POLYMER ANALYSIS WITH MATHEMATICAL MORPHOLOGY

[1]*Alessandro Ledda,* [2]*Pieter Samyn,* [2]*Jan Quintelier,* [2]*Patrick De Baets and* [1]*Wilfried Philips*

`ledda@telin.UGent.be`
[1]TELIN, Sint-Pietersnieuwstraat 41, B-9000 Ghent, Belgium
[2]Laboratory Soete, Sint-Pietersnieuwstraat 41, B-9000 Ghent, Belgium

ABSTRACT

The debris particles obtained from wear experiments on polymers are analysed by size and shape with image processing techniques. The Fourier spectrum and morphological pattern spectra, like the classical pattern spectrum, the area pattern spectrum and the opening tree, are very promising techniques. They can be used to extract parameters that relate to particle size and shape. We determine the usefulness of the spectra in the field of material science. The tests in this paper were done on a polymer called "polyimide". We performed wear experiments with different temperatures, loads and frequencies. In the future we will perform experiments on composite materials.

1. INTRODUCTION

We perform tribological experiments on polymers and composite materials (these are in essential polymers with fibre strengthening). These materials can be used as dry friction bearing materials, like in machinery for food and pharmaceutical processes.

1.1. Tribology

Tribology is the science and technology of interacting surfaces in relative motion and the practices related thereto [1]. Traditional parameters studied in tribological investigations are the normal and the friction forces, which allow us to determine one of the most important physical parameters: the *friction coefficient*. This parameter is important because friction produces heat, and thus the friction coefficient should be minimized. It depends on many parameters such as normal load, speed and temperature.

Another interesting parameter to measure is the *vertical displacement*, which gives an indication of the wear rate.

Nowadays it is not enough to know how quick materials wear, but we want to understand how they wear, what the major processes are, all in order to pick up the better material combination. In this paper we investigate what role image processing techniques can play.

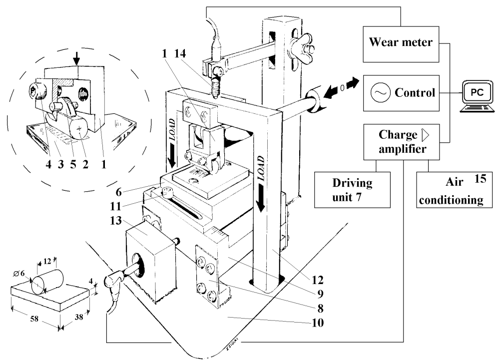

Figure 1: *Experimental setup of the pin-on-plate experiment.*

1.2. Experimental Setup

The test samples are installed onto a *pin-on-plate* setup (Plint TE-77) (fig. 1) where a steel pin applies a normal force on the sample while moving forward and backward. This will wear the material. For the experiments described in this paper we have used polymers called "polyimide" (Vespel-SP1).

The debris particles resulting from the wear process are collected and photographed. The analysis of the shape and size of these particles in the pictures will hopefully help us with the understanding of the wear process, and with the correlation between all the parameters collected during a test.

The wear particles look like the ones in fig. 2. The original pictures were colour images with resolution 1300×1030, but are converted to 8 bit grayscale pictures, since our image processing algorithms only work on monochrome images. For display purposes, we also performed a histogram equalisation to improve the contrast between the background and the particles.

Three parameters are varied in the experiment: *temperature*, *load* and *frequency*.

The friction causes the plate to heat up. The temperature, which initially equals room temperature, can be regulated to stay at a certain value, or it can be left unregulated, resulting in further heating up of the plate ("free

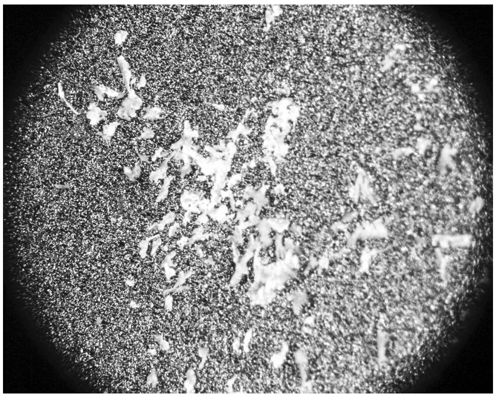

Figure 2: *Grayscale image of the debris particles from a polymer.*

T" in the table). The load of the pin on the sample varies between 50 N and 200 N. We use different frequencies for the movement speed of the plate.
We obtained pictures of debris particles for the following experimental conditions:

	Temperature (°C)	Load (N)	Frequency (Hz)
1	free T	100	10
2	free T	100	20
3	free T	100	40
4	100	50	10
5	100	100	10
6	100	150	10
7	100	200	10
8	180	50	10
9	180	100	10
10	180	150	10

2. METHODS

Different possibilities are available to extract information from an image. We use the parameters from spectral techniques like the Fourier transform and morphological spectra.

2.1. Fourier Analysis

The *Fourier transform* [2] of an image yields an energy spectrum that reflects the grayscale periodicity (spatial frequency spectrum) in the image. Large image objects will constitute to the low frequency energy, small objects to the energy of higher frequencies. The Fourier transform lacks the ability of spatial localisation, but in our case this is not a problem, since our debris particles are randomly distributed over space.
An image is transformed with the 2D-FFT. This results in a two dimensional (real) spatial frequency energy spectrum. In order to calculate some first-order histogram features and to remove the orientation dependence of the FFT-spectrum, we transform the 2D-spectrum into a 1D-spectrum (FFTh) with on the abscissa the spatial frequency $\sqrt{\nu_x^2 + \nu_y^2}$.
From the histogram, we compute the following parameters, which will be discussed further on:

- **Mean**: the average frequency value;
- **Standard deviation**: the spread of the values;
- **Skewness**: the asymmetry of the histogram curve;
- **Kurtosis**: the curve shape relative to a normal distribution;
- **Energy**: the non-uniformity of the spectrum;
- **Entropy**: the uniformity of the distribution.

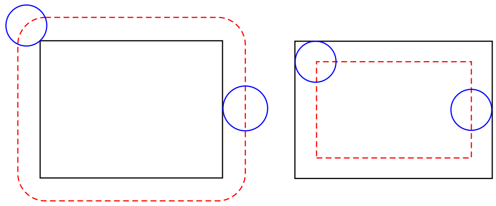

Figure 3: *Schematic example of the basic morphological operators. Solid line: original object; Dashed line: result object; Circle: structuring element. Left:* dilation*; Right:* erosion.

2.2. Mathematical Morphology

Mathematical morphology (MM) [3] is based on set theory. The shapes of objects in a binary image are represented by object membership sets. This theory can be extended to grayscale images. Morphological operations can simplify image data, preserving the objects' essential shape characteristics, and can eliminate irrelevant objects. Mathematical morphology is based on two basic operations, defined in terms of a *structuring element* (short: *strel*), a small window that scans the image and alters the pixels in function of its window content: a *dilation* of image A with strel B ($A \oplus B$) blows up the object, an *erosion* ($A \ominus B$) lets it shrink (see fig. 3).
Other operations, like the *opening* (an erosion followed by a dilation) and the *closing* (a dilation followed by an erosion), are derived from the basic operators.

2.2.1. *(Area) Pattern Spectrum*

If we take a strel and use it to perform an opening on an image, some elements will disappear. If we take a bigger strel, then more elements in the image will vanish.

In this way we can determine how the number of eliminated pixels increases when the image is morphologically opened using strels $nB = B \oplus B \oplus \ldots \oplus B$ (n times) of increasing size n. The resulting plot of the number of eliminated pixels versus the strel size n is called the *pattern spectrum* (PS) or the *granulometric curve* [4].

The pattern spectrum is a histogram of the distribution of the sizes of various objects displayed in an image. Formally, it is defined as follows:

$$PS(A;B)(n) = \sharp[A \circ nB \setminus A \circ (n+1)B], n \geq 0 \quad (1)$$

where $\circ$ is the opening symbol, $\setminus$ is the pixelwise difference and $\sharp$ is the count of grayscale pixels. Note that $0B = \{0\}$.

Similarly, we define the *Area Pattern Spectrum* (APS). Instead of using a structuring element, we now define increasing threshold values λ_n which will decide which connected components will be removed from the image.

There is an analogy with the Fourier spectrum: the low frequencies in the Fourier spectrum and the big structuring elements in the pattern spectrum relate to the global features of the image, the other end of the spectrum relates to the details in the image.

From the pattern spectrum we can extract different parameters [4], [5] that provide statistical information about the content of the image:

- **Mean object size**: the mean strel size or mean area;
- **Average roughness**: the *entropy*, a quantification of the shape-size complexity;
- **Normalised average roughness**: the entropy divided by $\log_2(N_{max} + 1)$;
- **B-shapiness**: a quantitative measure of the resemblance of the objects to the strel shape;
- **Maximal n** (N_{max}): the last bin (the highest n-value, when all image objects are sieved out) of the pattern spectrum histogram.

Notice that a different strel or λ-series results in different (area) pattern spectra, and thus in other values for the parameters.

Since the calculation time of the PS is very high, there is need for other, similar techniques: besides the APS (using a very fast algorithm [6], [7]), we will investigate a *block based technique* and the *pseudo-granulometry by minima of linear openings* [8].

2.2.2. Block Based Technique

The *block based technique* (BBT) divides the images into smaller blocks that are processed independently. This should lower the total calculation time, but introduces a potential problem: a particle at a block boundary will be treated as multiple particles or as a smaller particle. Or in some cases even as a bigger particle. We investigate the resulting deteriorations experimentally. Different sizes for the subimages are used and verified.

2.2.3. Opening Tree

The *opening tree* [9] makes it possible to approximate the pattern spectrum for grayscale images, with the calculation time about three orders of magnitude faster than the traditional pattern spectrum calculation [8]. The opening tree is a hierarchical technique where the image pixels are the *leaves* and the *nodes* are made of pairs (h, n), where h is a grayscale value and n is an opening size. The algorithm (for details, see [8], [9]) works row-per-row or column-per-column, or even diagonal-per-diagonal, but can be extended to a two dimensional version. In that case we get the so-called *pseudo-granulometries by minima/maxima of linear openings/closings*. The pseudo-granulometric curve should be a good approximation of a square granulometry because of similar image features.

3. RESULTS

3.1. Calculation Time

In this section we compare the calculation times for the different algorithms used. Seventeen 8 bit grayscale images with resolution 1300×1030 were processed.

The average results for one image are: the FFTh algorithm only takes a few seconds, the APS about five seconds, for the opening tree (1D and 2D) tens of seconds are needed, the calculation time for the BBT goes from a few hours (image split into 20×20 different images) to a few days (3×4 subimages), while the calculation of the classical PS takes a few days.

The calculations were made on a PC with an AMD Athlon XP processor (CPU-speed about 1.6 GHz) and with 1 GB of RAM. Other processes were running on the computers at the same time, though. The opening tree and pattern spectrum algorithms are written in C.

For the block based technique, the calculation time is the sum of the different block images. We have not taken into account the time necessary to split the images and merge the results.

3.2. Fourier Analysis

Table 1 shows the correlation coefficients P between the different experimental parameters (frequency, temperature and load) and the different spectral parameters. It

Table 1: Correlation coefficients for the FFTh parameters

		Mean	**Std**	**Skewness**	**Kurtosis**	**Energy**	**Entropy**
Frequency	P	0.60	0.37	-0.64	-0.61	-0.51	0.78
free T, L = 100 N	$\tilde{P}$	0.59	0.76	0.55	0.59	0.66	0.43
Temperature	P	1	1	1	1	1	-1
L = 50 N	$\tilde{P}$	—	—	—	—	—	—
Temperature	P	1	1	-1	-1	-1	1
L = 100 N	$\tilde{P}$	—	—	—	—	—	—
Temperature	P	1	1	1	1	-1	1
L = 150 N	$\tilde{P}$	—	—	—	—	—	—
Load	P	-0.14	0.62	0.87	0.86	-0.0087	-0.67
T = 100 °C	$\tilde{P}$	0.86	0.38	0.14	0.14	0.99	0.33
Load	P	-0.83	-0.25	0.44	0.41	-0.94	-0.63
T = 180 °C	$\tilde{P}$	0.38	0.84	0.71	0.73	0.23	0.57

also shows the value $\tilde{P}$, the probability of getting a correlation as large as the observed value by random chance, when the true correlation is zero. So it is important that the value for P is close to 1 and the value for $\tilde{P}$ is low. Since there are too few pictures available to do the statistical analysis, the $\tilde{P}$ value will be quite large in most of the reported results. In the case of the correlation of the temperature with the spectral parameters no value of $\tilde{P}$ is listed, since only two observations were available.
From the correlation table we observe the following trends for the parameters from the FFTh:

- **Frequency** increases with *entropy*;
- **Temperature** increases with *mean* and *standard deviation*;
- **Load** increases with *kurtosis* and *skewness* when T = 100 °C;
- **Load** decreases with *energy* and *mean* when T = 180 °C;
- **Load** increases with *entropy* in both cases but with lesser probability.

Overall, *entropy* correlates best with the physical parameters.

3.3. (Area) Pattern Spectrum

We calculated the classical spectrum by using a square structuring element, starting at size 2×2, in order to be able to compare with the pseudo-granulometry (section 2.2.3). Here we calculated five parameters. Table 2 shows the correlation coefficients P for the different spectral parameters with the different experimental parameters (frequency, load and temperature).

From the correlation table we observe the following trends for the parameters from the PS:

- **Frequency** decreases with *roughness*, *normalised roughness* and *shapiness*;
- **Temperature** decreases with N_{max} and *size*;
- **Load** increases with *normalised roughness* when T = 100 °C;
- **Load** increases with *size* when T = 180 °C;
- **Load** increases with *roughness* and to N_{max} in both cases.

For the generation of the area pattern spectrum we used a 4-connectivity and λ-series $\{4, 9, 16, 25, \ldots\}$, so comparison with the PS with a 2×2 strel is possible.
From table 3 we observe the following trends for APS:

- **Frequency** decreases with *roughness*, *normalised roughness* and *shapiness* and increases with *size*;
- **Load** increases with all parameters, except *shapiness* when T = 100 °C.

The area pattern spectrum gives a similar correlation as the classical PS, except for the load when T = 180 °C.
Overall, for PS and APS, *roughness* and *normalised roughness* correlate best with the physical parameters, and they show the same behaviour.
Compared to the FFTh parameters (section 3.2), there is a better correlation between the experimental parameters and those from the (area) pattern spectrum.
As stated before, the number of pictures that were available is small. Therefore the presented results are preliminary. More data are needed.

Table 2: Correlation coefficients for the PS parameters

		Size	**Roughness**	**Norm. roughness**	**N$_{max}$**	**Shapiness**
Frequency	P	-0.68	-0.88	-0.94	-0.10	-0.90
free T, L = 100 N	$\tilde{P}$	0.53	0.32	0.23	0.94	0.28
Temperature	P	-1	1	1	-1	-1
L = 50 N	$\tilde{P}$	—	—	—	—	—
Temperature	P	-1	-1	1	-1	-1
L = 100 N	$\tilde{P}$	—	—	—	—	—
Temperature	P	-1	-1	-1	-1	1
L = 150 N	$\tilde{P}$	—	—	—	—	—
Load	P	0.55	0.96	0.93	0.88	-0.79
T = 100 °C	$\tilde{P}$	0.45	0.037	0.074	0.12	0.21
Load	P	0.98	0.95	0.48	0.93	0.51
T = 180 °C	$\tilde{P}$	0.14	0.20	0.68	0.23	0.66

Table 3: Correlation coefficients for the APS parameters

		Size	**Roughness**	**Norm. roughness**	**N$_{max}$**	**Shapiness**
Frequency	P	0.90	-0.86	-0.86	-0.76	-0.97
free T, L = 100 N	$\tilde{P}$	0.29	0.34	0.34	0.45	0.17
Temperature	P	1	1	1	1	1
L = 50 N	$\tilde{P}$	—	—	—	—	—
Temperature	P	1	1	1	1	-1
L = 100 N	$\tilde{P}$	—	—	—	—	—
Temperature	P	-1	-1	-1	-1	-1
L = 150 N	$\tilde{P}$	—	—	—	—	—
Load	P	0.95	0.93	0.91	0.95	0.61
T = 100 °C	$\tilde{P}$	0.049	0.067	0.093	0.051	0.39
Load	P	0.016	0.13	0.65	-0.0010	-0.34
T = 180 °C	$\tilde{P}$	0.99	0.92	0.55	0.99	0.78

3.4. Block Based Technique

When the number of subimages increases, the values of *size*, N_{max} en *roughness* decrease. The values of *shapiness* and *normalised roughness* increase. Fig. 4 shows the differences between the pattern spectra. At lower n-values, the spectra from the images divided into more blocks show higher PS-values, but lower PS-values at higher n and lower N_{max}-values. Notice from the insert in fig. 4 that N_{max} for the 3×4-case is higher than the one for the original image. This is due to boundary effects.

Concerning the correlation we notice the following: when the number of subimages increases the correlation between the frequency and the spectral parameters improves, but it worsens between the load and the spectral parameters. Also, the parameter N_{max} tends to decrease with the load.

3.5. Opening Tree

If we compare the pattern spectrum with the 2D granulometries (obtained with horizontal/vertical, diagonal or horizontal/vertical/diagonal opening trees), then we get the following result: `min.hv`'s (minimal opening with horizontal and vertical trees) parameters agree more with PS than the other opening trees, although values can differ strongly and other 2D granulometries visually look more like the exact PS than `min.hv`. *Roughness* and *normalised roughness* from `min.hv` agree most with the value from the PS (over 90 % resemblance), followed by the parameter *size* (about 30 % too large). The other parameters differ very much.

For the pseudo-granulometry by minima of linear openings we observe the following trends:

- **Frequency** decreases with *roughness*, *normalised roughness* and *size*;

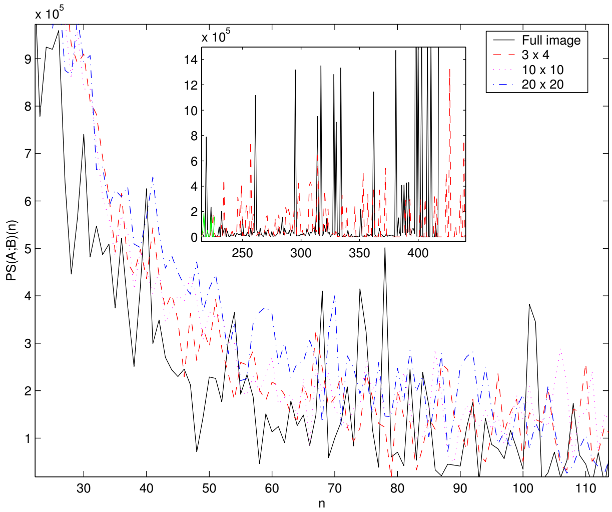

Figure 4: *Part of the Pattern Spectrum of the original image and the into subimages divided (and afterwards merged) images.*

- **Temperature** decreases with *size*;
- **Load** increases with *roughness* and *normalised roughness* when T = 100 °C;
- **Load** increases with *shapiness* when T = 180 °C;
- **Load** increases with *size* in both cases.

Overall, *roughness* and *normalised roughness*, which show almost exactly the same correlation, and *size* correlate best with the physical parameters.
For the other types of opening trees similar results are obtained. There are slight differences between the diagonal opening trees and the other trees, but they are not significant.
It seems that the parameters from the PS (section 3.3) are better correlated with the experimental parameters than the parameters from the opening tree are.

4. CONCLUSIONS

Although the images don't contain enough debris particles and there are too few pictures to draw definite and reliable conclusions, some indications for correlation between the parameters of the FFTh/PS/APS/opening tree and the physical parameters from the experiment can be seen.
The Fast Fourier Transform histogram is a fast technique where the parameter *entropy* is the best to classify the differently treated images. For the pattern spectrum algorithms the parameters *roughness* and *normalised roughness* are the best. The PS is very slow, but the APS is very fast and gives similar results as the classical PS when a proper λ-series has been chosen. The opening tree algorithm is very fast compared to the exact pattern spectrum algorithm and gives similar results (especially `min.hv`). *Size* is a useful parameter.
Concluding from the obtained results, the opening tree and the APS are preferred to the FFTh because of their correlation accuracy and to the PS because of their huge speed improvement.

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