Image Interpolation using Mathematical Morphology

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Abstract—**We present a new method for interpolating binary images that outperforms existing techniques. Bitmapped images have a specific horizontal and vertical resolution. When we wish to magnify such an image then the resolution will be increased, allowing more details in the image. These extra details are not present in the original image. A simple blowup of the image will introduce jagged edges, also called "jaggies". Interpolation techniques calculate the values of the new pixels that become available. These values are derived from their neighbouring picture elements. Our interpolation technique "mmINT" is based on mathematical morphology, a theoretical framework to alter an image while preserving the image objects' geometry. The algorithm detects jaggies in the blown up image and removes them, making the edges smoother. This is done by replacing specific black pixels with white pixels, and vice versa.**

Keywords—**Mathematical Morphology, Binary Interpolation**

I. Introduction

A digital bitmapped image consist of picture elements, *pixels*, aligned on a grid. The image resolution is the number of pixels present in the horizontal and vertical direction. When the image is magnified M times, then the number of pixels is also increased (M^2 times). But we only know the pixel values of the original pixels. The values of the new created pixels must be calculated. This value has to be chosen in some intelligent way.

The easiest way is simply to copy the existing pixel values to the new neighbouring pixels (figure 1). This is called *pixel replication* or *nearest neighbour interpolation*. Every old pixel looks like a blown up pixel, introducing unwanted jagged edges, called *jaggies*.

(a) Original image (b) 2× magnified image

Fig. 1. Pixel replication creates "jaggies".

Other techniques are for example the *bilinear* and *bicubic interpolation* [1]. Here, the (weighted) mean of respectively the 4 or 16 closest neighbours is taken for the new pixel values. Other linear methods use higher order polynomials, B-splines, windowed sinc functions, etc. Most of them create extra artifacts, like blurring and/or ringing. Additional techniques can reduce some unwanted artifacts [2].

Non-linear interpolation methods are designed to tackle jagged edges, such as NEDI [3] and AQua [4] but the results often look segmented or yield important visual degradation in fine textured areas.

In this paper we present a novel non-linear interpolation technique that works very well on black-and-white images. We use mathematical morphology to interpolate images. Therefore, we will call our new method *mmINT* (*Mathematical Morphological INTerpolation*).

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II. Theoretical Background

Mathematical morphology [5] [6] is a framework of image processing techniques based on set theory. An object can be represented as a set, i.e., the set of the coordinates of all the pixels of the object. Objects are connected areas of pixels with value 1, the background pixels have value 0. Morphological operations can simplify image data, preserving the objects' essential shape characteristics, and can eliminate irrelevant objects.

Binary mathematical morphology is based on two basic operations: the *dilation* and *erosion*. They are defined in terms of a *structuring element*, a small window that scans the image and alters the pixels in function of its window content. A *dilation* of image A with structuring element B ($A \oplus B$) enlarges the objects (more 1-pixels will be present in the image), an *erosion* ($A \ominus B$) shrinks objects (the number of 1-pixels in the image decreases) (see figure 2).

The basic morphological operators on sets A and B are defined as:

$$\begin{array}{ll} dilation: & A \oplus B = \bigcup_{b \in B} T_{-b}(A) \\ erosion: & A \ominus B = \bigcap_{b \in B} T_{-b}(A) \end{array} \quad (1)$$

with $T_{-b}(A)$ the translation of image A over vector $-b$.

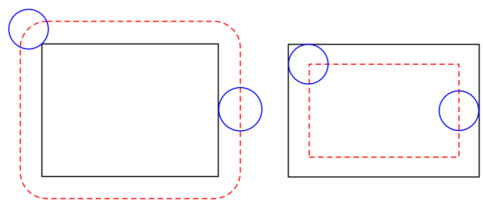


Fig. 2. Schematic example of the basic morphological operators. Solid line: original object; Dashed line: result object; Circle: struct. element. Left: *dilation*; Right: *erosion*.

The structuring element B can be of any size or shape (square, cross, disk, line, …). The choice of this element is based on the content of the image and on the purpose of the morphological operation.

The *hit-miss transform* is a morphological method used in our algorithm. With this transform it is possible to detect specific shapes of corners in the image. Its formula is:

$$A \otimes (B, C) = (A \ominus B) \cap (A^c \ominus C) \qquad (2)$$

with A^c the complement of A (1 becomes 0 and vice versa).

III. Methodology

The purpose of *mmINT* is to remove the jagged edges from a pixel replicated magnified image, by changing specific object pixels to background and vice versa.

mmINT is an iterative procedure that stops when all possible changes are made. It is possible to stop at a lower step of the iteration to reduce the calculation time, but then lines with angles near $0°$ and $90°$ will not be completely smoothed.

The hit-miss transform is used to detect corners in the magnified image that are possibly jaggies. This is done for both the white and the black pixels. For every iteration step, different structuring elements are used.

Not all found corners need to be changed, some are real corners from image objects, not an artifact of the pixel replication. A real corner has no complementary (opposite colour) corner in its neighbourhood.

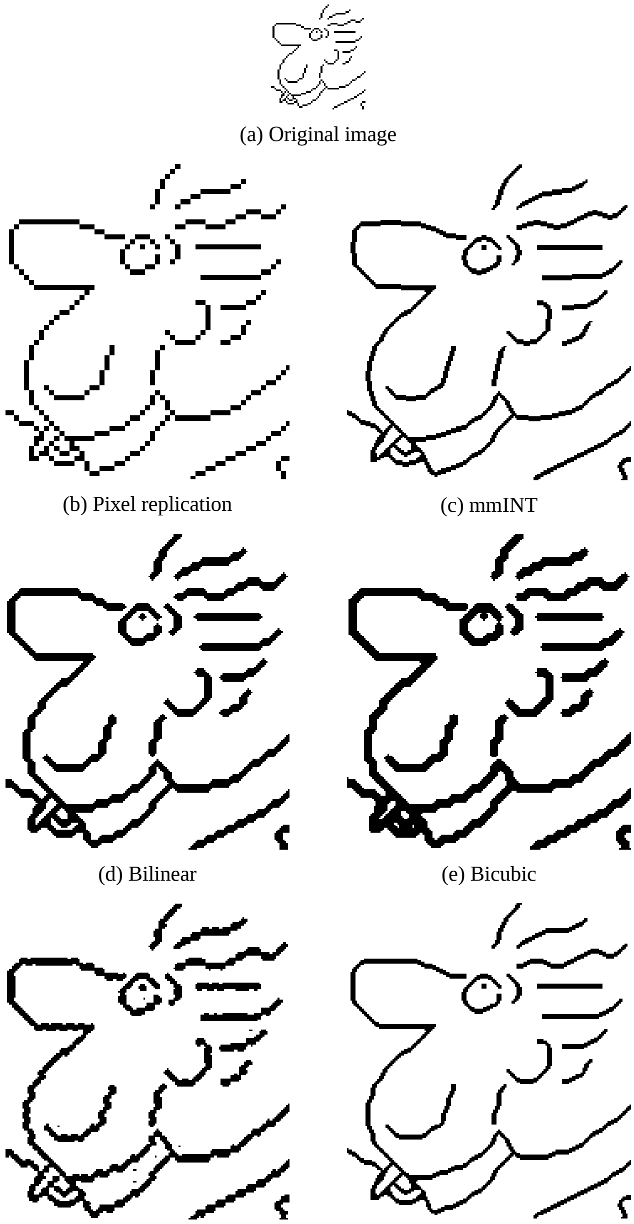
We then change the values of the found corners and surrounding neighbours with a morphological dilation, with the structuring element depending on the magnification used, the current iteration step and of course the orientation of the corner.

This procedure is repeated, using different structuring elements, until no more jaggies are detected.

IV. Results

We will compare our technique with the method of [7] and others [1]. As these methods produce a grayscale image, and our technique *mmINT* only produces a binary result, binarisation is needed in order to compare. We use the well known *Otsu threshold* method [8] to define the ideal gray value for the threshold.

As can be seen in the results in figure 3 for a three times magnified binary image, our technique is visually better than other methods. The method HQ [7] also produces good results, but still more jaggies are visible. The contours in the figure interpolated with *mmINT* are smoother.



(a) Original image

(b) Pixel replication (c) mmINT

(d) Bilinear (e) Bicubic

(f) sinc (Blackman-Harris) (g) hq

Fig. 3. Interpolation results with $3\times$ magnification.

V. Conclusions

Our interpolation method *mmINT* is a technique based on mathematical morphology that works very well on binary images, like logos, text and cartoons. We have shown that it outperforms existing techniques.

In the future we plan to extend this method to grayscale and colour images.

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