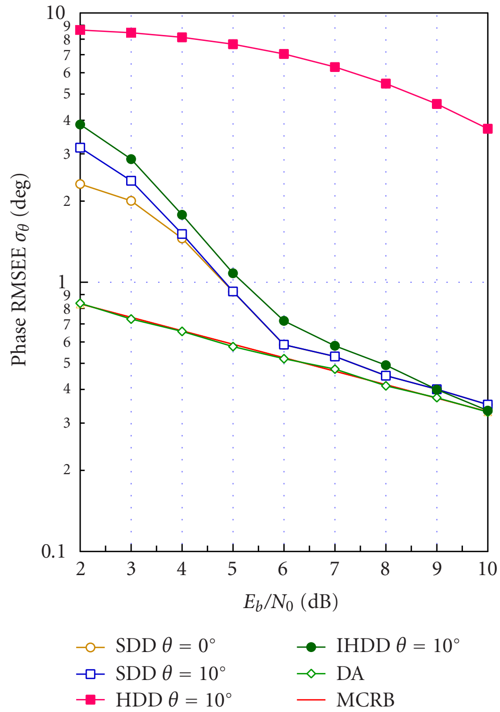

FIGURE 6: RMSEE curves for single-block SDD and IHDD, DA, HDD phase recovery, 16-QAM, $\lambda = 3/4$, $L = 1500$, $I = 12$.

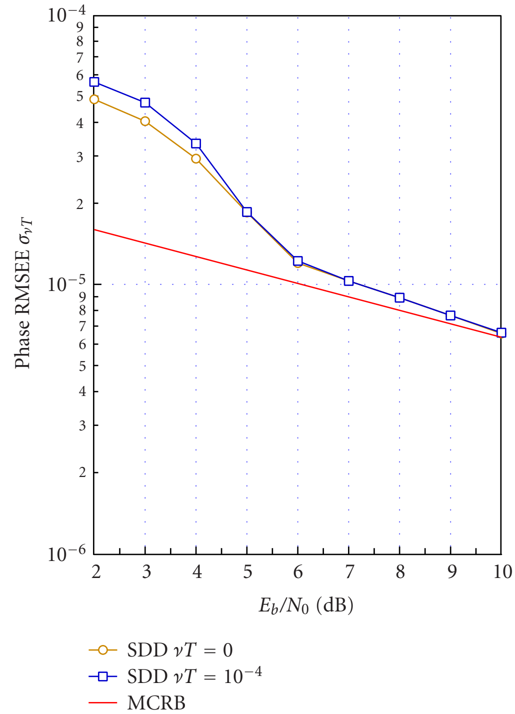

FIGURE 7: RMSEE curves for single-block joint SDD phase-frequency recovery, 16-QAM, $\lambda = 3/4$, $L = 1500$, $I = 12$.

The curves for the frequency RMSEE $\sigma_{\nu T}$ (i.e., $\sqrt{E\{(\hat{\nu} - \nu)^2 T^2\}}$) in Figure 7 follow the same general pattern as those for the phase. As noted for SDD phase recovery, the frequency MCRB bound [31] is attained for $E_b/N_0 \geq 6$ dB, and negligible performance degradation is observed both for the frequency offsets $\nu T = 0$ and $\nu T = 10^{-4}$.

4.3. BER performance

To get a picture about the overall performance of the 16-QAM turbo receiver equipped with the proposed SDD carrier synchronizer, the BER curves can be evaluated as a function of the signal-to-noise ratio E_b/N_0. For each curve all simulation runs were stopped upon the detection of 100 frame error events. Specifically, Figure 8 shows the BER curves with time-recursive SDD phase recovery and with $I = 10$ iterations. The curves with a phase offset $\vartheta = 20°, 40°$ exhibit a negligible performance degradation with respect to the one with ideal phase recovery. These curves motivate the departure of the RMSEE curves of SDD synchronization from the MCRB. The "knee point" of the RMSEE curves, which roughly corresponds to $E_b/N_0 = 6$ dB, is in fact located in the so-called "waterfall region" (abrupt BER decrease). The associated BER is then sufficiently decreased and the synchronization algorithm performance tends to that of a DA synchronizer. Further, for the sake of completeness, is worthy to point out that a similar behavior is found even with a lower rate, namely 1/2, encoder combined with a 4-QAM modulation format, as shown in the results presented in [11, 27].

The BER curves for joint SDD phase-frequency recovery are illustrated in Figures 9 and 10 in the case of single-block and time-recursive estimation, respectively. The main result which has to be pointed out is that the performance of single-block-based joint SDD phase-frequency recovery algorithm gets worse for increasing frequency offsets to be estimated, while the time-recursive approach enables to achieve turbo decoding with a negligible degradation with respect to ideal synchronization for a frequency offset up to about $\nu T = 3 \cdot 10^{-4}$. The increased robustness of the time-recursive version of the proposed synchronizer is coherent with what was already observed above in Section 4.1. Indeed, with the iteration of (23) the carrier offset estimation error is progressively reduced despite a nonnegligible initial value due to, for instance, the choice of employing a shorter preamble to achieve a better efficiency.

5. COMPUTATIONAL COMPLEXITY

In this section we focus on the computational complexity of the turbo receiver (whose performance has been evaluated in Section 4) performing soft-decision-based iterative carrier synchronization. In particular, we perform a comparison with the complexity of the turbo receiver with ideal synchronization.

The computational load of both the *iterative* SDD and the *ideal* receiver is dominated by matched filtering, turbo decoding, and carrier synchronization (for the latter only). Depending on the different arrangements for decoding/synchronization, the above functions contribute differently to the overall complexity. For simplicity, we assume that

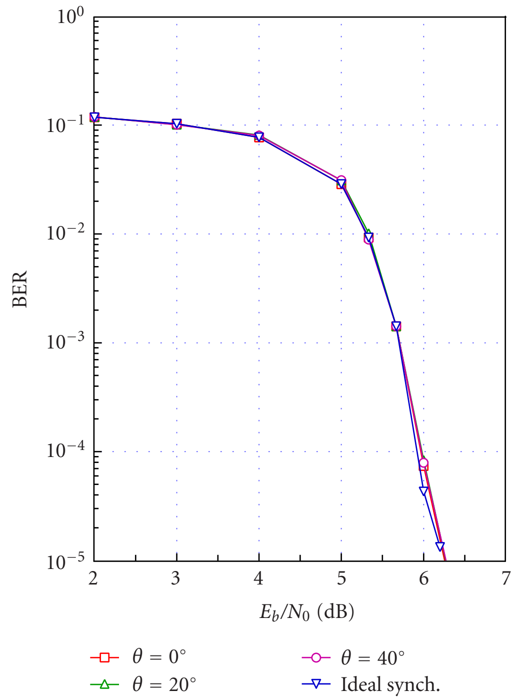

Figure 8: BER for time-recursive SDD phase recovery, 16-QAM, $\lambda = 3/4$, $L = 1500$, $I = 10$.

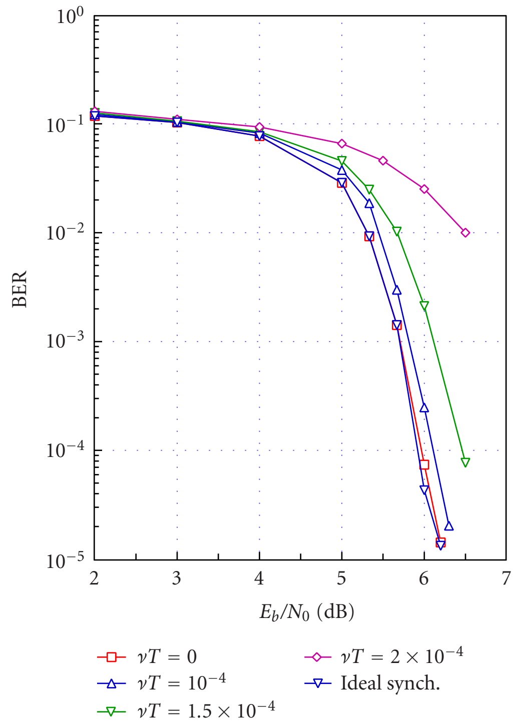

Figure 9: BER for single-block joint SDD phase-frequency recovery, 16-QAM, $\lambda = 3/4$, $L = 1500$, $I = 10$.

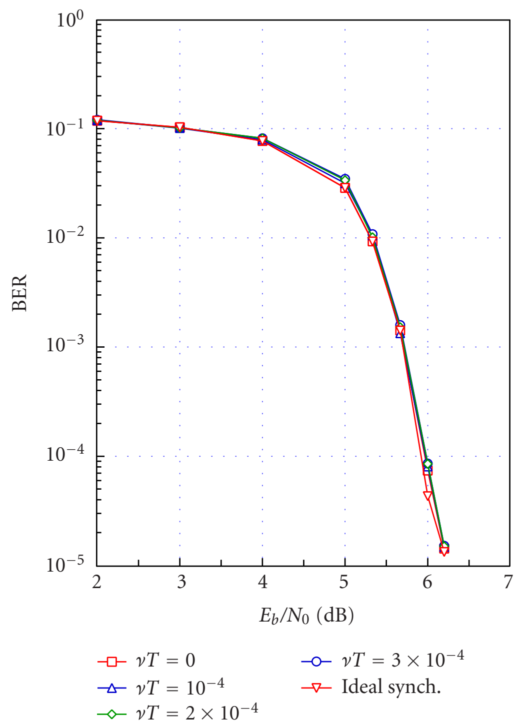

Figure 10: BER for time-recursive joint SDD phase-frequency recovery, 16-QAM, $\lambda = 3/4$, $L = 1500$, $I = 10$.

(as adopted in Section 4) the SDD receiver performs at each EM iteration only 1 detection/decoding iteration, whereas for both receivers the matched filtering is carried out only once before applying the decoding and synchronization procedures. Our complexity evaluation is performed on the basis of the number of required floating point (FP) operations, namely additions and multiplications, thereby leaving out (in a first approximation) operations such as comparisons and table lookups.

We denote with L the block length of information bits, with S the number of states of the rate $-1/2$ RSC component decoder and with N the number of transmitted 16-QAM symbols, respectively. The following basic operations have to be performed.

(OP_0) Matched filtering is based on an FIR filter with an operating frequency equal to $2/T$, where T is the signaling interval. Taking as impulse response a root cosine Nyquist function in the range $(-5T, 5T)$, which corresponds to 20 samples, the relevant computational complexity amounts to $C_0 \cong 80N$.

(OP_1) Each SISO constituent decoder accomplishes MAP decoding by evaluating the APPs for the systematic bits according to the BCJR algorithm [25]. Specifically, to limit the decoder complexity and avoid multiplications, we adopt the Max-Log-MAP approach illustrated in [30]. This involves the computation of the metrics (related to the states transitions) $\alpha_l(s)$, $\beta_l(s')$, and $\gamma_l(s, s')$ through forward and backward recursions, with s and s' enumerating the trellis states and $1 \le l \le L$. As for each decoding iteration two SISO decoders are employed, assuming a total of I iterations, the complexity of this operation is approximately $C_1 = 24S \cdot L \cdot I$.

(OP_2) In the iterative synchronizer, the APPs for the parity bits are required as well. This means that additional calculations have to be carried out within the two constituent SISO decoders, for a total additional load of roughly $C_2 = 12S \cdot L \cdot I$ operations.

(OP_3) The matched filter output samples after frequency and phase compensation are exploited to evaluate the initial metrics required by the turbo decoder. Denoting with y_k, $1 \leq k \leq N$, the sequence at the decoder input, the metrics for the four bits associated to the kth transmitted symbol are computed as follows [28]:

$$\begin{aligned} \Gamma_k^{(1)} &= \operatorname{Re}\{y_k\}, & \Gamma_k^{(2)} &= |\operatorname{Re}\{y_k\}| - 2, \\ \Gamma_k^{(3)} &= \operatorname{Im}\{y_k\}, & \Gamma_k^{(4)} &= |\operatorname{Im}\{y_k\}| - 2. \end{aligned} \tag{24}$$

According to (24), the relevant computational complexity can be regarded roughly negligible, that is, $C_3 = 2N \cdot I$.

(OP_4) The APPs of both the systematic bits and the parity bits provided by the turbo decoder are required in the computation of the a posteriori mean value of the transmitted symbols defined in (16a). After some algebra it can be proved that

$$\begin{aligned} &\eta_k(\mathbf{r}, \hat{\mathbf{b}}^{(n-1)}) \\ &\quad = \tanh\frac{L_n(a_1^{(k)})}{2}\left[2 + \tanh\frac{L_n(a_2^{(k)})}{2}\right] \\ &\qquad + j\tanh\frac{L_n(b_1^{(k)})}{2}\left[2 + \tanh\frac{L_n(b_2^{(k)})}{2}\right], \end{aligned} \tag{25}$$

where $[a_1^{(k)}, a_2^{(k)}, b_1^{(k)}, b_2^{(k)}]$ are the four coded bits associated with the kth transmitted symbol, and $L_n(a_1^{(k)})$, $L_n(a_2^{(k)})$, $L_n(b_1^{(k)})$, and $L_n(b_2^{(k)})$ are the corresponding APPs at the nth decoding iteration, with $1 \leq n \leq I$. Note that in (25) the evaluation of the hyperbolic tangent may be carried out via a proper lookup table, and consequently the complexity of this operation amounts to $C_4 = 4N \cdot I$.

(OP_5) Using the a posteriori averages $\eta_k(\mathbf{r}, \hat{\mathbf{b}}^{(n-1)})$, the frequency and phase estimates are updated according to (22). The maximization can be carried out through an exhaustive grid search on a small number N_g of trial values (since the operating range of the estimator is narrow), so that the corresponding complexity equals $C_5 = 6N \cdot I \cdot N_g$.

The complexity concerning (OP_0), (OP_3), (OP_4), and (OP_5) is proportional to the block length N, but not to the number of states S. This is the reason why the overall complexity is dominated by (OP_1) and (OP_2). To sum up, the complexity of the SDD receiver is given by

$$C_{\text{SDD}} = \sum_{i=0}^{5} C_i \cong 80N + 36S \cdot L \cdot I + 6N \cdot I + 6N \cdot I \cdot N_g. \tag{26}$$

When synchronization is known a priori, (OP_2), (OP_4), and (OP_5) do not apply since they are needed for SDD synchronization. The metrics calculation (OP_3) is carried out only once, and the matched filtering (OP_0) is clearly the same. Therefore, the overall complexity of the ideal receiver is

$$C_{\text{ideal}} = \sum_{i=0}^{5} C_i = 80N + 24S \cdot L \cdot I + 2N \cdot I, \tag{27}$$

and the additional complexity introduced by the SDD iterative receiver is

$$\begin{aligned} \Delta C &= \frac{C_{\text{SDD}} - C_{\text{ideal}}}{C_{\text{ideal}}} \cdot 100 \\ &\approx \frac{12S \cdot L \cdot I + 4N \cdot I + 6N \cdot I \cdot N_g}{80N + 24S \cdot L \cdot I + 2N \cdot I} \cdot 100 \approx 50\%. \end{aligned} \tag{28}$$

Summing up, the complexity of the receiver with iterative synchronization is greater, namely around 50%,than that of the receiver with ideal synchronization, because the additional complexity is due mainly to the evaluation of the APPs of the parity bits (OP_2). This price to be paid can be avoided whether one accepts to evaluate them only once at the decoder input. This approximate solution entails a negligible performance degradation in the case of high code rate, that is, when the parity bits are substantially less in number than the information bits, as is shown by the performance results illustrated in Section 4. However, it has to be remarked that for other channel coding schemes also suited to iterative decoding, such as SCCC (serially-concatenated convolutional codes) and LDPC (low-density parity check), at each iteration APPs are available for both systematic and parity bits in a code block, and consequently the incremental complexity due to synchronization is relatively smaller than the complexity of the whole iterative decoder.

6. CONCLUSIONS

The main conclusion of the paper is that ad hoc iterative schemes adopted in the context of joint synchronization and decoding can be justified in a theoretical framework based on the well-known EM algorithm. The resulting estimation procedure can also be easily interpreted as a form of *iterative soft-decision-directed* synchronization, as opposed to conventional *hard-decision-directed* estimation that fails in a condition of low signal-to-noise ratio. Iterative synchronization comes natural in the context of decoding of channels codes suited to iterative detection, such as turbo codes with parallel and serial concatenation, and LDPC codes. This fully justifies the formulation of the so-called *turbo synchronization concept*, that is, soft-decision-directed synchronization and parameter estimation within a turbo (iterative) receiver. As a case study, we demonstrated the application of the proposed mathematical formulation to the particular case of joint carrier phase and frequency offsets estimation in a turbo-coded 16-QAM system. We showed negligible performance degradation with respect to the ideal coherent system down to low signal-to-noise ratios.

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