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Estimation and detection from coded signals

Joint research :
- UGent (N. Noels, H. Wymeersch, H. Steendam, M. Moeneclaey)
- UCL (C. Herzet, L. Vandendorpe)

Presented by Marc Moeneclaey, UGent - TELIN dept.
Marc.Moeneclaey@telin.UGent.be

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Outline

- Coding + linear modulation,
  AWGN channel, unknown parameter θ (carrier phase, ...)
- MAP detection of information bits
  * θ known
  * θ unknown
- Low-complexity alternatives to MAP detection for unknown θ
  * iterative ML estimation of θ (EM algorithm)
  * simplified sum-product algorithm
- Numerical results
- Conclusions

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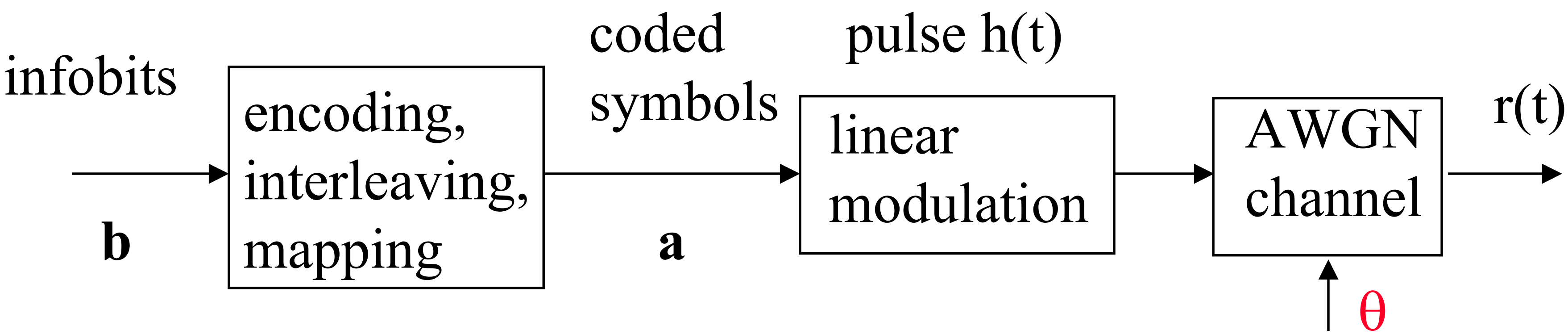
Coding + linear modulation, AWGN channel



$\mathbf{b} \rightarrow \mathbf{a} = \chi(\mathbf{b})$: turbo code, LDPC code, BICM, ...

h(t) : square-root Nyquist pulse

θ : unknown parameter (carrier phase, time delay, freq. offset, gain)

received signal $\quad r(t) = e^{j\theta} \sum_{k} a_k h(t - kT) + w(t)$

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MAP detection, θ known

MAP detection (on individual infobits) achieves minimum BER

$$\hat{b}_k = \arg \max_{b_k \in \{0,1\}} p(b_k \mid \mathbf{r})$$

$\mathbf{r}$: vector representation of r(t)

Assuming $\theta = \theta_0$ is known, bit APP $p(b_k|\mathbf{r})$ is given by

$$p(b_k \mid \mathbf{r}) \propto \sum_{\mathbf{b}:b_k} p(\mathbf{r} \mid \mathbf{a} = \chi(\mathbf{b}), \theta_0)$$

(many terms !)

Efficient computation of APPs : *sum-product (SP) algorithm + message-passing in factor graph (FG)*)

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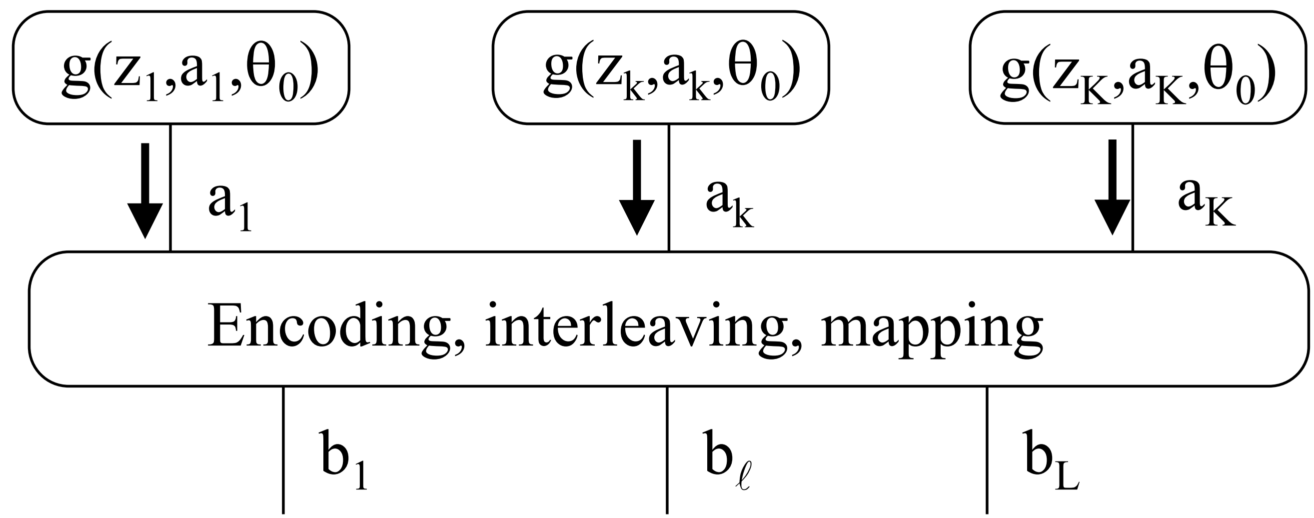
Factor graph for known θ



$\downarrow$ $g(z_k,a_k,\theta_0)$ channel observation

$$g(z_k, a_k, \theta_0) = \exp\left(\frac{2}{N_0}\mathrm{Re}\left[a_k^* z_k e^{-j\theta}\right]\right)$$

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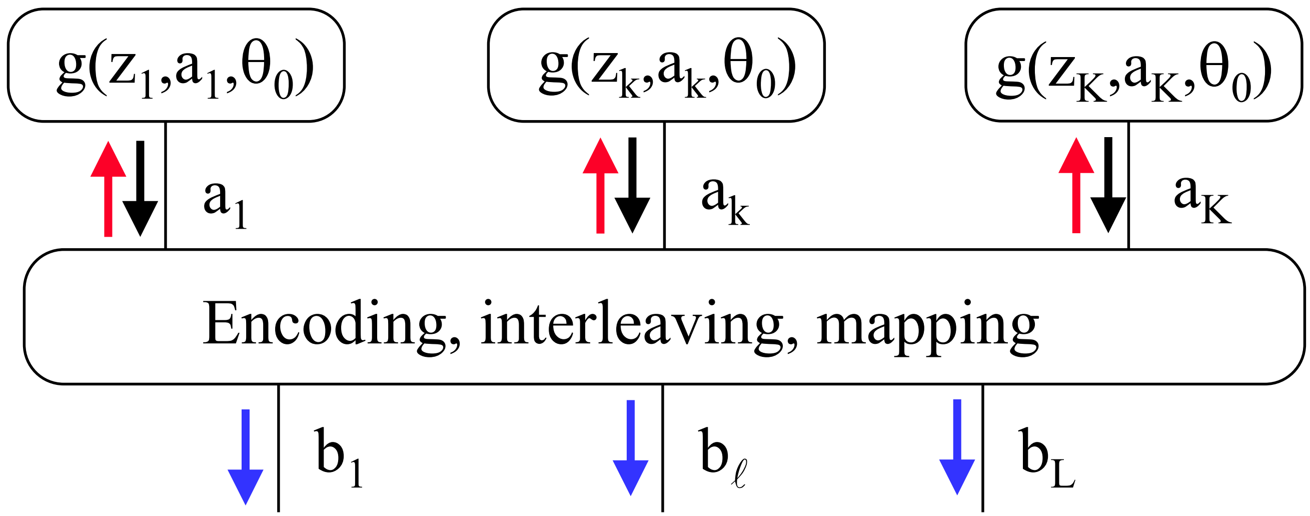
Factor graph for known θ



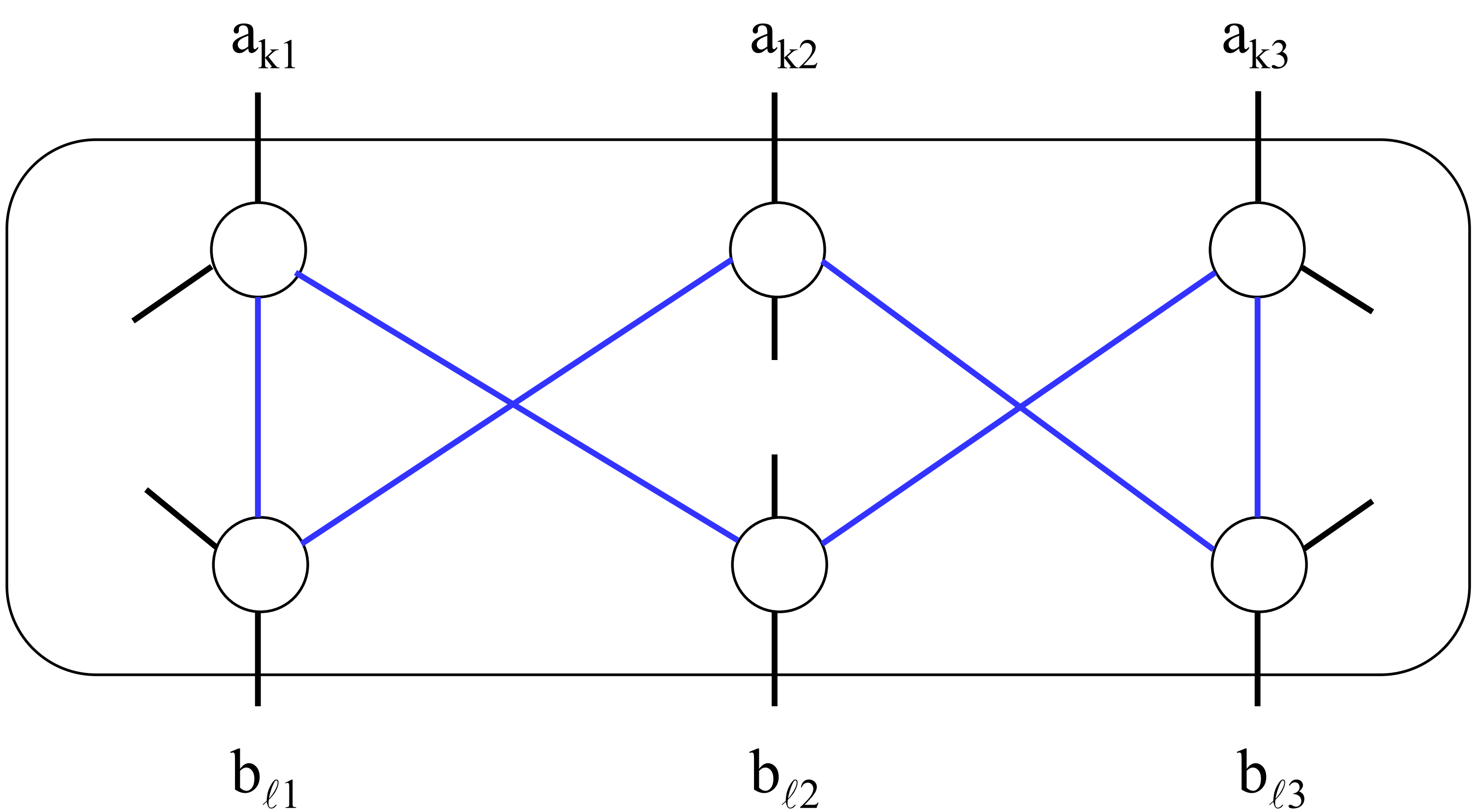
$g(z_k,a_k,\theta_0)$ channel observation $$g(z_k,a_k,\theta_0)=\exp\left(\frac{2}{N_0}\mathrm{Re}\left[a_k^* z_k e^{-j\theta}\right]\right)$$

$p_e(a_k)$ extrinsic information on a_k

$p(b_\ell|\mathbf{r})$ information bit APP

Factor graph for known θ

Because of cycles in FG, MAP detection is *iterative*



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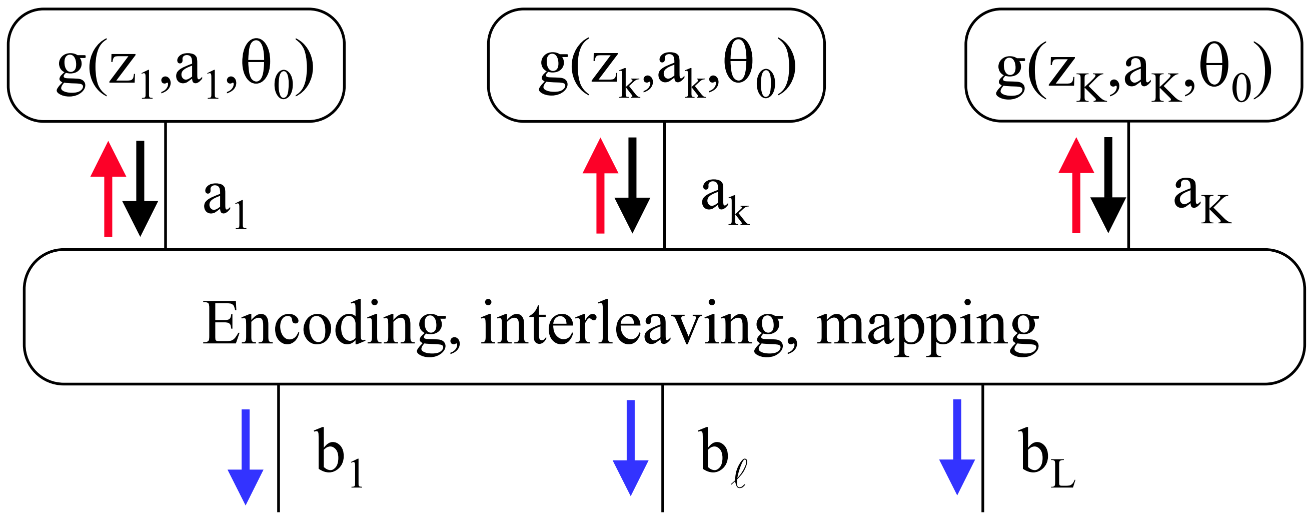
Factor graph, θ known



↓ $g(z_k,a_k,\theta_0)$ channel observation $\quad g(z_k,a_k,\theta_0) = \exp\left(\frac{2}{N_0}\mathrm{Re}\left[a_k^* z_k e^{-j\theta}\right]\right)$

↑ $p_e^{(n)}(a_k)$ extrinsic information on a_k

↓ $p^{(n)}(b_\ell|\mathbf{r})$ information bit APP

n : iteration index of MAP detector

iterate until convergence

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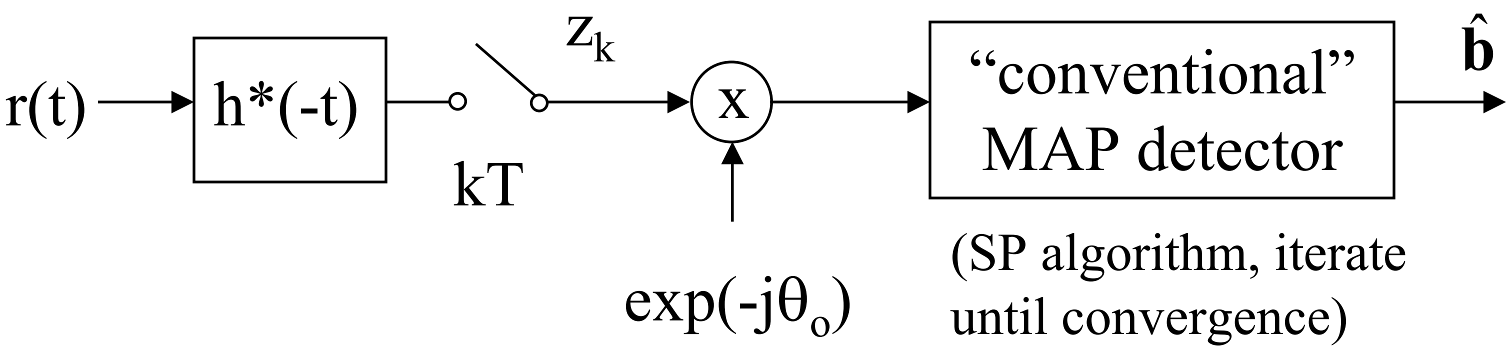
Receiver structure, θ known



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MAP detection, θ unknown

When θ is unknown (uniform distrib.), bit APP is given by

$$p(b_k \mid \mathbf{r}) \propto \sum_{\mathbf{b}:b_k} \left(\int p(\mathbf{r} \mid \mathbf{a} = \chi(\mathbf{b}), \theta) d\theta \right)$$

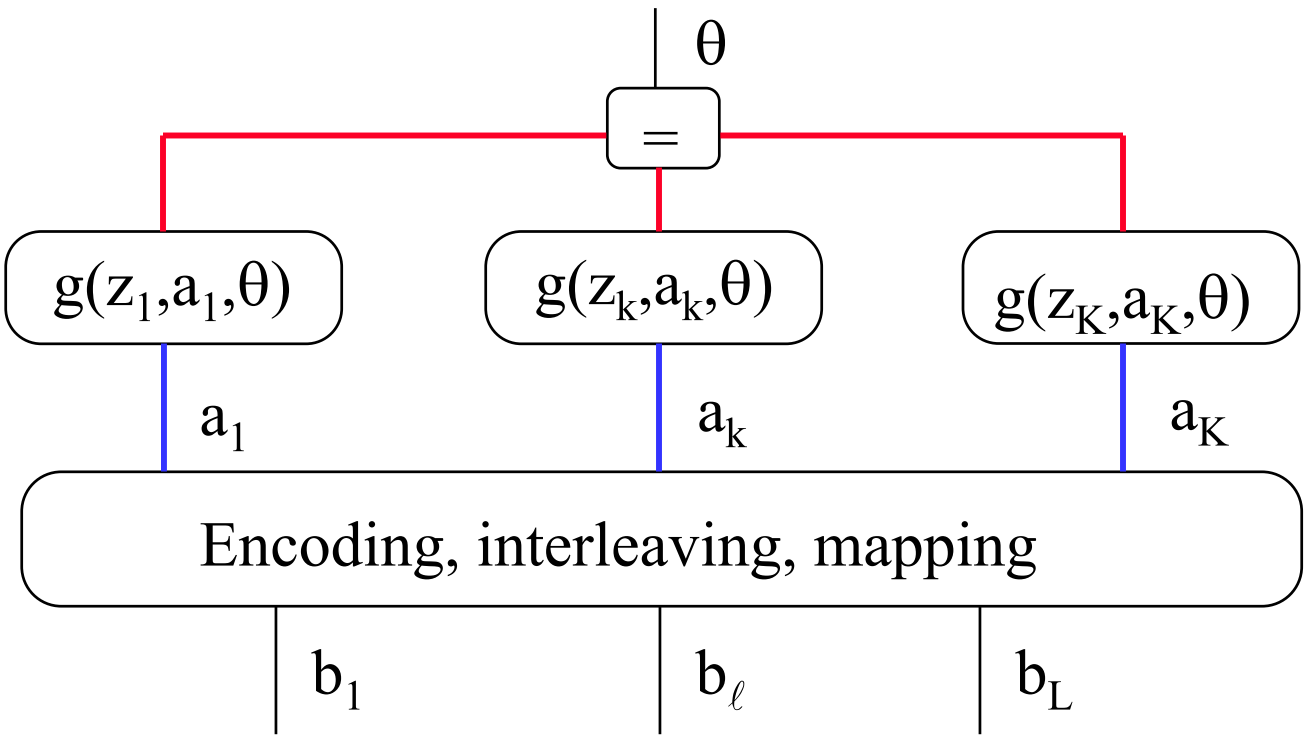
≠ conventional MAP detector

hard to compute, because of integration over θ

Factor graph, θ unknown

messages are functions of θ

downward messages involve integration over θ



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Alternatives to exact MAP detection when θ is unknown

Alternative 1 : ML parameter estimation
Provide estimate of θ to conventional MAP detector

$$p(b_k \mid \mathbf{r}) = \int p(b_k \mid \mathbf{r}, \theta) p(\theta \mid \mathbf{r}) d\theta$$
$$\approx p(b_k \mid \mathbf{r}, \hat{\theta}(\mathbf{r}))$$

$$p(\theta \mid \mathbf{r}) \approx \delta(\theta - \hat{\theta}(\mathbf{r}))$$

Alternative 2 : Simplified sum-product algorithm
Approximation of θ-dependent messages

Alternative 1 :
ML parameter estimation

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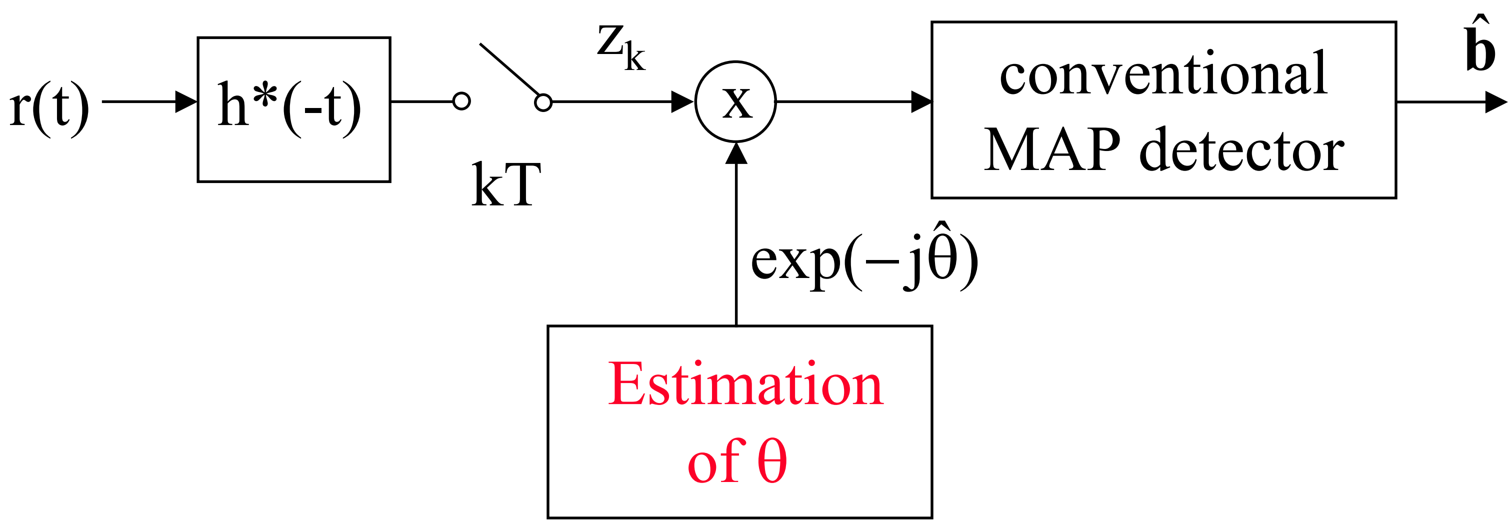
Receiver structure when estimating θ

Strategy : use conventional MAP detector, but provide *estimate* (instead of correct value) of θ



ML parameter estimation

ML estimation of θ : $\hat{\theta}_{ML} = \arg\max_{\theta} \; p(\mathbf{r} \mid \theta)$

For long observation intervals, mean-square error (MSE) of ML estimate converges to Cramer-Rao lower bound (CRB) on MSE

Computation of CRB is hard when data symbols are not a priori known to receiver.

Simpler but less tight bound : modified CRB (MCRB)
(assumes data symbols are known to receiver)

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Cramer-Rao bound

$$\mathrm{CRB}^{-1} = \mathrm{E}_{\mathbf{r}}\left[\left(\frac{\partial}{\partial\theta}\ln \mathrm{p}(\mathbf{r};\theta)\right)^2\right] = -\mathrm{E}_{\mathbf{r}}\left[\frac{\partial^2}{\partial\theta^2}\ln \mathrm{p}(\mathbf{r};\theta)\right]$$

$$\mathrm{p}(\mathbf{r};\theta) = \sum_{\mathbf{a}} \mathrm{p}(\mathbf{r}\,|\,\mathbf{a};\theta)\mathrm{p}(\mathbf{a}) = \mathrm{E}_{\mathbf{a}}\left[\mathrm{p}(\mathbf{r}\,|\,\mathbf{a};\theta)\right]$$

average over (coded) symbol sequence **a** (*many terms !!*)

analytical difficulty : $\mathrm{E}_{\mathbf{a}}[.]$ "inside" ln(.), so that $\ln(\mathrm{p}(\mathbf{r};\theta))$ is not a simple function of **r** and θ

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Modified Cramer-Rao bound

$$\mathrm{MCRB}^{-1} = \mathrm{E}_{\mathbf{r},\mathbf{a}}\left[\left(\frac{\partial}{\partial\theta}\ln \mathrm{p}(\mathbf{r}\,|\,\mathbf{a};\theta)\right)^2\right] = -\mathrm{E}_{\mathbf{r},\mathbf{a}}\left[\frac{\partial^2}{\partial\theta^2}\ln \mathrm{p}(\mathbf{r}\,|\,\mathbf{a};\theta)\right]$$

$\mathrm{E}_{\mathbf{r},\mathbf{a}}[.]$ "outside" ln(.) $\Rightarrow$ MCRB easier to evaluate than CRB,
when $\ln(\mathrm{p}(\mathbf{r}|\mathbf{a};\theta))$ is simple function of **a** and **r**
(e.g., linear modulation on AWGN channel)

MCRB for phase estimation

$$\mathbf{r} = \mathbf{a}\exp(j\theta) + \mathbf{w} \qquad E[\mathbf{w}\mathbf{w}^T] = \frac{N_0}{2}\mathbf{I}$$

$$\ln p(\mathbf{r}\,|\,\mathbf{a};\theta) = \frac{2}{N_0}\sum_k \mathrm{Re}[a_k^* r_k \exp(-j\theta)]$$

$$\frac{\partial^2}{\partial\theta^2}\ln p(\mathbf{r}\,|\,\mathbf{a};\theta) = \frac{-2}{N_0}\sum_k \mathrm{Re}[a_k^* r_k \exp(-j\theta)]$$

$$\mathrm{MCRB}^{-1} = -E_{\mathbf{r},\mathbf{a}}\left[\frac{\partial^2}{\partial\theta^2}\ln p(\mathbf{r}\,|\,\mathbf{a};\theta)\right] = \frac{2}{N_0}\sum_k E_{a_k}\left[|\,a_k\,|^2\right] = \frac{2E_sK}{N_0}$$

CRB for phase estimation : uncoded transmission

$$p(\mathbf{a}) = \prod_k p(a_k) \qquad \text{(i.i.d. symbols)}$$

$$\Rightarrow \quad p(\mathbf{r};\theta) = \prod_k p(r_k;\theta) = \prod_k E_{a_k}[p(r_k \mid a_k;\theta)]$$

$$CRB^{-1} = K \cdot E_{r_k}\left[\left(\frac{\partial}{\partial\theta}\ln\left(E_{a_k}[p(r_k \mid a_k;\theta)]\right)\right)^2\right]$$

$E_{a_k}[\cdot]$: 1-D summation

$E_{r_k}[\cdot]$: 1-D numerical integration

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CRB for phase estimation : coded transmission

$$\frac{\partial}{\partial\theta}\ln p(\mathbf{r};\theta) = \sum_{\mathbf{a}}\left(\frac{\partial}{\partial\theta}\ln p(\mathbf{r}\,|\,\mathbf{a};\theta)\right)p(\mathbf{a}\,|\,\mathbf{r};\theta)$$

$$\mathbf{r} = \mathbf{a}\exp(j\theta) + \mathbf{w} \quad \Rightarrow \quad \frac{\partial}{\partial\theta}\ln p(\mathbf{r}\,|\,\mathbf{a};\theta) = \frac{2}{N_0}\sum_k \mathrm{Im}[a_k^* r_k \exp(-j\theta)]$$

$$\frac{\partial}{\partial\theta}\ln p(\mathbf{r};\theta) = \frac{2}{N_0}\sum_k \mathrm{Im}[\tilde{a}_k^*(\mathbf{r};\theta) r_k \exp(-j\theta)]$$

$$\tilde{a}_k(\mathbf{r};\theta) = \sum_{\mathbf{a}} a_k p(\mathbf{a}\,|\,\mathbf{r};\theta) = \sum_{a_k} a_k p(a_k\,|\,\mathbf{r};\theta)$$

marginal symbol APPs (from MAP detector operating on **r**)

$$\mathrm{CRB}^{-1} = E_{\mathbf{r}}\left[\left(\frac{\partial}{\partial\theta}\ln p(\mathbf{r};\theta)\right)^2\right]$$

$E_{a_k|\mathbf{r}}[\cdot]$: 1-D summation per symbol

$E_{\mathbf{r}}[\cdot]$: Monte Carlo simulation

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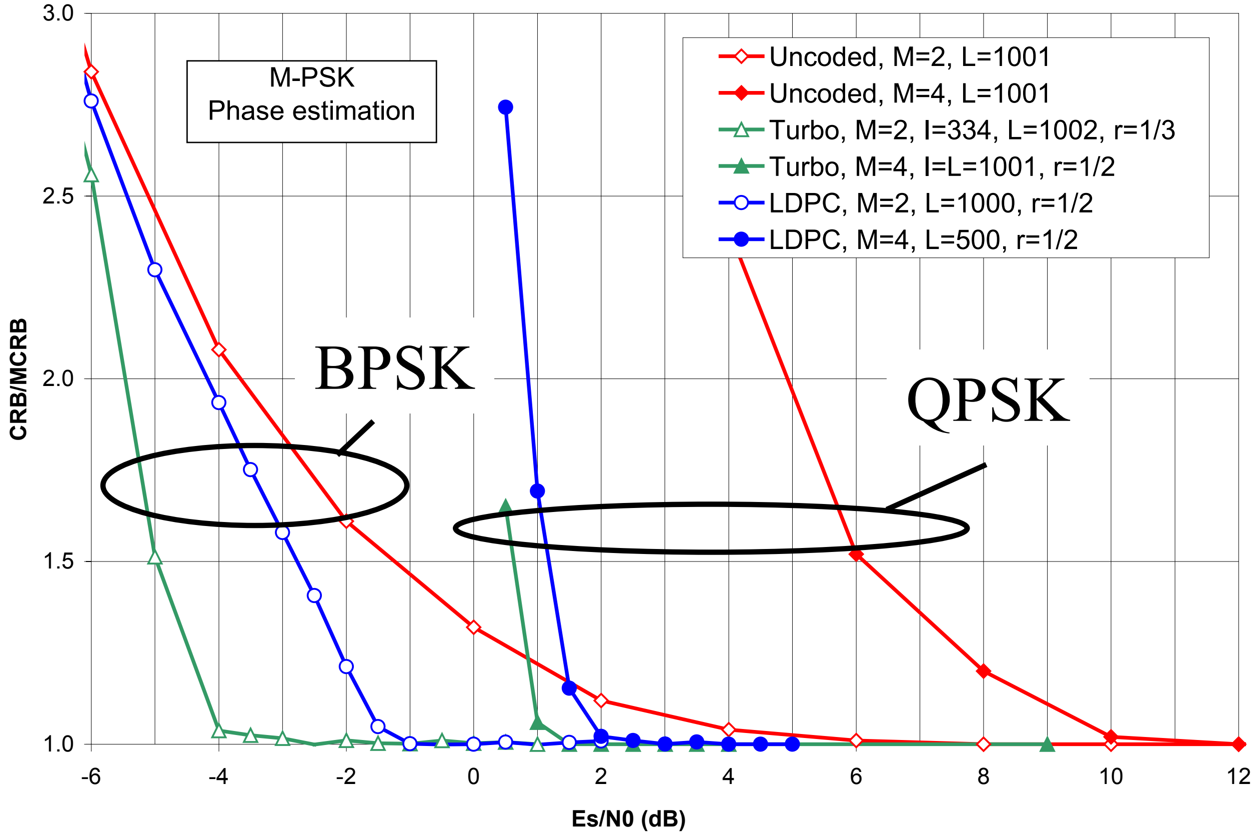
CRB, MCRB : numerical results



$$\mathrm{MCRB} = \frac{N_0}{2LE_s}$$

(phase estimation)

Large SNR : CRB → MCRB

Small SNR : CRBuncoded > CRBcoded > MCRB

⇒ code properties should be exploited during estimation

Computation of ML estimate

Direct application of ML estimation is complicated :

$$p(\mathbf{r} \mid \theta) \propto \sum_{\mathbf{b}} p(\mathbf{r} \mid \mathbf{a} = \chi(\mathbf{b}), \theta) \qquad \text{many terms !}$$

$\Rightarrow$ compute ML estimate *iterative*ly (EM algorithm) : $\hat{\theta}_{ML} = \lim_{i \to \infty} \hat{\theta}^{(i)}$

$$\hat{\theta}^{(i+1)} = \arg\max_{\theta} E\left[p(\mathbf{r} \mid \mathbf{a}, \theta) \mid \mathbf{r}, \hat{\theta}^{(i)}\right] = \arg\left(\sum_k \tilde{a}_k^{(i)*} z_k\right)$$

$\tilde{a}_k^{(i)} = E[a_k \mid \mathbf{r}, \hat{\theta}^{(i)}]$ soft decision (SD) on symbols

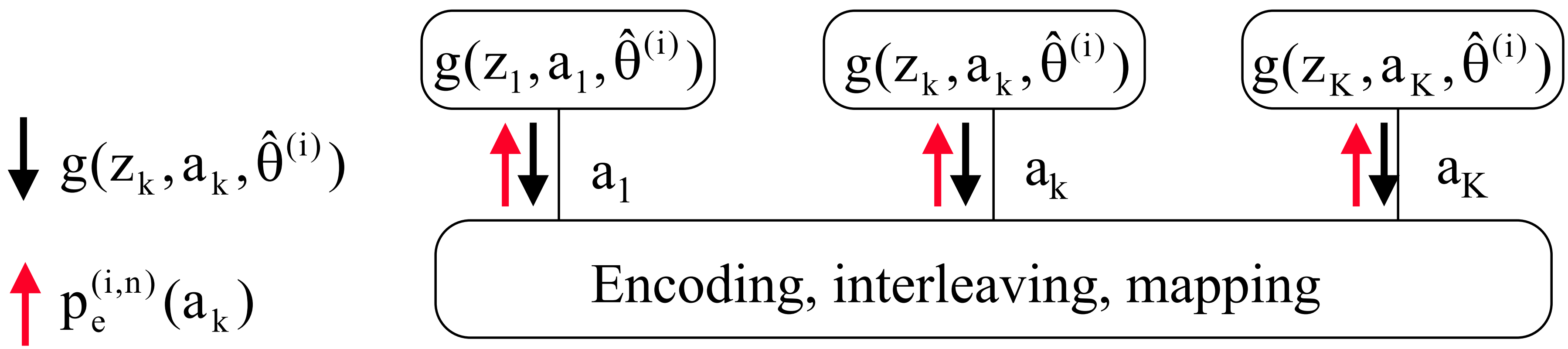
symbol APPs computed by *MAP detector*

EM algorithm : symbol APP computation

outer iteration index i (EM algorithm),
inner iteration index n (MAP detector)

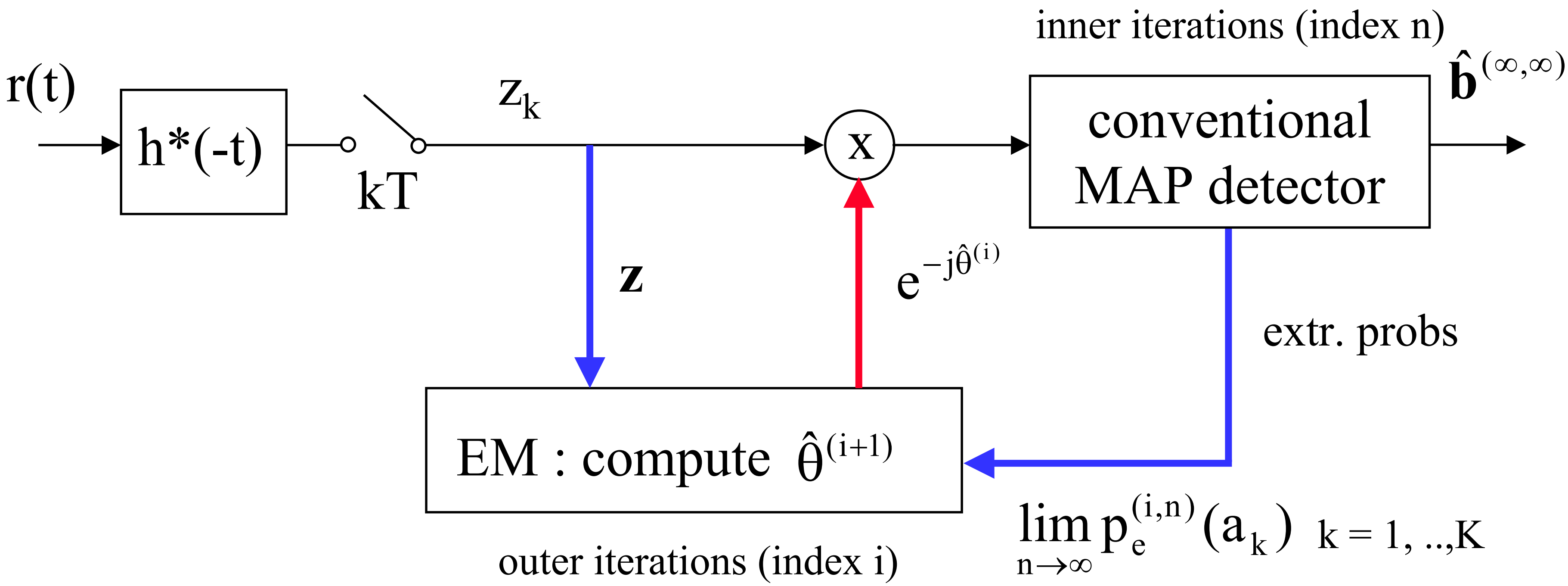
for each i :

MAP detector is reset and iterated *until convergence* ($n \to \infty$)



$$\text{symbol APP : } p(a_k \mid \mathbf{r}, \hat{\theta}^{(i)}) = g(z_k, a_k, \hat{\theta}^{(i)}) p_e^{(i,\infty)}(a_k) \quad \Rightarrow \quad \text{SD } \tilde{a}_k^{(i)}$$

Receiver structure for EM algorithm



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Reduced-complexity EM algorithm

For each EM iteration, MAP detector is iterated till convergence
$\Rightarrow$ computational complexity too high

Solution : "merging" of EM iterations and MAP detector iterations.
For each EM iteration, *only one* MAP detector iteration is performed, without resetting MAP detector
$\Rightarrow$ reduced complexity (at expense of reduced convergence speed)

$$\text{symbol APP}: \; p(a_k \mid \mathbf{r}, \hat{\theta}^{(i)}) \propto g(z_k, a_k, \hat{\theta}^{(i)}) p_e^{(i,1)}(a_k) \quad \Rightarrow \quad \text{SD} \;\; \tilde{a}_k^{(i)}$$

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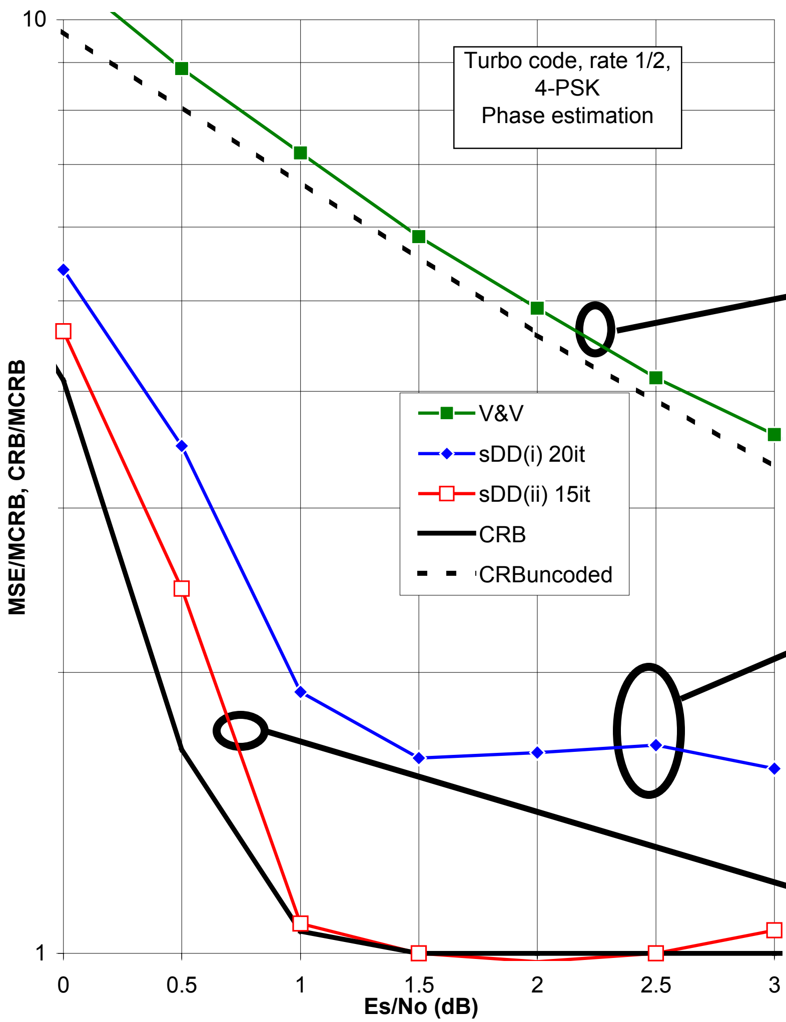
EM algorithm : exploiting code properties



partial (or no) exploitation of code properties $\Rightarrow$ degradation of MSE

does not exploit code properties

uses infobit APPs only, assumes parity bits are uncoded

uses infobit APPs and parity bit APPs

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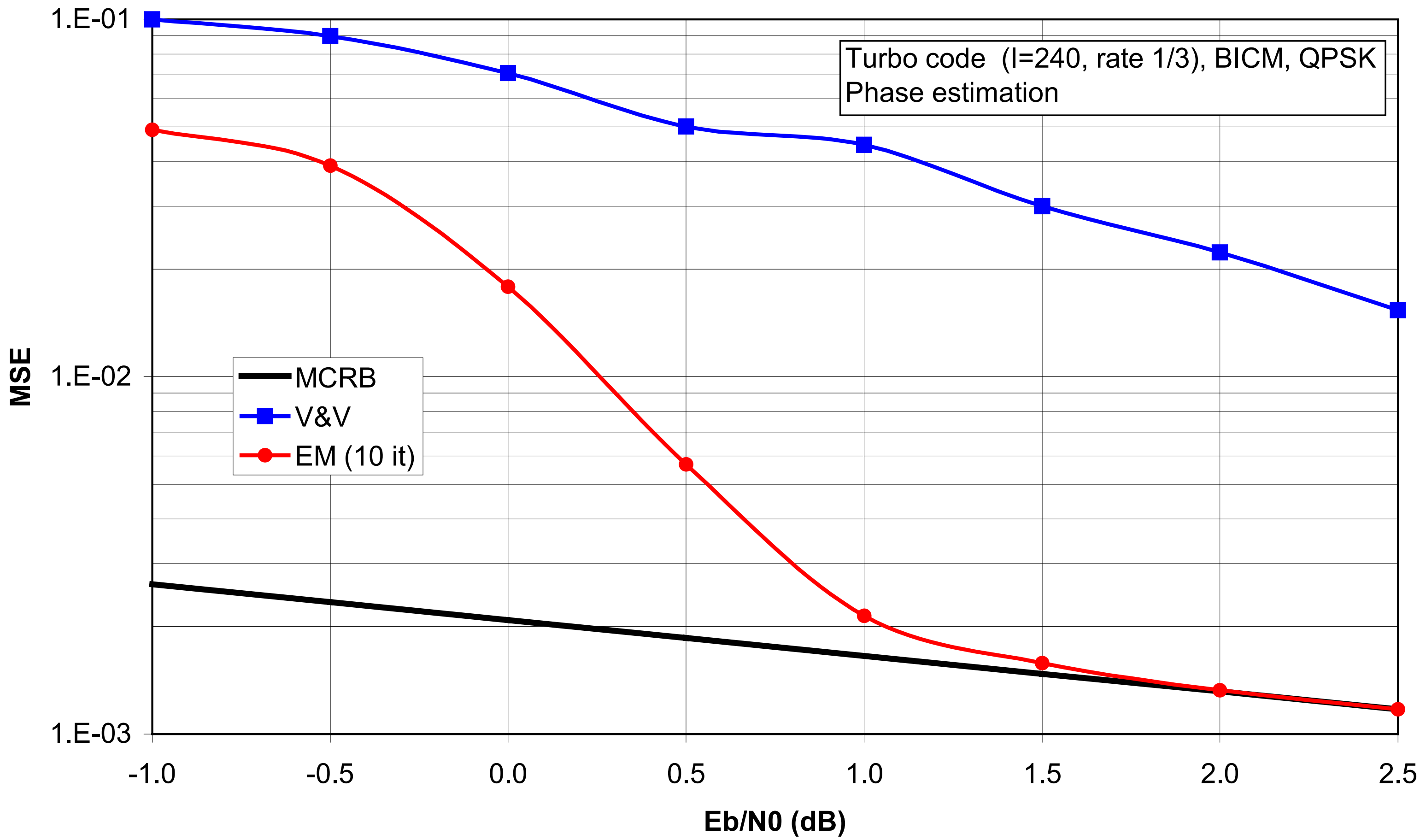
EM algorithm : phase estimation (1/2)



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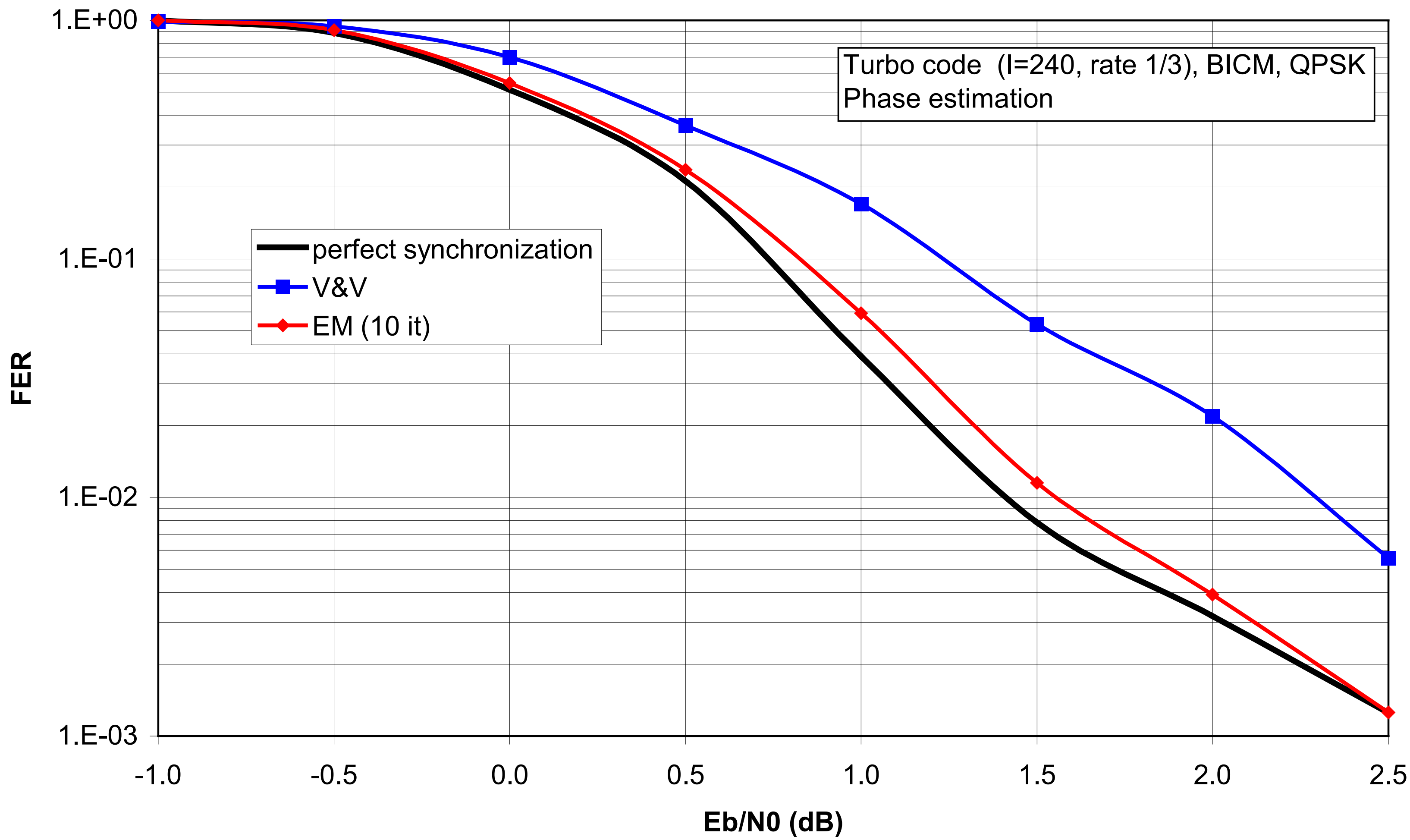
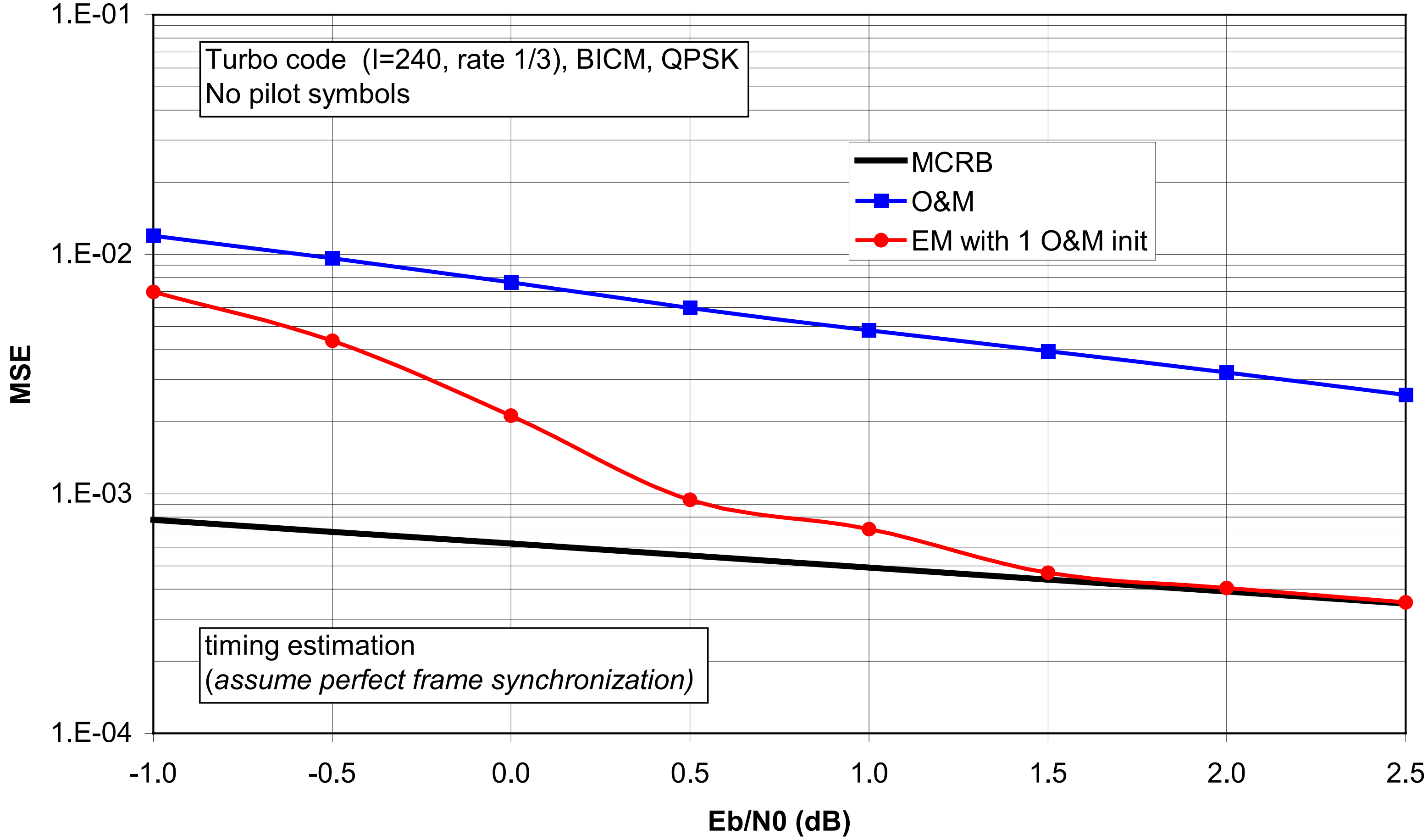
EM algorithm : phase estimation (2/2)



EM algorithm : timing estimation (1/2)



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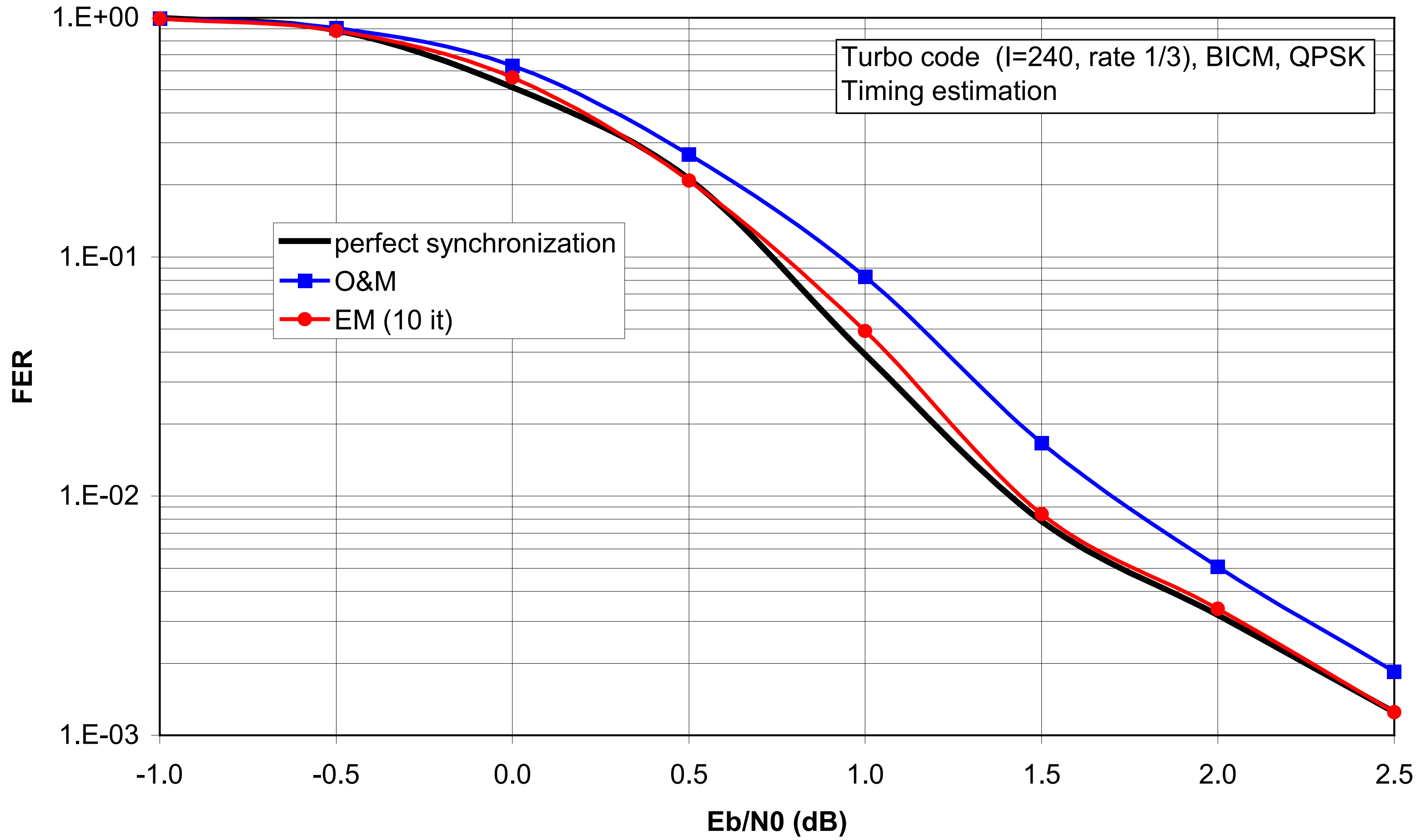
EM algorithm : timing estimation (2/2)



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Alternative 2 :
Simplified sum-product algorithm

Simplified sum-product algorithm

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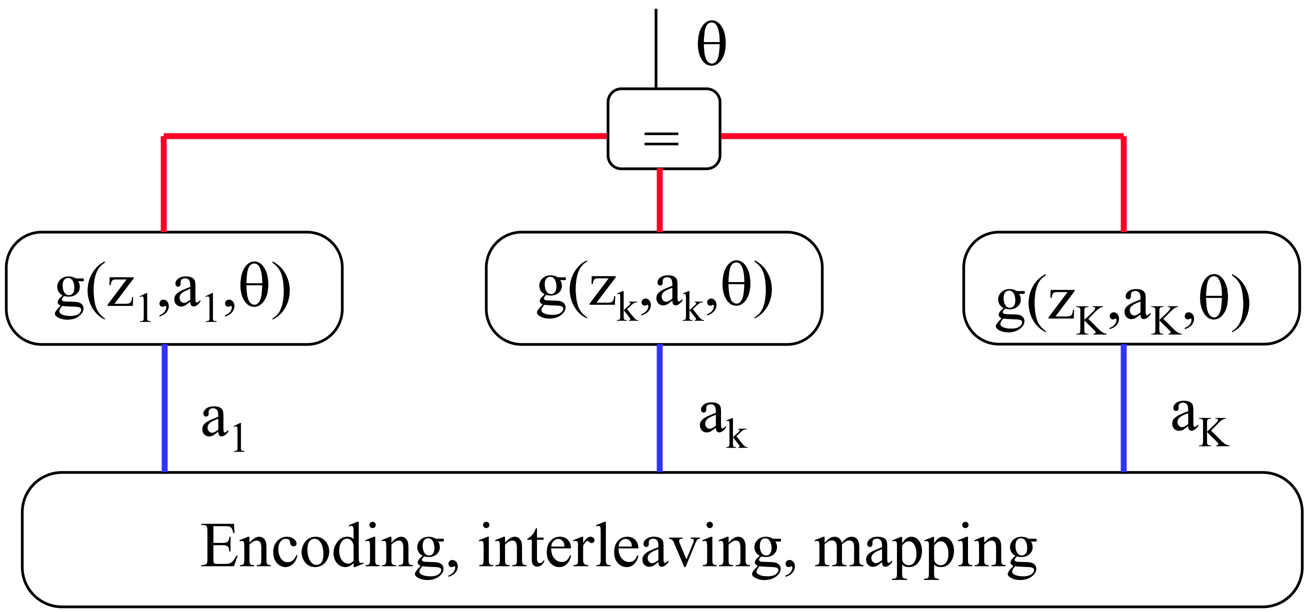
Factor graph corresponding to unknown θ

messages are functions of θ

downward messages involve integration over θ



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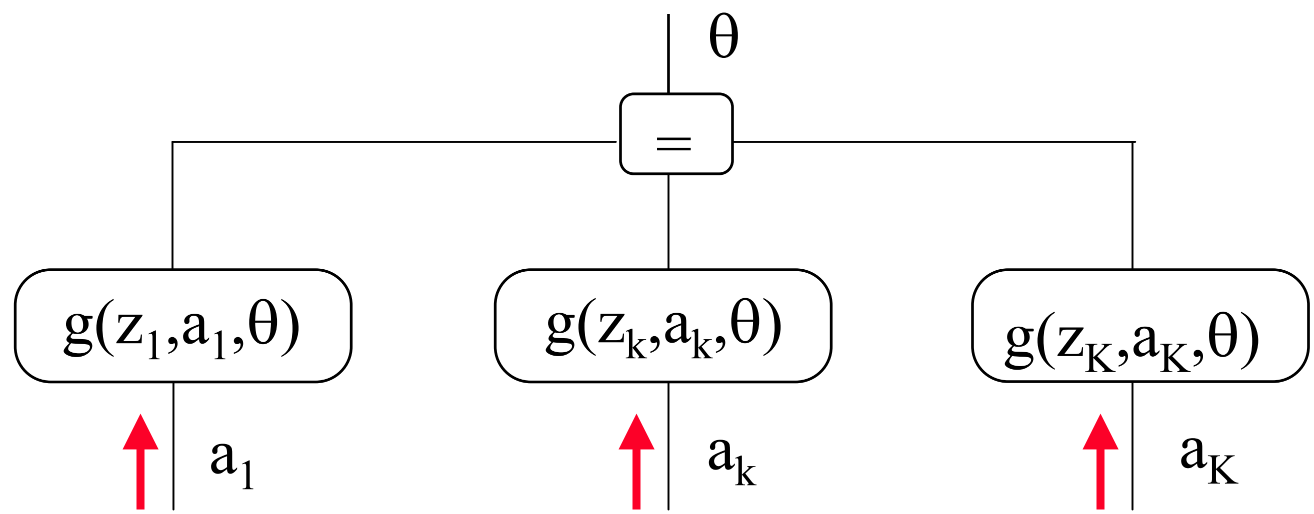
Simplified sum-product algorithm

Messages depending on θ are *approximated* by $\delta(\theta - \hat{\theta}^{(n)})$
(n : iteration index of MAP detector)

$$\hat{\theta}^{(n)} = \arg\max_{\theta} p^{(n)}(\mathbf{r} \mid \theta) = \arg\max_{\theta} \sum_{\mathbf{a}} p(\mathbf{r} \mid \mathbf{a}; \theta) p_e^{(n)}(\mathbf{a})$$

$$p(\mathbf{r} \mid \mathbf{a}; \theta) = \prod_k g(z_k, a_k, \theta) \qquad p_e^{(n)}(\mathbf{a}) = \prod_k p_e^{(n)}(a_k)$$

$\uparrow \; p_e^{(n)}(a_k)$



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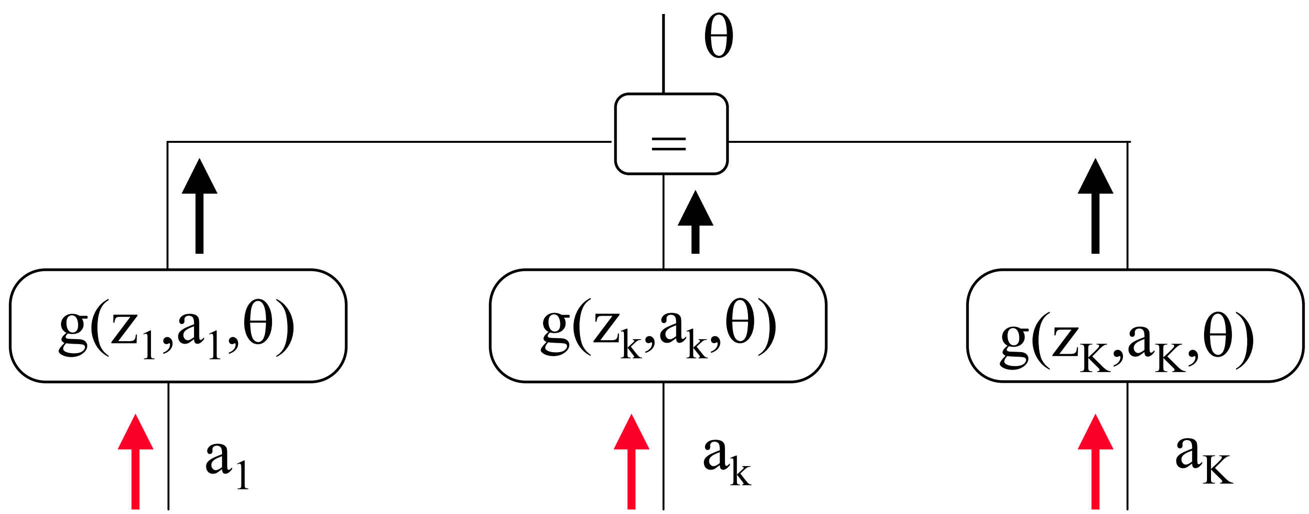
$$p(\mathbf{r}\mid\mathbf{a};\theta) = \prod_k g(z_k, a_k, \theta) \qquad p_e^{(n)}(\mathbf{a}) = \prod_k p_e^{(n)}(a_k)$$

$p_e^{(n)}(a_k)$



Simplified sum-product algorithm

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Messages depending on θ are *approximated* by $\delta(\theta-\hat{\theta}^{(n)})$
(n : iteration index of MAP detector)

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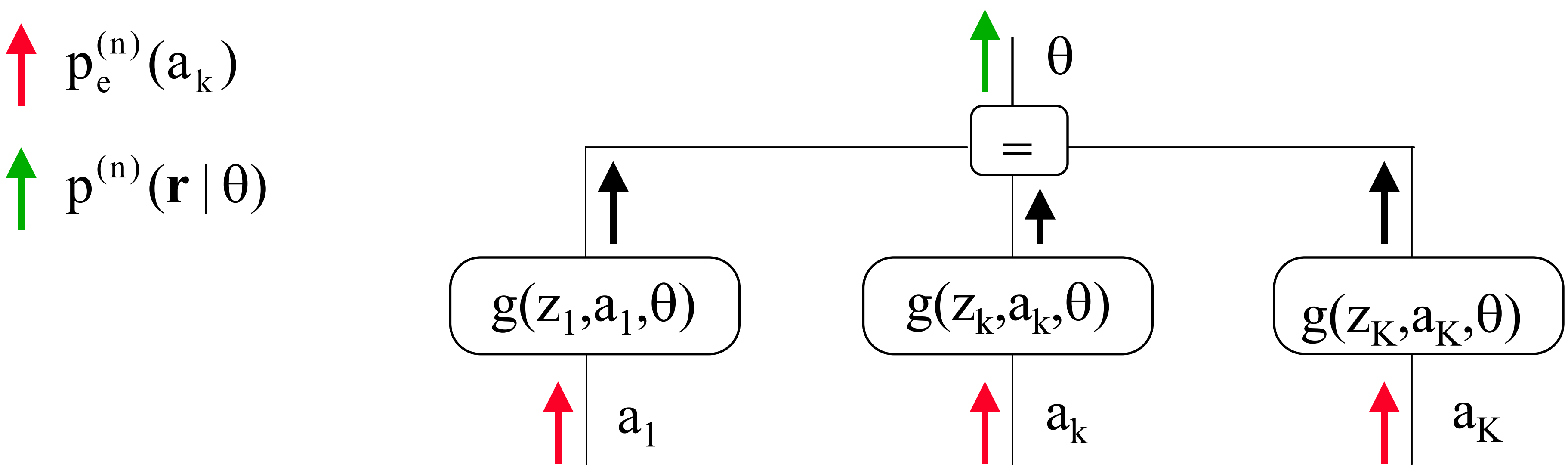
$$p(\mathbf{r}\,|\,\mathbf{a};\theta) = \prod_k g(z_k, a_k, \theta) \qquad p_e^{(n)}(\mathbf{a}) = \prod_k p_e^{(n)}(a_k)$$



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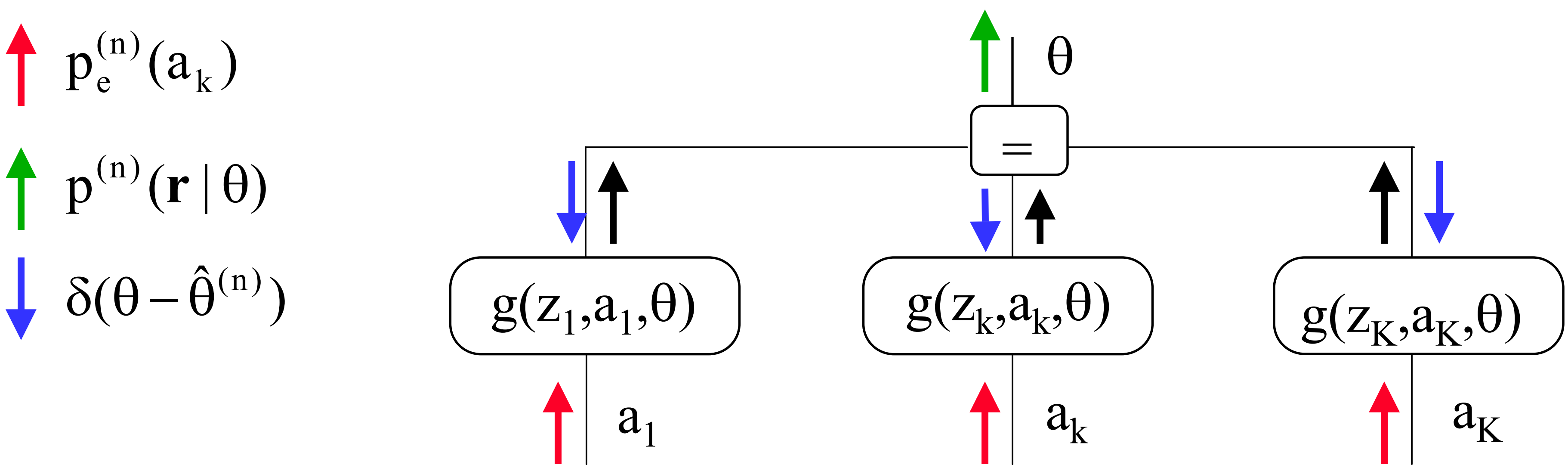
$$p(\mathbf{r}\,|\,\mathbf{a};\theta) = \prod_k g(z_k, a_k, \theta) \qquad p_e^{(n)}(\mathbf{a}) = \prod_k p_e^{(n)}(a_k)$$



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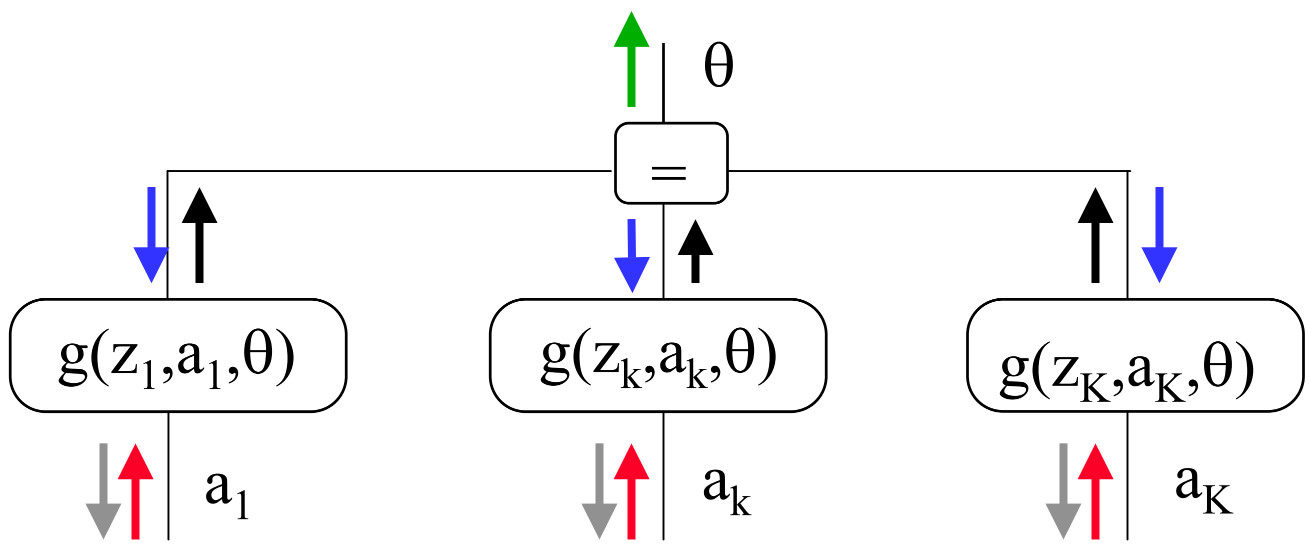
$$p(\mathbf{r} \mid \mathbf{a}; \theta) = \prod_k g(z_k, a_k, \theta) \qquad p_e^{(n)}(\mathbf{a}) = \prod_k p_e^{(n)}(a_k)$$

$p_e^{(n)}(a_k)$

$p^{(n)}(\mathbf{r} \mid \theta)$

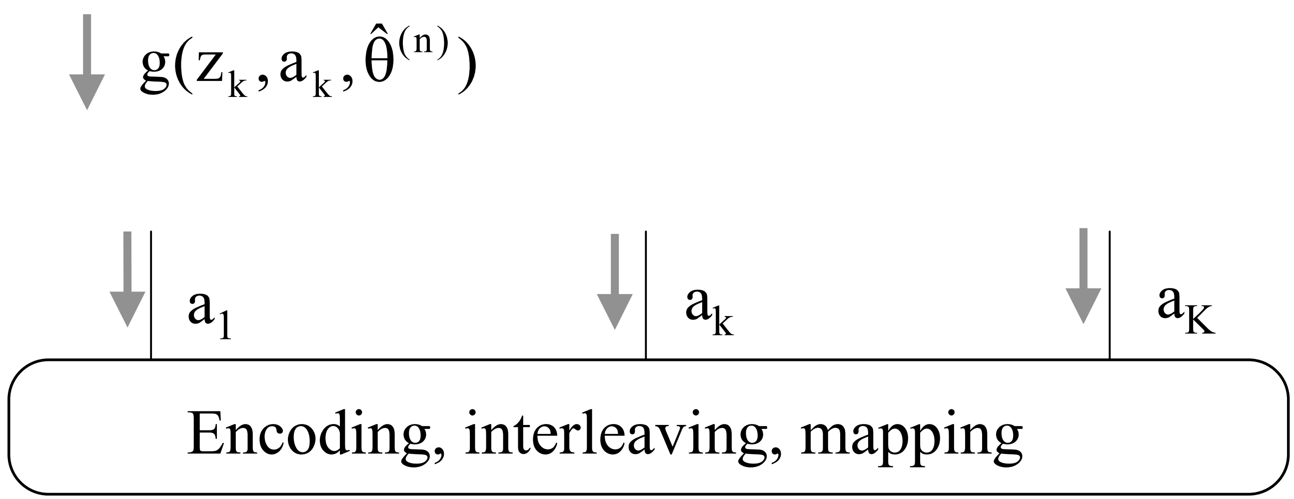
$\delta(\theta - \hat{\theta}^{(n)})$

$g(z_k, a_k, \hat{\theta}^{(n)})$



Simplified sum-product algorithm

(n+1)-th iteration of MAP detector



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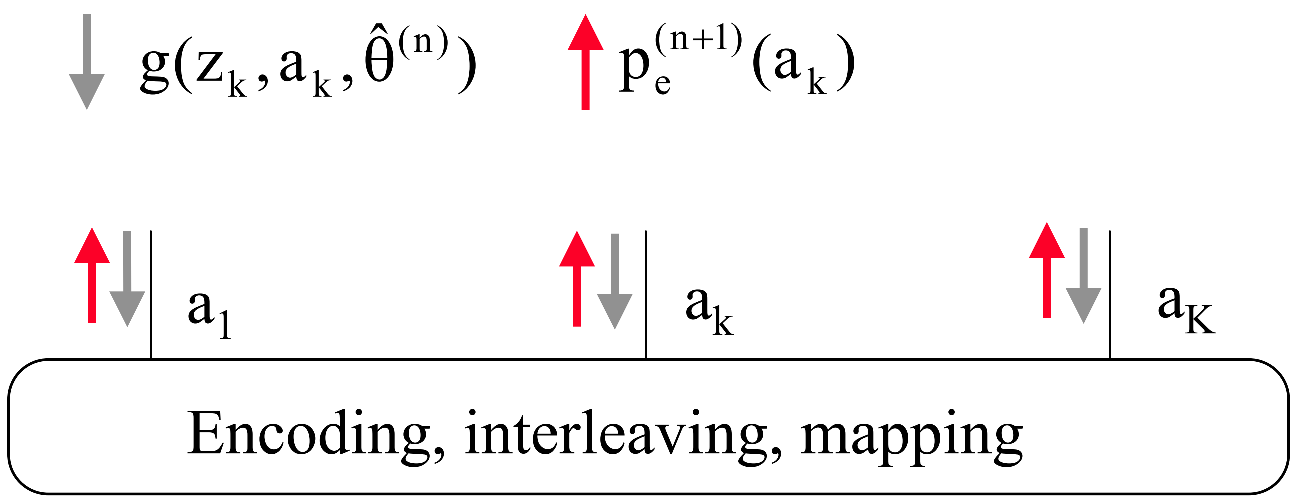
Simplified sum-product algorithm

(n+1)-th iteration of MAP detector



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Simplified sum-product algorithm : maximization of $p^{(n)}(\mathbf{r}|\theta)$

$$\hat{\theta}^{(n)} = \arg\max_{\theta} p^{(n)}(\mathbf{r} \mid \theta) = \arg\max_{\theta} \sum_{\mathbf{a}} p(\mathbf{r} \mid \mathbf{a}; \theta) p_e^{(n)}(\mathbf{a})$$

Similar to ML equation : $p(\mathbf{a})$ is replaced by $p_e^{(n)}(\mathbf{a})$

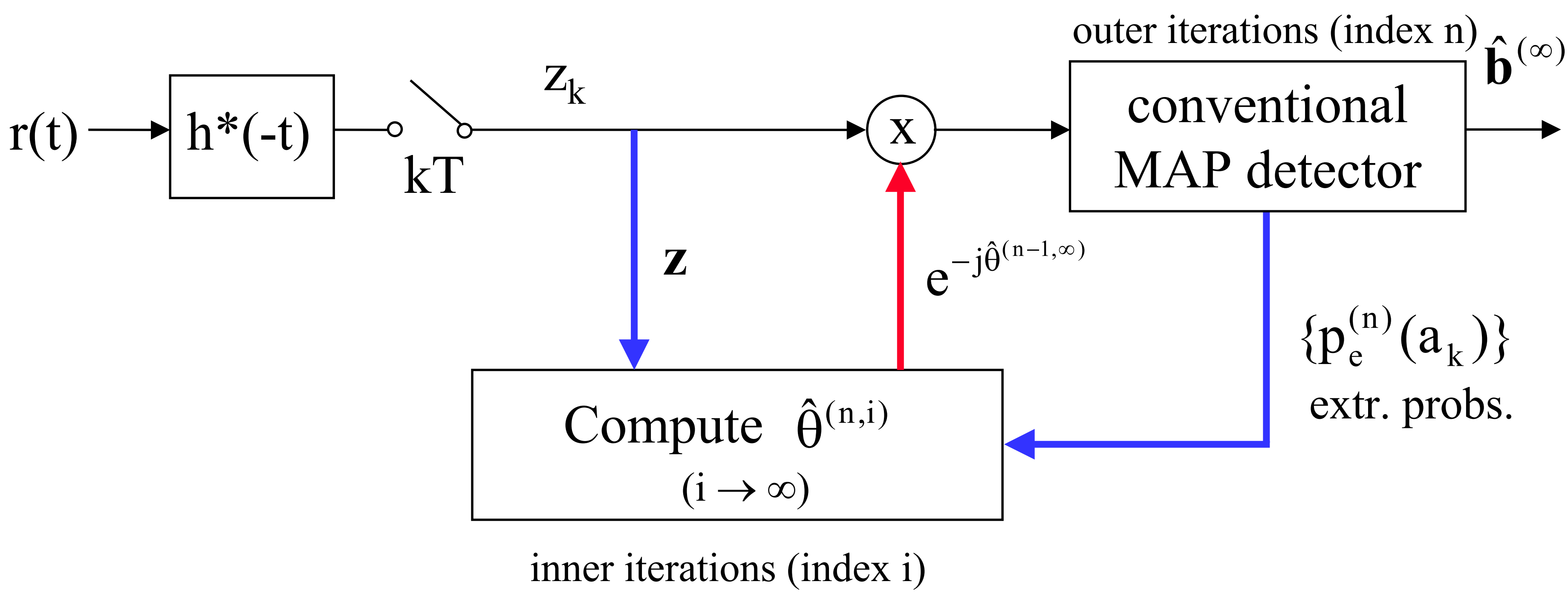
$\Rightarrow$ EM algorithm to maximize $p^{(n)}(\mathbf{r}|\theta)$ iteratively $\quad \hat{\theta}^{(n)} = \lim_{i\to\infty} \hat{\theta}^{(n,i)}$

$$\hat{\theta}^{(n,i+1)} = \arg\max_{\theta} \sum_{k} E[g(z_k, a_k, \theta \mid \mathbf{r}, \hat{\theta}^{(n,i)})] = \arg\left(\sum_{k} \tilde{a}_k^{(n,i)*} z_k \right)$$

symbol APP : $p^{(n)}(a_k \mid \mathbf{r}; \hat{\theta}^{(n,i)}) \propto g(z_k, a_k, \hat{\theta}^{(n,i)}) p_e^{(n)}(a_k) \;\Rightarrow\;$ SD $\tilde{a}_k^{(n,i)}$

Simplified sum-product algorithm : receiver stucture

for each n : iterate EM algorithm until convergence ($i \to \infty$)



symbol APP : $p^{(n)}(a_k \mid \mathbf{r}; \hat{\theta}^{(n,i)}) \propto g(z_k, a_k, \hat{\theta}^{(n,i)}) p_e^{(n)}(a_k) \Rightarrow \text{SD } \tilde{a}_k^{(n,i)}$

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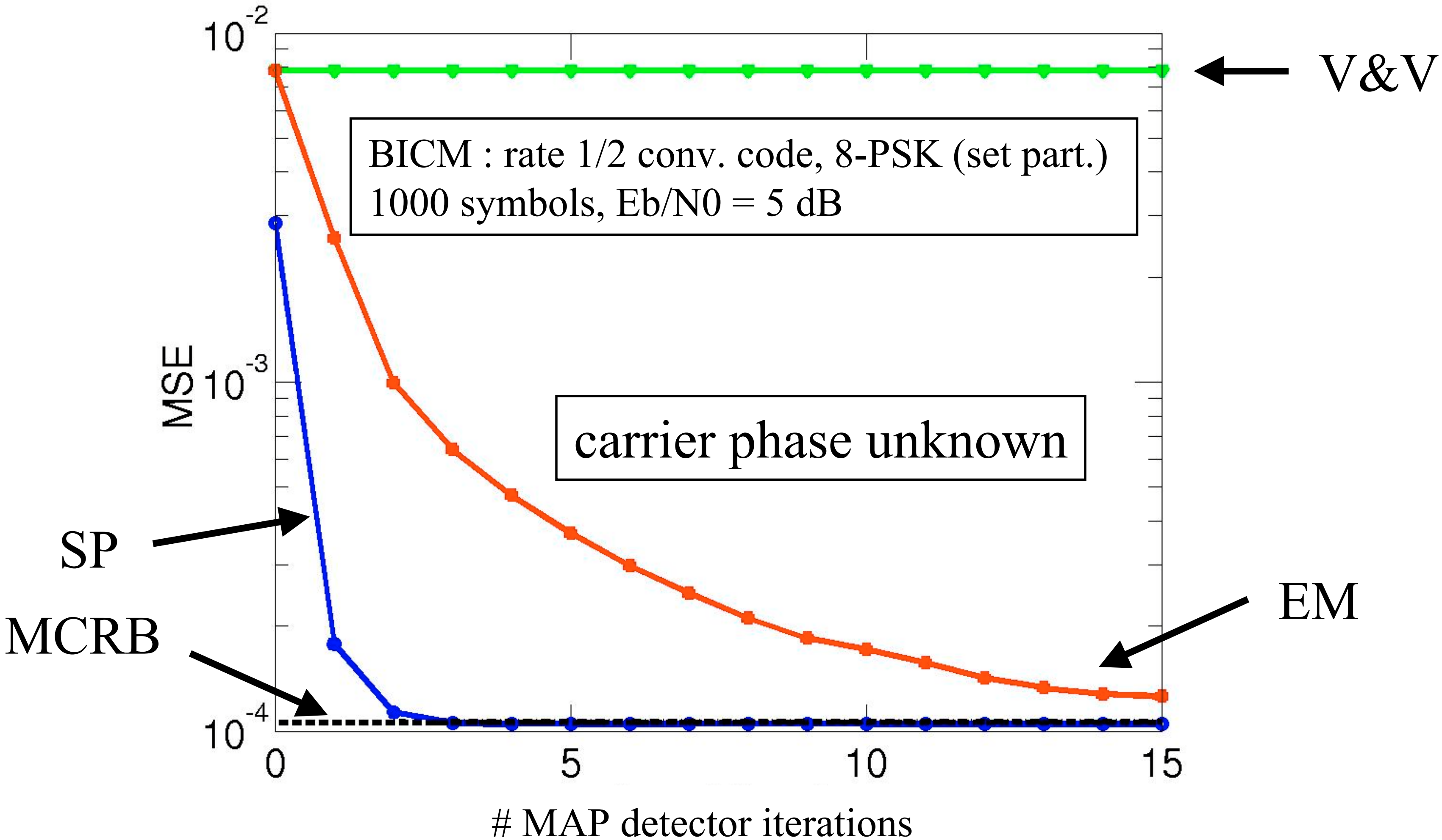
Simplified sum-product algorithm : estimator performance



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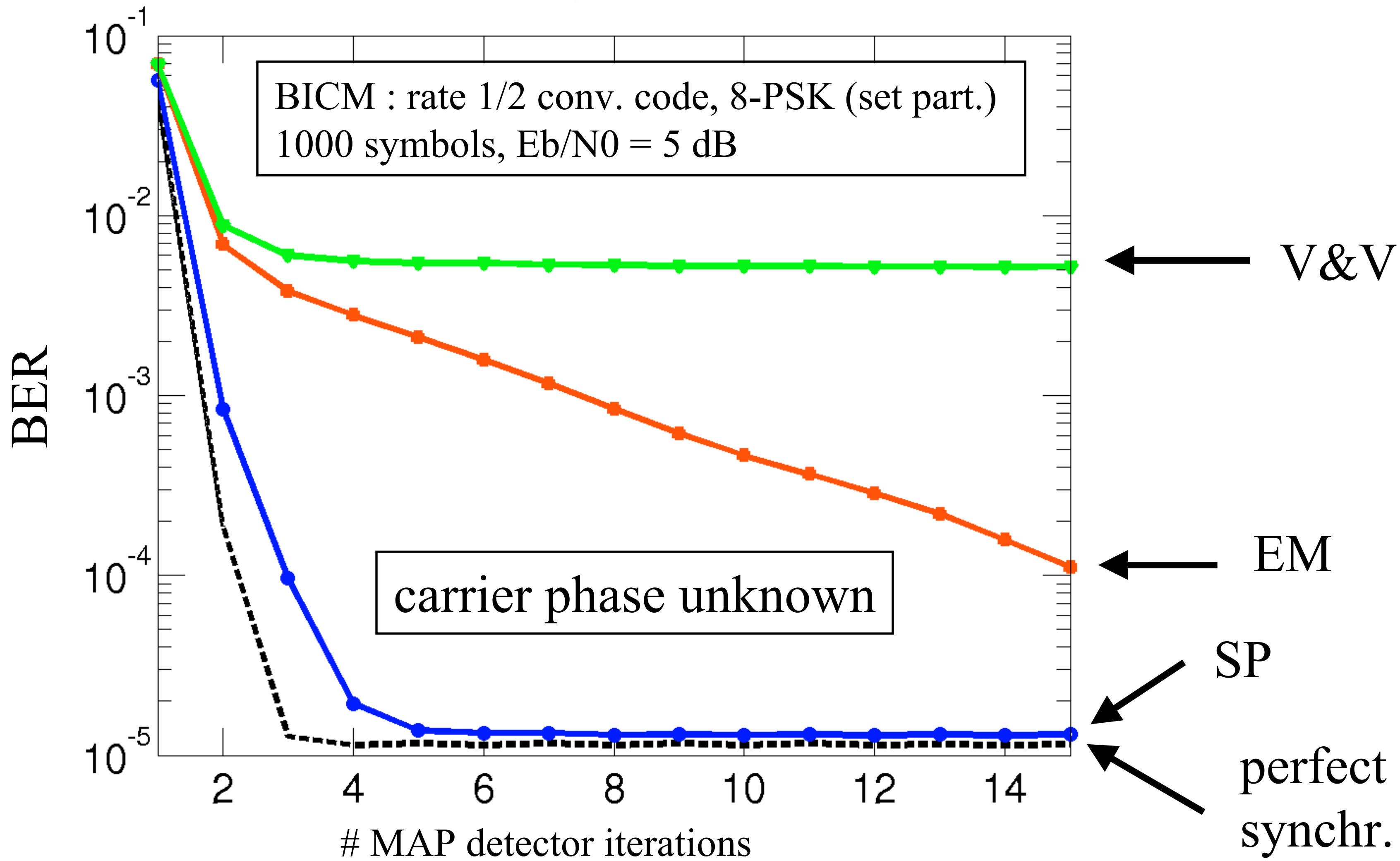
Simplified sum-product algorithm : BER performance



Conclusions

Two alternatives for MAP detection when θ is unknown

- Iterative ML estimation of θ by means of EM algoritm

symbol APP during i-th EM iteration

$$\propto g(z_k, a_k, \hat{\theta}^{(i)}) p_e^{(i,1)}(a_k)$$

one MAP detector iteration
for each EM iteration

- Simplified SP algorithm (combined with EM algorithm)

symbol APP during i-th EM iteration of n-th MAP decoder iteration

$$\propto g(z_k, a_k, \hat{\theta}^{(n,i)}) p_e^{(n)}(a_k)$$

EM algorithm iterated till convergence
for each MAP detector iteration

Both algorithms yield similar MSE and BER after convergence, but simplified SP algorithm requires considerably less MAP decoder iterations to converge.