DESIGN AND ANALYSIS OF THE UW-OFDM SIGNAL

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ABSTRACT

Unique word (UW-) OFDM is a novel multicarrier system that is shown to be suitable for cognitive radio systems. To construct the UW-OFDM signal, a code generation matrix is required to introduce redundancy in the frequency domain, implying that the code generation matrix must satisfy a number of conditions. In this paper, we use an algebraic construction method to separate the conditions imposed by the signal shape from the code design. The degrees of freedom resulting from this construction method are used to optimize different performance measures (i.e., the minimum Euclidean distance and the power) at the transmitter or at the receiver side. Based on the algebraic decomposition, a composite channel can be defined. Irrespective of whether the optimization is done at the transmitter or the receiver, we shown in the paper that always the strongest modes of this composite channel must be excited.

***Index Terms*—** multicarrier communication, error rate, Euclidean distance, tight frame

1. UW-OFDM

In multicarrier communication systems, typically a guard interval is used to combat the effect of intersymbol interference. In traditional multicarrier systems, this guard interval is added on top of the signal containing the data, resulting in a signal with extended symbol duration. Unlike these standard multicarrier systems, unique word (UW-) OFDM counteracts intersymbol interference without the necessity to extend the DFT block with a guard interval. In UW-OFDM, this guard interval is part of the DFT interval, i.e., the last N_u samples of the DFT interval are reserved for the unique word, which is a sequence of known samples. In [1], it is shown that UW-OFDM outperforms cyclic prefix (CP-) OFDM with respect to the bit error rate in fading channels. This is confirmed in [2] where a theoretical analysis of the error rate performance shows that UW-OFDM is always able to achieve full diversity, whereas in CP-OFDM only a diversity one is obtained unless precoding is used. Further, it is shown in [3] that UW-OFDM has much lower out-of-band radiation compared to CP-OFDM. Hence, UW-OFDM is an excellent candidate for cognitive radio systems.

To construct the UW-OFDM signal, we use the two-step approach. First, we generate the time-domain sequence corresponding to the data symbols, resulting in a block of N_u zeroes at the end of the DFT interval. In this block of zeroes, we add the unique word symbols during the second step. The construction of the data contribution containing a block of zeroes in the time domain requires the presence of redundancy in the frequency domain. Consequently, assuming the DFT size equals N, a maximum of $N - N_u$ data symbols can be transmitted per DFT block.

The UW-OFDM signal is generated as follows. First, the $N_d \le N - N_u$ data symbols $\mathbf{a}_d$ are fed to the $N_m \times N_d$ code generator matrix $\mathbf{G}$, in order to introduce the redundancy in the frequency domain. Because of the presence of guard bands, generally the number N_m of modulated carriers is smaller than the DFT size: $N_m \le N$. The $N \times N_m$ mapping matrix $\mathbf{B}$ maps the 'coded' symbols on the modulated carriers. This mapping matrix is a reduced version of the $N \times N$ identity matrix, where the columns corresponding to the carriers that are not modulated are removed. The resulting frequency-domain vector is modulated on the carriers using the inverse DFT, resulting in the time-domain samples $\mathbf{y}$:

$$\mathbf{y} = \mathbf{F}_N^H \mathbf{B}\mathbf{G}\mathbf{x}_d = \begin{pmatrix} * \\ \mathbf{0} \end{pmatrix} \tag{1}$$

where $\mathbf{F}_N$ is the DFT matrix with elements $(\mathbf{F}_N)_{k,\ell} = \frac{1}{\sqrt{N}} e^{-j2\pi \frac{k\ell}{N}}$.

To ensure that the last N_u samples of $\mathbf{y}$ are zero, the frequency-domain vector must be selected properly. The mapping matrix $\mathbf{B}$ is determined by the spectral band requirements of the communication system, such that the designer can only select the code generator matrix $\mathbf{G}$. Let us denote $\tilde{\mathbf{F}}$ as the $N_u \times N_m$ matrix containing the last N_u rows of $\mathbf{F}_N^H \mathbf{B}$. Imposing that the last N_u elements of $\mathbf{y}$ must be zero implies that the vector $\mathbf{G}\mathbf{x}_d$, $\forall \mathbf{x}_d$, must belong to the null space of $\tilde{\mathbf{F}}$. The rank-nullity theorem states that the dimension of the null space is related to the rank of the matrix. Considering the Gramian matrix of $\mathbf{F}_N^H \mathbf{B}$, i.e., $\mathbf{B}^H \mathbf{F}_N \mathbf{F}_N^H \mathbf{B} = \mathbf{I}_{N_m}$, it follows that $\mathbf{F}_N^H \mathbf{B}$ is full rank. Hence, the submatrix $\tilde{\mathbf{F}}$ will

The author gratefully acknowledges the financial support from the Flemish Fund for Scientific Research (FWO). This research has been funded by the Interuniversity Attraction Poles Programme initiated by the Belgian Science Policy Office.

also have full rank, implying that the null space of $\tilde{\mathbf{F}}$ has dimension $N_m - N_u$. Consequently, $N_m - N_u$ orthonormal basis vectors can be found spanning the null space. Such an orthonormal basis for $\tilde{\mathbf{F}}$ can easily be found using the singular value decomposition. Let us define the $N_m \times (N_m - N_u)$ matrix $\mathbf{U}$, where the columns contain the null-space vectors of $\tilde{\mathbf{F}}$, such that $\tilde{\mathbf{F}}\mathbf{U} = \mathbf{0}$. Taking into account that all linear combinations of these basis vectors belong to the null space, we propose to decompose the code generator matrix $\mathbf{G}$ as:

$$\mathbf{G} = \mathbf{U}\mathbf{W} \tag{2}$$

where the $(N_m - N_u) \times N_d$ matrix $\mathbf{W}$ can be selected freely. Note that to avoid ambiguity between the different data sequences $\mathbf{x}_d$, the number of data symbols is upper bounded by $N_m - N_u$: $N_d \leq N_m - N_u$.

2. EUCLIDEAN DISTANCE AT THE TRANSMITTER

In the previous section, we have decomposed the code-generator matrix $\mathbf{G}$ to simplify the design of the UW-OFDM signal. To satisfy the constraints on the signal shape, the system parameters specify both the mapping matrix $\mathbf{B}$ and the null-space matrix $\mathbf{U}$. The designer only has the freedom to select the linear combination matrix $\mathbf{W}$. In [2], it is shown that, in order to achieve full diversity, the code generator matrix must be full rank. Hence also $\mathbf{W}$ should be full rank.

In this section, we restrict our attention to the case where the minimum Euclidean distance at the transmitter is maximized. As this Euclidean distance is given by $d_T = \mathbf{e}^H\mathbf{G}^H\mathbf{B}^H\mathbf{B}\mathbf{G}\mathbf{e}$, where $\mathbf{e} = \mathbf{x}_d - \mathbf{x}'_d$, it can be verified that maximization of the minimum Euclidean distance requires $\mathbf{G}^H\mathbf{B}^H\mathbf{B}\mathbf{G} = \mathbf{I}_{N_d}$ [2]. Besides the maximization of the minimum Euclidean distance, this requirement results in every data symbol being equally represented in the transmitted signal, i.e., assuming the energy per data symbol equals E_s, the time-domain signal corresponding to a single data symbol has energy E_s, and the transmit power $P_T = E_s\text{trace}(\mathbf{G}^H\mathbf{B}^H\mathbf{B}\mathbf{G}) = N_d E_s$. Moreover, it is shown in [4] that this restriction results in the MSE of the BLUE and MMSE data detectors to be minimized, in frequency-flat channels.

The restriction $\mathbf{G}^H\mathbf{B}^H\mathbf{B}\mathbf{G} = \mathbf{I}_{N_d}$ enforces the matrix $\mathbf{W}$ to satisfy $\mathbf{W}^H\mathbf{U}^H\mathbf{B}^H\mathbf{B}\mathbf{U}\mathbf{W} = \mathbf{I}_{N_d}$. Let us look closer at this restriction to find a solution for the matrix $\mathbf{W}$. Taking into account that $\mathbf{B}^H\mathbf{B} = \mathbf{I}_{N_m}$ and $\mathbf{U}^H\mathbf{U} = \mathbf{I}_{N_m - N_u}$, it follows that

$$\mathbf{W}^H\mathbf{W} = \mathbf{I}_{N_d}. \tag{3}$$

In the case where $N_m - N_u = N_d$, $\mathbf{W}$ is a square matrix, and the condition (3) states that $\mathbf{W}$ must be a unitary matrix. In general, when $N_m - N_u \leq N_d$, the condition (3) corresponds to a $(N_m - N_u) \times N_d$ matrix describing a finite equal-norm Parseval frame [5], also known as a tight frame. Unfortunately, no general analytical solution exists for this problem. Yet, one analytical solution exits for the general case, where $N_m - N_u$ and N_d can take any value with $N_d \leq N_m - N_u$, i.e., the generalized harmonic frame $\mathbf{W} = \frac{1}{\sqrt{N_m - N_u}}[b_0\mathbf{c}^0 \quad b_1\mathbf{c}^1 \ldots b_{N_d-1}\mathbf{c}^{N_d-1}]$, where $\mathbf{c} = [1 \quad \alpha \ldots \alpha^{N_m-N_u-1}]^T$, α is the $(N_m - N_u)$-th root of 1, i.e., $\alpha = \exp\left(\frac{j2\pi}{N_m - N_u}\right)$, and the coefficients b_i, $i = 0, \ldots, N_d - 1$ with $|b_i| = 1$ can be selected freely. Further, some other analytical solutions exist for special values of $N_m - N_u$ and N_d. To find other solutions for the tight frame $\mathbf{W}$, several construction methods were proposed in the literature, e.g., based on a generalization of the Gram-Schmidt orthogonalization method [6]-[8].

Many matrices satisfy condition (3), implying we can use this degree of freedom to optimize other performance measures. Two measures of interest are the minimum Euclidean distance at the receiver d_R and the received power P_R:

$$\begin{aligned} d_R &= \min_{\mathbf{e}} \mathbf{e}^H\mathbf{G}^H\mathbf{B}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{B}\mathbf{G}\mathbf{e} \\ P_R &= E_s\,\text{trace}(\mathbf{G}^H\mathbf{B}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{B}\mathbf{G}). \end{aligned} \tag{4}$$

where $\tilde{\mathbf{H}}_{k,k'} = \delta_{k,k'}\mathcal{H}(k)$ is the channel frequency response matrix with $\mathcal{H}(k) = \sum_{m=0}^{L} h(m)e^{j2\pi\frac{km}{N}}$, and $\mathbf{h} = [h(0)\ldots h(L)]$ is the channel impulse response. Both the minimum Euclidean distance at the receiver and the received power have an influence on the error rate performance, and should preferably be as large as possible. Note that both performance measures depend on the matrix $\mathbf{G}^H\mathbf{B}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{B}\mathbf{G}$. Within this matrix product, the matrix $\mathbf{B}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{B}$ is a diagonal matrix with as diagonal elements the spectral response of the channel at the modulated frequencies, i.e., $\mathbf{B}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{B} = \text{diag}(|\mathcal{H}(n_\ell)|^2)$, and n_ℓ, $\ell = 1, \ldots, N_m$, are the carrier indices of the modulated carriers. Using the decomposition $\mathbf{G} = \mathbf{U}\mathbf{W}$, the matrix $\mathbf{G}^H\mathbf{B}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{B}\mathbf{G}$ can be rewritten as $\mathbf{W}^H\mathbf{U}^H\mathbf{B}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{B}\mathbf{U}\mathbf{W} \equiv \mathbf{W}^H\mathbf{S}\mathbf{W}$, where $\mathbf{S} = \mathbf{U}^H\mathbf{B}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{B}\mathbf{U}$ is the 'composite' channel matrix and $\mathbf{W}$ is a tight frame, i.e., $\mathbf{W}^H\mathbf{W} = \mathbf{I}_{N_d}$. Hence, the two optimization problems can be written as

$$\begin{aligned} &\max_{\mathbf{W}} d_R = \max_{\mathbf{W}} \min_{\mathbf{e}} \mathbf{e}^H\mathbf{W}^H\mathbf{S}\mathbf{W}\mathbf{e} \\ &\text{subject to } \mathbf{W}^H\mathbf{W} = \mathbf{I}_{N_d} \end{aligned} \tag{5}$$

and

$$\begin{aligned} &\max_{\mathbf{W}} P_R = \max_{\mathbf{W}} E_s\,\text{trace}(\mathbf{W}^H\mathbf{S}\mathbf{W}) \\ &\text{subject to } \mathbf{W}^H\mathbf{W} = \mathbf{I}_{N_d}. \end{aligned} \tag{6}$$

Let us define the transmit modes of the composite channel $\mathbf{S}$ according to its different eigenvalues: the strongest mode corresponds to the largest eigenvalue of $\mathbf{S}$ and the weakest mode to the smallest eigenvalue. Both optimization problems result in the same optimal $\mathbf{W}$: the columns of the matrix $\mathbf{W}$ should be selected as an orthonormal basis for the eigenspace corresponding to the N_d algebraically largest eigenvalues of $\mathbf{S}$, or equivalently, we should excite the strongest modes of the composite channel. Both solutions are a direct consequence of the Courant-Fisher theorem [9], and are commonly used in dimensionality reduction methods [10] and principal component analysis [11]. The solution is not unique as any orthonormal basis can be selected. However, the selected orthonormal basis has neither influence on the received power, nor on the minimum Euclidean distance at the receiver, and therefore also not on the error rate performance.

Let us consider the special case where $N_d = N_m - N_u$, implying $\mathbf{W}$ is a square, unitary matrix. As a result, the received power P_R and the minimum Euclidean distance become independent of the selected unitary matrix $\mathbf{W}$: $P_R = E_s\text{trace}(\mathbf{S})$ and d_R equals the minimum eigenvalue of $\mathbf{S}$. Although this result can be derived straightforwardly, it also follows from the general case above. In that solution, the matrix $\mathbf{W}$ had to be selected according to an orthonormal basis for the eigenspace of the N_d largest (out of $N_m - N_u$) eigenvalues. However, as $N_d = N_m - N_u$, all eigenvalues of $\mathbf{S}$ must be considered. This not only implies that the smallest eigenvalue of $\mathbf{S}$ determines the minimum Euclidean distance at the receiver, but also that any unitary matrix could be used, without having an influence on the performance. As a conclusion, if $N_d = N_m - N_u$, the received power and the minimum Euclidean distance only depend on the channel and the system parameters, but not on the selected unitary matrix $\mathbf{W}$. Moreover, if we assume that all carriers are modulated, i.e., $N_m = N$, such that $\mathbf{B} = \mathbf{I}_N$, it can be verified that the null-space matrix $\mathbf{U}$ can be written as $\hat{\mathbf{F}}$, where $\hat{\mathbf{F}}$ corresponds to the $N - N_u$ first columns of $\mathbf{F}_N$, such that the received power reduces to

$$P_R = E_s \frac{N - N_u}{N} \sum_{\ell=0}^{N-1} |\mathcal{H}(\ell)|^2 . \qquad (7)$$

Until now, it was implicitly assumed that the channel is priorly known, as this knowledge is required to find the eigen decomposition of the matrix $\mathbf{S}$. Let us now consider the case where the channel is not priorly known. When $N_d = N_m - N_u$, it is shown earlier in this section that both the received power and minimum Euclidean distance at the receiver are independent of the matrix $\mathbf{W}$, irrespective of the knowledge of the channel. However, for the general case, when $N_d < N_m - N_u$, we need information about the channel to find the eigenspace of $\mathbf{S}$. As a solution to this problem, we suggest to select the matrix $\mathbf{W}$ such that, on the average, the received power is maximized:

$$\begin{aligned} \max_{\mathbf{W}} \overline{P}_R &= \max_{\mathbf{W}} E_{\mathbf{h}}[E_s \,\text{trace}(\mathbf{W}^H \mathbf{S} \mathbf{W})] \\ &= \max_{\mathbf{W}} E_s \,\text{trace}(\mathbf{W}^H \overline{\mathbf{S}} \mathbf{W}) \\ \text{subject to } & \mathbf{W}^H \mathbf{W} = \mathbf{I}_{N_d}. \end{aligned} \qquad (8)$$

where $E_{\mathbf{h}}[\cdot]$ denotes the averaging over the channel statistics, $\overline{P}_R = E_{\mathbf{h}}[P_R]$ is the average received power and $\overline{\mathbf{S}} = \mathbf{U}^H \mathbf{B}^H E_{\mathbf{h}}[\tilde{\mathbf{H}}^H \tilde{\mathbf{H}}] \mathbf{B} \mathbf{U}$. Similarly as for the known channel, the optimal matrix $\mathbf{W}$ can be obtained by determining the eigenspace corresponding to the largest eigenvalues of $\overline{\mathbf{S}}$. Unfortunately, finding the matrix $\mathbf{W}$ that maximizes the minimum Euclidean distance at the receiver, averaged over the channel, does not result in a simple analytic solution: in contrast with the received power where the objective function to be maximized is linear in the matrix $\mathbf{S}$, the objective function for d_R is not linear. Hence, finding the optimal matrix $\mathbf{W}$ in the latter case is an intractable problem. As an example of the optimization (8), we consider the special case where the channel taps are identically distributed and uncorrelated: $E_{\mathbf{h}}[\mathbf{h}\mathbf{h}^h] = \alpha_L^2 \mathbf{I}_{L+1}$, with α_L a normalization parameter. In that case, $E_{\mathbf{h}}[\tilde{\mathbf{H}}^H \tilde{\mathbf{H}}] = \alpha_L \mathbf{I}_N$ and $\overline{\mathbf{S}} = \mathbf{I}_{N_m - N_u}$. Consequently, $\overline{P}_R = \alpha_L N_d E_s$, independent of the selected matrix $\mathbf{W}$.

3. EUCLIDEAN DISTANCE AT THE RECEIVER

In section 2, we selected the matrix $\mathbf{W}$ to normalize the transmit power and at the same time to maximize the minimum Euclidean distance at the transmitter. The available degree of freedom was used to maximize the received power and the minimum Euclidean distance at the receiver. However, with respect to the error rate performance, the above solution is not optimal. The error rate performance is optimized if the minimum Euclidean distance at the receiver is as large as possible. Hence, this section concentrates on the matrix $\mathbf{W}$ that maximizes the minimum Euclidean distance at the receiver.

A similar analysis as for the transmitter side shows that the matrix $\mathbf{W}$ maximizing the minimum Euclidean distance at the receiver is a solution of the equation $\mathbf{W}^H \mathbf{S} \mathbf{W} = \mathbf{I}_{N_d}$. Note that this requirement not only maximizes the minimum Euclidean distance at the receiver, but also it normalizes the received power: $P_R = N_d E_s$. The matrix $\mathbf{W}$ satisfying $\mathbf{W}^H \mathbf{S} \mathbf{W} = \mathbf{I}_{N_d}$ can easily be obtained as follows. Let us consider the eigenvalue decomposition $\mathbf{S} = \mathbf{V} \boldsymbol{\Lambda} \mathbf{V}^H$. Assuming the channel does not contain spectral nulls at the frequencies of the modulated carriers, i.e., $\mathbf{S}$ is full rank, the matrix $\mathbf{W}$ can be decomposed as $\mathbf{W} = \mathbf{V} \boldsymbol{\Lambda}^{-1/2} \mathbf{X}$, resulting in $\mathbf{X}^H \mathbf{X} = \mathbf{I}_{N_d}$. Hence, the matrix $\mathbf{X}$ should form a tight frame. In the previous section, it was shown that many solutions exist for $\mathbf{X}$. Therefore, we use this degree of freedom to minimize the transmit power required to obtain the requested received power. The transmit power can be rewritten as:

$$\begin{aligned} P_T &= E_s \,\text{trace}(\mathbf{W}^H \mathbf{W}) = E_s \,\text{trace}(\mathbf{X}^H \boldsymbol{\Lambda}^{-1} \mathbf{X}) \\ &= E_s \,\text{trace}(\tilde{\mathbf{X}}^H \mathbf{S}^{-1} \tilde{\mathbf{X}}) \end{aligned} \qquad (9)$$

where $\tilde{\mathbf{X}} = \mathbf{V}^H \mathbf{X}$. Note that $\tilde{\mathbf{X}}$ also is a tight frame. Similarly as the optimization problem (6), the solution of the maximization of P_T is a direct consequence of the Courant-Fisher theorem. The columns of $\mathbf{X}$ ($\tilde{\mathbf{X}}$) should be selected to form an orthonormal basis for the eigenspace of the algebraically smallest eigenvalues of $\mathbf{\Lambda}^{-1}$ ($\mathbf{S}^{-1}$), or equivalently, the largest eigenvalues of $\mathbf{S}$.

Let us consider the effect of the matrix $\mathbf{W}$ on the minimum Euclidean distance at the transmitter:

$$\begin{aligned} d_T &= \min_{\mathbf{e}} \mathbf{e}^H \mathbf{W}^H \mathbf{W} \mathbf{e} = \min_{\mathbf{e}} \mathbf{e}^H \mathbf{X}^H \mathbf{\Lambda}^{-1} \mathbf{X} \mathbf{e} \\ &= \min_{\mathbf{e}} \mathbf{e}^H \tilde{\mathbf{X}}^H \mathbf{S}^{-1} \tilde{\mathbf{X}} \mathbf{e} \end{aligned} \quad (10)$$

Interestingly, the matrix $\mathbf{W}$ that minimizes the transmit power also minimizes the minimum Euclidean distance at the transmitter. Although this might sound counter-intuitive, it can be explained using the modes op the 'composite' channel, defined earlier in this paper. To obtain the same amount of energy out of each mode, less energy has to be assigned to the strongest mode, such that the Euclidean distance between the transmitted sequences corresponding to this mode can reduce. As the matrix $\mathbf{W}$ is selected to only excite the strongest modes corresponding to the largest eigenvalues, the minimum Euclidean distance is the smallest possible, as it is determined by the largest eigenvalue.

In the special case where $N_d = N_m - N_u$, the matrix $\mathbf{W}$ is a square matrix, and $\mathbf{X}$ is a unitary matrix. Similarly as in the previous section, the transmit power and the minimum Euclidean distance at the transmitter become independent of the selected matrix $\mathbf{X}$, as all eigenvalues should be taken into account.

To construct the matrix $\mathbf{W}$, knowledge on the eigenvalues of $\mathbf{S}$ is required. In the case that the channel is not known, this knowledge is not available. However, similarly as in the previous section, we can opt to select the matrix $\mathbf{W}$ such that the average received power is normalized. In that case, $\mathbf{S}$ should be replaced by $\overline{\mathbf{S}}$, defined in the previous section. Note that selecting the matrix $\mathbf{W}$ to optimize the minimum Euclidean distance after averaging over the channel characteristics is an intractable problem, similarly as in the previous section.

4. NUMERICAL RESULTS

To evaluate the obtained optimal code generator matrices, we consider the transmission of the signal over a channel with channel impulse response $h(\ell) = \nu e^{-\ell}$, where ν is selected such that the channel is normalized: $\mathbf{h}^H \mathbf{h} = 1$. Further, we assume that at both sides of the frequency band, γ carriers are not used, i.e., the number of modulated carriers equals $N_m = N - 2\gamma$. We compare the code generator matrix according to both construction methods given in sections 2 and 3, where in the first case, the transmit power is normalized, and in the second case the received power. Figure 1 shows the ratio of the transmit power to the received power, for the two cases. We can observe that the ratio is larger for the case when the received power is normalized than for the case where the transmit power is normalized. Hence, for given transmit power, normalizing the transmit power results in a larger received power than normalizing the received power.

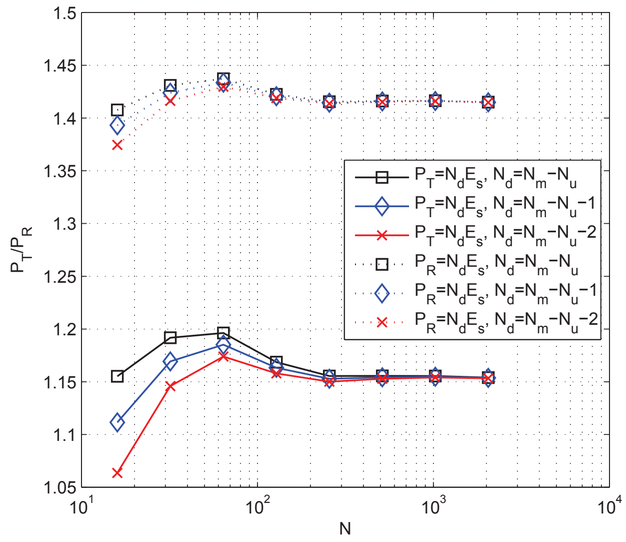

Fig. 1. Ratio of the transmit power to the received power, P_T/P_R, solid lines correspond to a normalized transmit power $P_T = N_d E_s$ and dashed lines to a normalized received power $P_R = N_d E_s$, $\gamma = \lceil 0.05N \rceil$, $N_u = \lceil 0.1N \rceil$, $L = N_u$.

This can be explained as follows. Assume that λ_k are the N_d largest eigenvalues of the matrix $\mathbf{S}$. In that case, the ratio P_T/P_R, assuming the transmit power is normalized, is given by

$$\frac{P_T}{P_R} = \frac{N_d}{\sum_{k=1}^{N_d} \lambda_k} \quad (11)$$

and when the received power is normalized, by

$$\frac{P_T}{P_R} = \frac{1}{N_d} \sum_{k=1}^{N_d} \frac{1}{\lambda_k}. \quad (12)$$

In the special case where all eigenvalues are equal, both ratios are equal, implying that optimization of the minimum Euclidean distance at the transmitter or at the receiver side results in the same transmit and received powers. For example, this occurs when $\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} = \lambda \mathbf{I}_N$, i.e., the channel is frequency-flat. For the general case, where the eigenvalues are not equal, it can be shown using the Cauchy-Schwarz inequality that the first ratio (11) is always smaller than or equal to the second ratio (12). Hence, the power efficiency is always better when the power is normalized at the transmitter side. However, by normalizing the transmit power, the eigenvalues of $\mathbf{W}^H \mathbf{S} \mathbf{W}$ will in general not be equal, which may result in a reduction of the minimum Euclidean distance at the receiver.

Furthermore, we can observe in Figure 1 that, except for small values of N, the ratio is essentially independent of the DFT size N. This effect is caused by the normalization of the channel, and the fact that the channel in the example gives rise

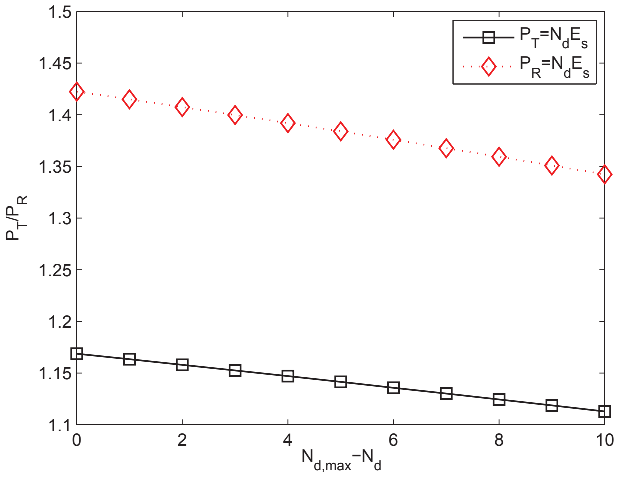

Fig. 2. Ratio of the transmit power to the received power, $P_T/P_R, \gamma = \lceil 0.05N \rceil, N_u = \lceil 0.1N \rceil, L = N_u$.

to a matrix $\mathbf{S}$ that is diagonally dominant. This effect is not observed for other channels. Finally, from the figure it follows that reducing the number of data symbols N_d compared to the maximum allowable data symbols $N_{d,\max} = N_m - N_u$ has only a small effect on the ratio. To further investigate the effect of reducing the number of data symbols, we plotted in Figure 2 the ratio of the transmit power to the received power as function of the difference between the maximum number of data symbols and the actual number of data symbols $N_{d,\max} - N_d$. Reducing the number of data symbols results in a lower ratio, thus a higher power efficiency, for both cases. This can be explained by evaluating expressions (11) and (12). When N_d decreases linearly, the smallest eigenvalues λ_k in the sums in (11) and (12) will be dropped. As a result, the sum in (11) will reduce in a less than linear way, whereas the sum in (12) will reduce in a faster than linear way. Consequently, both ratios will reduce. The pace of the reduction depends on the magnitude of the eigenvalues. The ratio (11) will drop faster when the eigenvalues are close to each other, whereas the ratio (12) will have a stronger dependency on N_d when the eigenvalues are more widely spread.

5. CONCLUSIONS

In this paper, we considered an algebraic construction method for the code generator matrix in UW-OFDM. Using this construction method, we can design the UW-OFDM signal to optimize the minimum Euclidean distance and the power, at the transmitter or at the receiver. Based on the analysis in this paper, we can see many parallels between the optimization of the minimum Euclidean distance at the transmitter or at the receiver. It turns out that no matter which minimum Euclidean distance is maximized, we should always excite the strongest modes of the composite channel, i.e., the matrix $\mathbf{W}$ should always be selected according to an orthonormal basis for the eigenspace of the algebraically largest eigenvalues of the composite channel matrix $\mathbf{S}$, and if the channel is not priorly known, of its average $\overline{\mathbf{S}}$.

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