The Effect of Narrowband Interference on ML Fractional Frequency Offset Estimator for OFDM

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Abstract - In orthogonal frequency division multiplexing (OFDM) systems affected by carrier frequency offsets, the estimation of the frequency offset corresponding to a fractional part of the carrier spacing is a crucial issue. The proper action of the fractional frequency estimator can be strongly affected by the presence of disturbances, like narrowband interference (NBI) signals. In this paper, we derive the data-aided maximum-likelihood (ML) fractional frequency estimator in the presence of (NBI). Based on the ML algorithm which has a high complexity, we propose a number of simplifications to develop a lower complexity algorithm. The susceptibility of the simplified fractional frequency estimator to NBI signals is investigated in an analytical way. The analytical results are verified by means of simulations. Although the simplified estimator turns out to be essentially independent of the bandwidth and the location of interferers, the performance of the estimator is very sensitive to the signal to interference ratio and the number of interferers. In contrast with the simplified estimator, simulation results indicate that the exact ML estimator is essentially independent of the (NBI) signals.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has been of major interest for both wire line and wireless applications due its high data rate transmission capability and its robustness to multipath delay spread. The ease of implementation and the fine granularity that OFDM provides have led to its acceptance in many standards such as Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), IEEE 802.11a, IEEE 802.16a, and Multimedia Access Communications (MMAC) [1]–[3]. However, OFDM is not without drawbacks. One important drawback in OFDM is its sensitivity to carrier frequency offset (CFO) arising from Doppler shifts and/or oscillator instabilities [4]. A CFO results in a shift of the received signal spectrum in the frequency domain. The CFO can be divided into an integer and a fractional part with respect to the OFDM sub-carrier spacing δf. An integer part of the CFO yields a different bit error rate from zero to 0.5 [5]. The fractional CFO leads to a reduction of the signal amplitude and to a loss of sub-carrier orthogonality. This loss introduces inter-carrier interference (ICI) which results in a degradation of the global system performance. To maintain a signal to inter-carrier interference ratio of 20 dB or more, the fractional CFO is limited to 4% or less of the sub-carrier spacing [6].

In some applications, the OFDM system must coexist with narrow band interference signals. To illustrate, broad-band OFDM system is now applied in some unlicensed frequency band, e.g. Industrial-Scientific-Medical (ISM) band in 2.4 GHz and Unlicensed National Information Infrastructure band (UNII) in 5.2 GHz [7], etc. However, since the frequency bands are unlicensed, there may exist many different wireless systems sharing the same frequency band. If a part of the spectrum of an OFDM system is overlapped by a relatively narrow band transmission signal, the latter will introduce narrow band interference to the OFDM system. Due to the spectral leakage effect caused by the DFT demodulation at OFDM receiver, many subcarriers near the interference frequency will suffer serious interference. Moreover, the broadband very high frequency (B-VHF) project [8], [9], which aims to develop a new integrated broadband VHF system for aeronautical voice and data link communications based on multi-carrier technology, is a good example of an overlay system. In this project, the multi-carrier (MC) system is intended to share the parts of the VHF spectrum with are currently used by narrowband (NB) systems such as voice DSB-AM signal and VHF digital links. These NB systems are considered as interference to the MC system which hampers coverage and capacity, and limits the effectiveness of the multi-carrier system.

The presence of the NBI signals can hamper the proper action of the synchronization algorithms used to synchronize the OFDM system. For example, in [10], [11], we have investigated the effect of NBI signals on timing and integer CFO synchronization of the OFDM system. The result of indicates that the performance of the estimators are strongly affected by the NBI.

In this paper, the effect of narrowband interference on the performance of data aided maximum likehood fractional CFO estimator for OFDM system is investigated. The paper is organized as follows. The system model is addressed in section 2. In section 3, we derive the ML fractional CFO estimator for OFDM system in the presence of NBI. Also, the susceptibility of the simplified ML fractional frequency estimator is investigated in an analytical way. The simulation and analytical results are discussed in section 4. Finally, conclusions are given in section 5.

II. System Description

The model of the OFDM system including narrowband interference signals (NBI) is depicted in fig. 1. In the OFDM

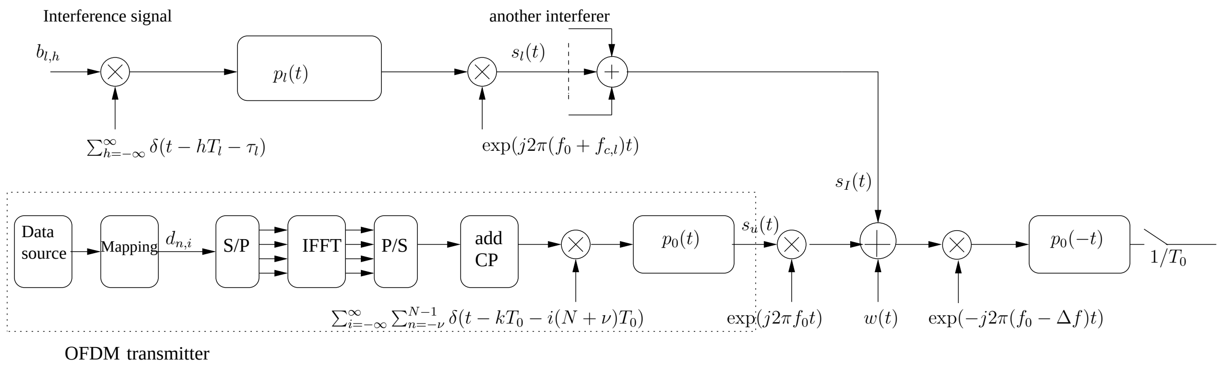

Fig. 1. block diagram of OFDM system including interfering signals

transmitter, frames consisting of M_s OFDM symbols are transmitted. Each frame is generated as follows (see fig. 1) : a block $\underline{\mathbf{z_i}}= [z_{0,i}, \ldots, z_{N_z-1,i}]^T$ of N_z information-bits is mapped to a sequence of N_d symbols, belonging to a unit-energy 2^q-point constellation Ω (with $N_d = N_z/q$). We will denote this sequence $\underline{\mathbf{d}}_i= [d_{0,i}, \ldots, d_{N_d-1,i}]^T$. Now, $\underline{\mathbf{d}}_i$ is broken down into M_s blocks of length N_u. The M_s blocks are buffered and converted, one at a time, to OFDM symbols by using N-point inverse fast Fourier transform (IFFT). A ν-point cyclic prefix (CP) is pre-appended, resulting in $N+\nu$ time-domain samples $[s_{-v,i}, \ldots, s_{-1,i}, s_{0,i}, \ldots, s_{N-1,i}]^T$ where $s_{l,i} = s_{l+N,i}$ for $l = -v, \ldots, -1$ and $i = 0, \ldots, M_s - 1$. The kth time domain sample of the ith OFDM data block can be written as

$$s_{k,i}^D = \sqrt{\frac{1}{N+\nu}} \sum_{n\in I_u} d_{n,i}\, e^{\frac{j2\pi kn}{N}} \quad -\nu \leq k \leq N-1 \qquad (1)$$

where I_u is a set of N_u carrier indices and $d_{n,i}$ is the nth data symbol of the ith OFDM block; the data symbols are i.i.d[1] random values with zero mean and variance $E\left[|d_{n,i}|^2\right] = E_s$.

Each data frame is preceded by one OFDM pilot block, to be able to synchronize the receiver. The pilot symbol has two identical halves in time domain, which will remain identical after passing though the channel, except that there will be a phase difference between them caused by the carrier frequency offset. The symmetry of the pilot symbol in the time domain is easily generated by transmitting a pseudonoise (PN) sequence on the even frequencies, while zeros are used on the odd frequencies. In order to maintain an approximately constant energy for each OFDM symbol, the frequency components of this training symbol are multiplied by $\sqrt{2}$ at the transmitter.

Finally, the time domain signal of the baseband OFDM signal $s_u(t)$ consists of the concatenation of all time domain data and pilot blocks :

$$s_u(t) = \sum_{k=-\nu}^{N-1} s_{k,i}^P\, p_0(t-kT_0) + \sum_{i=0}^{M_s-1}\sum_{k=-\nu}^{N-1} s_{k,i}^D\, p_0(t-kT_0-(i+1)(N+\nu)T_0) \qquad (2)$$

where $p_0(t)$ is the unit-energy transmit pulse of the OFDM system and $1/T_0$ is the sample rate. The baseband signal (2) is up-converted to the radio frequency f_0. At the receiver, the signal is first down-converted to $-(f_0 - \Delta f)$, where Δf represents the frequency difference between transmitter and receiver oscillator, then fed to the matched filter and finally sampled at rate $1/T_0$. The OFDM signal is disturbed by additive white Gaussian noise with uncorrelated real and imaginary parts, each having variance σ_n^2. The signal to noise ratio (SNR) at the output of the matched filter is defined as $\frac{\sigma_s^2}{\sigma_n^2}$, where σ_s^2 is the variance of the real/imaginary part of $s_{k,i}^D$.

Further, the signal is disturbed by narrowband interference residing within the same frequency band as the wideband OFDM signal as shown in Fig. 2. The interfering signal $s_I(t)$ may be described as

$$s_I(t) = \sum_{l=1}^{N_I} s_l(t) e^{j2\pi(f_0+f_{c,l})t} \qquad (3)$$

where $s_l(t)$ is a baseband narrowband signal and $f_{c,l}$ is the carrier frequency deviation from f_0 for the lth interferer. The baseband interference $s_l(t)$ is modeled as digitally modulated signal

$$\begin{aligned} s_l(t) &= \sum_{h=-\infty}^{\infty} b_{h,l} p_l(t-hT_l-\tau_l) \qquad (4) \\ &\simeq \sum_{h=0}^{q} b_{h,l} p_l(t-hT_l-\tau_l) \end{aligned}$$

where $p_l(t)$ is the time domain response of the transmit filter of the lth interferer, $b_{h,l}$ is the hth interfering data symbol of the lth interferer, τ_l is its delay, and $1/T_l$ its sample rate. Because $p_l(t)$ degrades rapidly in the time axis, symbol $\{b_{h,l}\ \forall h < 0\ or\ h > q\}$ have little effect on the signal $s_l(t)$ $(0 \leq t \leq T_{FFT})$. Therefore, the approximation in (4) is valid. Let B_l be the bandwidth of $p_l(t)$ and B_0 is the bandwidth of OFDM signal. In an OFDM symbol duration T_{FFT}, there are q symbols of $s_l(t)$, where q is an integer equal to or less $\frac{B_l}{B_0}$. The total NBI signal may be seen at the output of the matched filter of the OFDM receiver as

$$r_I(t) \simeq \sum_{l=1}^{N_I}\sum_{h=0}^{q} b_{h,l}\, e^{j2\pi f_{c,l} hT_l}\, g_l(t-hT_l) \qquad (5)$$

where $g_l(t)$ is the convolution of $p_0(-t)$ and $p_l(t-\tau_l)\exp\left(j2\pi\left(f_{c,l}+\Delta f\right)t\right)$. The normalized location of the

[1] i.i.d = independently and identically distributed

interferer within the OFDM spectrum may be defined as $f'_{c,l} = \left(\frac{f_{c,l}+\Delta f}{B_0}\right)$. It is assumed that the interfering symbols are uncorrelated with each other, i.e. $E[b_{h,l}b^*_{h',l'}] = E'_l \delta_{ll'}\delta_{hh'}$, where E'_l is the energy per symbol of the lth interferer. Further, the interfering data symbols $b_{h,l}$ are statistically independent of the OFDM data symbols $a_{n,i}$. The signal to interference ratio (SIR) at the input of the receiver is defined as [10], [11]

$$SIR = \frac{2\sigma_s^2/T_0}{\sum_{l=1}^{N_I} \frac{E'_l}{T_l}} \quad (6)$$

III. ML Fractional CFO Estimator

As the frequency offset Δf is generally larger than the sub-carrier spacing, it is useful to split it into an integer part m and fractional part ϵ where $\epsilon \in [-.5, .5[$ with respect to the carrier spacing $\delta f = \frac{1}{NT_0}$, i.e. $\Delta f = \frac{m}{NT_0} + \frac{\epsilon}{NT_o}$. We assume that the parameter m has already been estimated and is corrected. The received time domain samples outside the CP are given by

$$y(k) = e^{j(2\pi\epsilon k/N)}x(k)+w(k)+r_I(k) \quad \forall k = 0, ..., N-1 \quad (7)$$

where $x(k)$ is the training signal contribution from the channel, $w(k) \sim N(0, 2\sigma_n^2)$ [2] and $r_I(k)$ is the interference contribution at sample time kT_0 as given in (5). We introduce two vectors of length $N/2$: $\underline{\mathbf{Y_1}} = [y(0), ..., y(N/2-1)]^T$ consists of the first half of the pilot symbol. Similarly, $\underline{\mathbf{Y_2}} = [y(N/2), ..., y(N-1)]^T$ consists of the second half of the pilot symbol. Taking into account that $x(k) = x(k + N/2)$, we find

$$\begin{aligned} Y_2(k) = \;& Y_1(k)\, e^{j\pi\epsilon} + w\,(k + N/2) - w(k)e^{j\pi\epsilon} \\ & + r_I\,(k + N/2) - r_I\,(k)\, e^{j\pi\epsilon} \end{aligned} \quad (8)$$

where $k = 0, ..., N/2-1$. Assuming that noise and interference components are Gaussian random variables, distribution of $\underline{\mathbf{y_2}}$ given $\underline{\mathbf{y_1}}$ and ϵ yields

$$\begin{aligned} P\left(\underline{Y_2}\,|\underline{Y_1}, \tilde{\epsilon}\right) = \;& \frac{1}{\pi^{N/2}|C_x|} \cdot \\ & exp\left\{-\left(\underline{Y_2} - \underline{Y_1}e^{j\pi\tilde{\epsilon}}\right)^H C_x^{-1}\left(\underline{Y_2} - \underline{Y_1}e^{j\pi\tilde{\epsilon}}\right)\right\} \end{aligned} \quad (9)$$

where $\tilde{\epsilon}$ is the trial value of ϵ and C_x is the covariance matrix given by

$$\begin{aligned} [C_x]_{k,k'} &= E\left[\left(Y_2(k) - Y_1(k)\, e^{j\pi\epsilon}\right)\cdot\left(Y_2(k') - Y_1(k')\, e^{j\pi\epsilon}\right)^*\right] \\ &= 4\sigma_n^2\delta_{kk'} + A(k,\, k') + A(k + N/2,\, k' + N/2) \quad (10) \\ &= -\left\{A(k',\, k + N/2) + A^*(k,\, k' + N/2)\right\} e^{j\pi\epsilon} \end{aligned}$$

where $E[x]$ is the expectation of x, and $A(k,\, k')$ is defined as

$$A(k,\, k') = \sum_{l=1}^{N_I} E_l \sum_{h=-\infty}^{\infty} g_l(kT_o - hT_l)g_l^*(k'T_o - hT_l) \quad (11)$$

[2] $w(k) \sim N(0,\, 2\sigma_n^2)$ means that $w(k)$ is Gaussian distributed with average 0 and variance $2\sigma_n^2$.

Dropping irrelevant terms from (9), we get the likelihood function $\Lambda(\tilde{\epsilon})$:

$$\Lambda(\tilde{\epsilon}) = Re\left\{Y_1^H C_x^{-1} Y_2 e^{-j\pi\tilde{\epsilon}}\right\} \quad (12)$$

The estimated value $\hat{\epsilon}$ of ϵ maximizes $\Lambda(\tilde{\epsilon})$. Taking the derivative of (12) with respect to $\tilde{\epsilon}$ and setting the result to zero yields

$$\hat{\epsilon} = \frac{1}{\pi} tan^{-1} \frac{Im\left\{Y_1^H C_x^{-1} Y_2\right\}}{Re\left\{Y_1^H C_x^{-1} Y_2\right\}} \quad (13)$$

This estimator (13) requires the knowledge about the second order statistics of (NBI) to compute the matrix C_x. Further, it requires the inversion of C_x. Therefore, the complexity of this estimator might be to high. To simplify the estimator (13), we take a closer look to the matrix C_x. The function $A(k, k')$ represents the sum of the product of the interference transmit pulse with its delayed version over an integer number of sample intervals T_0. In fig. 3, C_x matrix is shown as an image for $N = 1024$, one interferer with $f'_{c,1} = 0$, $E_1 = 1$, normalized interference bandwidth $NBW = 0.024$, and $SNR = \infty$. Lighter color corresponds to larger values of C_x elements and black color corresponds to small values (nearly zero) of C_x elements. It is clear from the figure that the larger C_x elements occur at diagonals around main diagonal. Hence, the elements of C_x (10) that are not close to the main diagonal will be much smaller than the diagonal elements and the further the element is located from the main diagonal, the smaller the element will be. Therefore, we suggest two types of approximations to simplify the inversion of the matrix C_x to simplify the implementation of exact ML estimator (13). In the first approximation, we can consider most of the C_x elements are zeros except the sub-diagonal elements around the main diagonal. In the second approximation, we neglect the interference components in the covariance matrix. In this case, C_x becomes diagonal matrix with diagonal value $4 * \sigma_n^2$. Therefore, the estimator (13) can be simplified as

$$\hat{\epsilon} = \frac{1}{\pi} tan^{-1} \frac{Im\left\{\sum_{k=0}^{N/2-1} Y_1^*(k)Y_2(k)\right\}}{Re\left\{\sum_{k=0}^{N/2-1} Y_1^*(k)Y_2(k)\right\}} \quad (14)$$

It is worth to state that the estimator (14) looks like the ML fractional frequency estimator derived by [6]. However, the ML estimator in [6] is based on two pilot symbols, while the ML estimator given (14) is based on only one pilot OFDM symbol.

Implementation Aspects:

- From fig. 3, we notice that $[C_x]_{k,k'}$ sharply drops when $k \neq k'$. Therefore, $A(k,\, k') + A(k + N/2,\, k' + N/2)$ is larger than $\{A(k',\, k + N/2) + A^*(k,\, k' + N/2)\}\, e^{j\pi\epsilon}$. Then, we can say that $[C_x]_{k,k'}$ is essentially independent of the fractional frequency offset ϵ, see (10). Therefore, if the statistics of the narrow band interference signals is know, the matrix C_x^{-1} can be precomputed and stored at the receiver. Accordingly, we do not need to reestimate C_x^{-1} every frame.
- The inversion of the C_x matrix can be simplified by neglecting all C_x elements except the $2M$ sub-diagonal

elements around the main diagonal, i.e. the elements $[C_x]_{k,k'} \neq 0$ if $k' = k \pm m$, $m = 0, ..., M$ and $[C_x]_{k,k'} = 0$ otherwise.
- If the statistical of narrow band interference signals is not known, we can use estimator (14).

Now, we evaluate analytically the effect of NBI on the simplified estimator (14). Due to the presence of the inverse of the tangent function, it is difficult to compute the mean and the variance of $\hat{\epsilon}$ directly. To simplify, we assume that $|\hat{\epsilon} - \epsilon| \ll 1/\pi$ as in [6]. Therefore, the tangent can be approximated by its argument. Further, consider the complex products $Y_2(k)Y_1^*(k)$ from which we estimate ϵ from (14). For a given ϵ, subtract the corresponding phase, $2\pi\epsilon$, from each product to obtain the tangent of the phase error

$$tan\left[\pi\left(\hat{\epsilon} - \epsilon\right)\right] \simeq \hat{\epsilon} - \epsilon = \frac{Im\left\{Y_1^H Y_2 e^{-j\pi\epsilon}\right\}}{Re\left\{Y_1^H Y_2 e^{-j\pi\epsilon}\right\}} \qquad (15)$$

At high SNR and SIR values, $Re\left[Y_2(k)Y_1^*(k)\, e^{-j\pi\epsilon}\right]$ can be approximated by $|x(k)|^2$ as the noise and interference components are much smaller than the signal component. Also, it is easily to prove that $E\left[Im\left\{Y_1^H Y_2 e^{-j\pi\epsilon}\right\}\right] = 0$. Then, $E\left[\hat{\epsilon}\right] = \epsilon$,i.e. the estimator is unbiased. The variance of the estimate is easily determined from (15) as

$$E\left[\left(\hat{\epsilon} - \epsilon\right)^2\right] = \frac{4}{\pi^2 N^2}\left(\frac{N}{2}\left(2\sigma_s^2\sigma_n^2 + \sigma_n^4\right) + \left(\sigma_s^2 + \sigma_n^2\right)\Upsilon + \Gamma\right) \qquad (16)$$

where Υ and Γ are given as

$$\Upsilon = \sum_{k=0}^{N/2-1} \left\{A(k,\, k) + A(k + N/2,\, k + N/2)\right\} \qquad (17)$$

$$\Gamma = \sum_{k,k'=0}^{N/2-1} A(k,\, k)A(k' + N/2,\, k' + N/2) \qquad (18)$$

In the next section, we evaluate the performance of the exact (13) and simplified ML estimator (14) by means of simulations.

IV. Numerical Results

The numerical results in this paper are obtained with the following OFDM and interference parameters:

- Transmit filters are square-root raised-cosine filters with roll off factors $\alpha_0 = 0.25$ and $\alpha_l = 0.5$ for OFDM and interfering signals, respectively.
- The total number of subcarriers is N = 1024.
- The number of active subcarriers is N_u = 1000.
- The guard interval is set to about 10 % of the useful part, $\nu = 102$.
- The bandwidth of the OFDM spectrum equals $B_0 = \frac{1}{T_0} = 1024$ kHz.
- We use QPSK modulation for the data symbols of the OFDM and the interferer signals.
- The time delay of the interferers equals $\tau_l = 0$.

Fig. 4 compares the analytical results (16) with the simulation of the variance of the simplified ML estimator (14), as function of the SIR. As can be observed, for a large range of SIR, the theoretical results and the simulation results agree well. At large SIR, the variance of the simplified ML estimator reaches an asymptote, corresponding to the case where no interference is present. This asymptote only depends on the SNR and is given by $\frac{2}{\pi^2 N\, SNR}$. At low SIR, the variance of the simplified estimator becomes independent of the SNR. The theoretical results are smaller than for the simulations when the SIR is small. Hence, the theoretical results can be considered as a lower bound on the performance for small SIR. Also, we note that the variance of the simplified ML estimator decreases for increasing SIR. Further, the variance at large SIR decreases with increasing SNR. This can easily be explained as at high SIR and SNR, the effect of the noise and interference is small.

Furthermore, the simulated variance of the exact ML estimator (13) is indicated in Fig. 4 as function of SIR at different values of SNR, curves (1, 2 , 3), where we assume that the characteristics of NBI signals, i.e SIR, BW, $f'_{c,l}$..., etc, are perfectly known. We note that the variance of exact ML estimator is independent of SIR. This is because the covariance matrix C_x is recalculated at each value of SIR. Also, we consider another case where there is an uncertainty in the estimation of the NBI parameters. Let us assume that the covariance matrix C_x is calculated at $f'_{c,l} = 0$, while the actual value of $f'_{c,l}$ is $.01B_1$. In this case, we observe that the simulated variance of the exact ML estimator is changed with SIR values as shown in curves (5, 6, 7). Hence, the exact estimator is very sensitive to uncertainties in the estimation of the NBI parameters.

Taking into account that most elements of the C_x matrix are zeros except the $2M$ sub-diagonals elements around main diagonal, fig. 5 shows the simulated variance of the exact ML estimators whose C_x have different number of sub-diagonals as function of SIR. It is clear from the figure that the exact ML estimator whose C_x is approximated as a diagonal matrix gives worse perfomance than the simplified estimator where the interference signal is neglected completely in C_x (14). As expected, when we increase the number of non-zero sub-diagonals in the matrix C_x, the performance improves. Moreover, depending on the operating SIR value, we can determine the necessary number of sub-diagonals. For example, at $SIR = 15$ dB, 200 sub-diagonals around main diagonal is acceptable.

Fig. 6 displays the analytical and simulation results for the variance of the simplified ML estimator as function of the interference bandwidth for different values of SIR. The results indicate that the dependency of the variance on the interference bandwidth increases when SIR decreases although the dependency is small. This is explained as at high SIR, the effect of interference diminishes and the variance mainly depends on noise. Furthermore, the variance of the exact ML estimator is indicated as function of NBW at different SIR. Results indicate that the variance of the exact ML estimator does not depend on NBI as we stated in the above section.

Fig. 7 illustrates the analytical and simulation results of the variance of the simplified ML estimator as function of the normalized interference carrier frequency deviation $f'_{c,l}$.

The shape of the variance may be explained with the aid of Fig. 2. As long as the interfering signal is located in the flat region of the OFDM spectrum, i.e $\left|f'_{c,l} + \frac{(1+\alpha_l)}{(1+\alpha_0)}\frac{T_0}{T_l}\right| < \frac{1-\alpha_0}{1+\alpha_0}$, the variance of the simplified ML estimator is constant and does not depend on the location of the interferer. When the interfering signal located outside OFDM bandwidth i.e. ($f'_{c,l} + \frac{(1+\alpha_l)}{(1+\alpha_0)}\frac{T_0}{T_l} > 1$ or $f'_{c,l} - \frac{(1+\alpha_l)}{(1+\alpha_0)}\frac{T_0}{T_l} < -1$), the interfering signal does not affect the OFDM signal anymore: in this region the variance is constant and its value depends on the SNR value only. Furthermore, the variance of the exact ML estimator is indicated as function of $f'_{c,l}$ at different SIR. Results indicate that the variance of the exact ML estimator does not depend on NBI as we stated above.

Fig 8. shows the variance of the estimator the simplified ML estimator as function of the number of interfering signals, N_I, in two cases. In case 'A', we consider that the SIR is fixed per interferer, so the total SIR decreases inversely proportional to N_I. In case 'B', we consider a fixed total SIR, i.e. the SIR per interferer decreases linearly as N_I increases. As the variance mainly depends on the total SIR, it follows that the variance will be an increasing function of the number N_I of interferers in case 'A', whereas it is independent of N_I in case 'B'. Furthermore, the variance of the exact ML estimator is indicated as function of N_I at different SIR. Results indicate that the variance of the exact ML estimator does not depend on NBI as we stated in above.

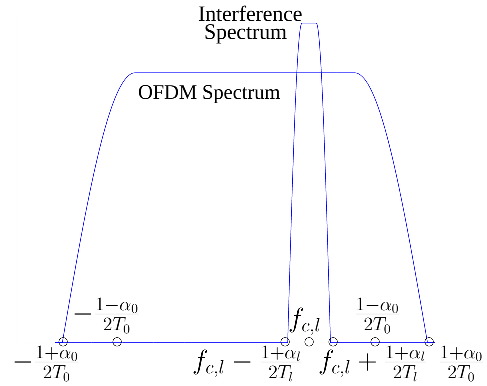

Fig. 2. Baseband OFDM and one interfering signal spectrum

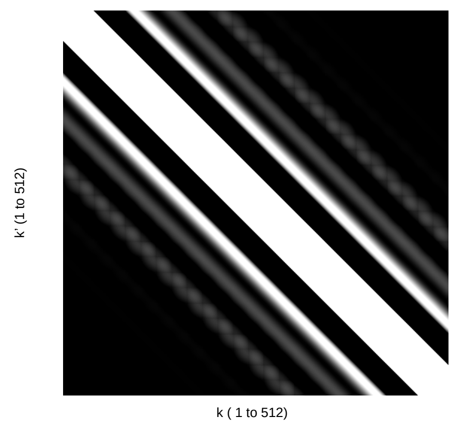

Fig. 3. A(k,k') at $N_I = 1$, $f'_{c,1} = 0$, $NBW = .0244$, and $E_1 = 1$

V. Conclusions

This paper evaluates the performance of ML fractional frequency estimator for OFDM system in the presence of narrowband interference for different interference parameters. We investigate the ML estimator based on a training symbol with two identical halves in time domain based on narrowband interfering (NBI) signal characteristic. The simplified estimator is also derived. Further, the statistical properties of the simplified estimator are evaluated analytically and by means of simulation in the presence of (NBI). The agreement between the theoretical and simulation results proves the validity of our analysis. Generally, the bandwidth of the interference and location of interferers do not have a large influence on the performance of the simplified estimator. However, it turns out that the signal to interference ratio has a large influence on the performance of the simplified estimator. However, results indicate that the performance of the exact estimator does not depend on the characteristics of (NBI).

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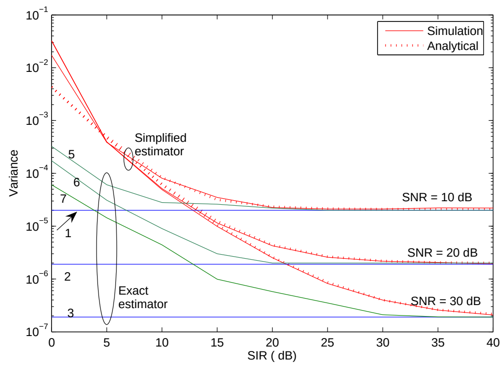

Fig. 4. Variance of the estimator versus signal to interference ratio (SIR) , NBW= .0244, $f^{'}_{c,1} = 0$, and N_I=1

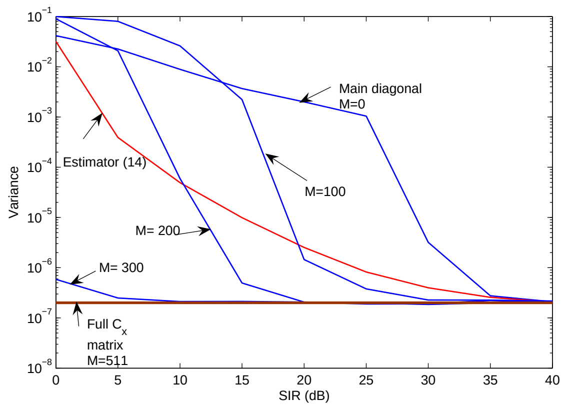

Fig. 5. Variance of exact ML estimator as function SIR at different number of sub-diagonal M ($N_I = 1$, $f'_{c,1} = 0$, $NBW = .0244$, and SNR = 30 dB)

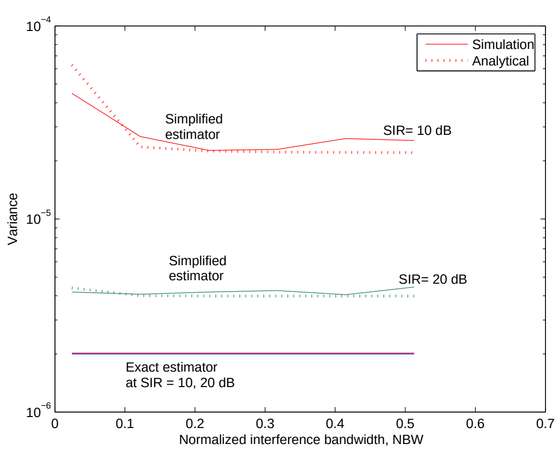

Fig. 6. variance versus normalized interference bandwidth, NBW , $f^{'}_{c,1} = 0$ and $N_I = 1$ and $SNR = 20$ dB

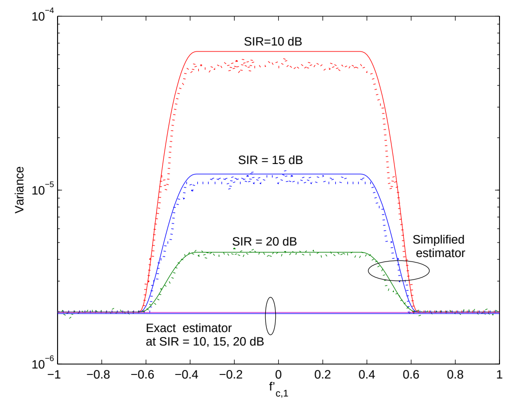

Fig. 7. Variance of the estimator versus normalized interference carrier frequency, $f'_{c,1}$, $SNR = 20$ dB, NBW= .0244, and N_I=1

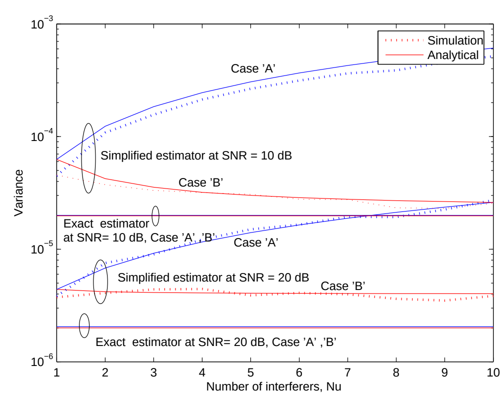

Fig. 8. variance versus number of interference signals, N_I, $NBW = .0244$, $SIR = 20$ and $SNR = 20$ dB